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Queuing Modelling of Air Transport Passengers of Nnamdi Azikiwe International Airport Abuja, Nigeria Using Multi Server Approach

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Abstract: The Multi Server approach of modeling was adopted in this study to develop a mathematical model to solve problem of queuing of air transport passengers at the Nnamdi Azikiwe International Airport (NAIA), Abuja, Nigeria's capital city. The airport just like similar facilities in the aviation industry of the country faces problems of many passengers queuing for boarding, departure with different arrival rate due to non availability of state of the art logistics management mechanisms for predicting the nature and service demands of travelers. A mathematical queuing model was developed in this study and simulated using data obtained from four different air transport companies that consisted of two domestic carriers (Arik Airline, Aero Contractor Limited) and two international carriers (British Airways and Ethiopian Airline). Result showed that there was shortage in total number of aircrafts available to effectively serve the monthly average of 21,863 domestic and international passengers at the airport. The system required modeling service factor of 0.5 (utilization factor of 0.4, 0.6 and 0.9) at 5% significance level. The simulation showed that in order to meet daily need of passengers at NAIA Abuja based on carriage capacity of available aircrafts; each international airline required one additional aircraft. Each domestic airline required five additional planes in her daily feet.

Key words: Multi server · Queuing modeling · Airport · Passengers · Utilization factor

INTRODUCTION

Preamble: Queuing which always takes place in the form of lining up, is rarely anyone's favourite activity. It is a cornerstone for measurement of efficiency and organizational capability for most service industries particularly in the airline industry. At any given moment, there may be more people or cases needing service, help or attention the industry can handle. This feature has become a regular and recurrent problem at Nnamdi Azikiwe International Airport (NAIA) which is the main and only airport that services Nigerian capital city, Abuja with both domestic and international terminal. As Abuja houses the seat of Government of Federal Republic of Nigeria, there is always a massive traffic of people commuting in and out of Abuja for multitude of purposes that include official, commercial, business, personal and all other purposes that necessitates arrival people of all shades humans into the city.

As a result of bad governance and monumental corruption in most third world countries like Nigeria (Masood, Khan and Naqvi, 2011) [1], road congestion and safety factors have contributed to better responses of air transport systems. Due to bad road network, bad driving attitudes, inadequate driver training and predominance of non-motorised traffic and pedestrian control and complete absence of transport waterways, those who can afford the fares prefer air transport. Air transport through the NAIA therefore becomes the quickest and safest means of movement of people to and out of Abuja to make their business transactions, ease journeys and meet up with promised deadlines. Continuous growth in population, urbanization and increased human activities resulting from modernization with increased standard of living (Odufuwa, 2008) [2] made commercial air transportation to witness some substantial developments in recent years. Consequently, there has been a substantial increase in number of airline operators in the industry (Ogwude, 1986)

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[3] that grew Nigerian air service industry from one airline before 1983 to three in 1988 and to fifteen by 2010 (Ukpere, Stephens, Ikeogu, Ibe and Akpan, 2012. Challenges of this continuous growth demand for improved logistics management mechanisms; the near absence of which resulted into passengers queuing for air transport system services that are limited in supply. Exorbitant flight fares brought by increased aviation fuel and maintenance costs Bofinger (2009) [4] have not dampened the zeal of air travel passengers.

The problems of passengers queuing for boarding, departure with different arrival rates is not only at the Nnamdi Azikiwe International Airport Abuja but also at all national and state government owned airport facilities in the country due to inadequate human and cargo traffic management technology. Thus queuing in NAIA, Abuja has become very complex to solve manually due to the patterns and irregularities in arrivals and departures or service processes. The airports have less capacity to serve all arrivals and departures promptly; resulting into rowdiness and randomness that result to some waiting line. According to Mehri, Djemel and Kammoun (2009), there are three basic components of a queuing process which are arrivals, service facilities and actual waiting lines. Queuing will be eliminated if irregularities can be solved without increasing overall service capacity or diminishing overall flow of arriving passengers. Waiting is therefore a consequence of irregularity in Nigeria's airport service for large population of travelers. Specifically, NAIA, Abuja faces problems of waiting line or queuing in its system in aspects like cargo handling, ticket clearance, departures and arrival rates. As a result of this cumbersome service most travelers that pass through the system are stressed up and uncontrollable. The situation is worse during airlifting of religious pilgrims to holy land when travelers sleep in airports awaiting flights.

Accurate prediction of various numbers of passengers in the system is necessary for minimizing the waiting line called queuing. In this way, the extent of delay of passengers waiting for departure will be minimized and therefore industry service will improve for higher efficiency and productivity. As part of a solution to this complex problem that frustrates travelers on arrivals through the airport, Ademoh and Anosike (2014) [5] used the Dearth and Birth Rate approach to model to model the waiting line at the airport and reported that more aircrafts are needed on daily basis both on the domestic and international routes for improved services. This study is a continuation of this past work aimed at developing a queuing model using alternative approach of the Multi-Server method to facilitate the prediction, processing of travelers and total aircraft needs at the NAIA per operational period for effectiveness. The main objectives of the work include; prediction of arrival rate of passengers into the system, prediction of departure rate of passengers from the system, prediction of overall expected number of passenger into the system, determination of system's service level of performance and development of queuing model using the Multi-Server approach so as to harmonize passengers' arrival and departure rates with capacity of handling facilities of the airport bearing in mind the expected passengers population in a system at any given time for improved service. The significance of the model when successfully developed is that it will reduce airline operators' inefficiency in the NAIA, Abuja. It will give Nigeria a better image as humility and efficiency of services at gateway airport to a country's capital gives firsthand impression of her integrity. The stressful experiences of travelers through the airport would be eliminated and lead to increased patronages of travelers. Thus there would be improved economic activities and financial earnings to stakeholders of the facility.

Model Development Procedure: In the study, a quantitative approach was adopted. Therefore existing passengers' data in the system were collected for analysis. The source of information and data for the study was mostly through questionnaire on sampled groups, companies and passengers. The collected data were analysed using queueing modelling for determining the waiting lines of the passengers. An appropriate queueing model was developed using the multiple server modelling approach based on data and information available from airline operators to handle the peculiar situation at NAIA, Abuja.

Definition of Terminologies: The key terminologies used in the work are hereby defined as related to their use.

MS	=	Multi Server modelling
NAIA, Abuja	=	Nnamdi Azikiwe International Airport (NAIA), Abuja.
Queue length	=	The total number of passengers in the air system.

Arrival rate	=	Number of passengers coming into NAIA for airline service per time.
Departure rate	=	Number of passengers being served per unit time.
Queuing Modeling	=	Analysis of arrival and departure rate of passengers in the system.
FCFS	=	(First come first served) the servicing rate of the system in accordance to arriving time.

Waiting Line in Air Transport System: Airports are nowadays multimodal, multi-service platforms, with intense non-aeronautical activities that cover several different industrial, economical and social aspects. Taxi services are a fundamental piece of the transportation diversity that airport requires, in order to become attractive and efficient. This transportation service gains special relevance when coupled to existing high-demand nodes like hospitals, monuments, shopping areas, hotels, or airports. Its "service profile" is highly compatible to the traditionally higher willingness to pay of passengers with trip urgency and high comfort needs or economic power, such as hospital patients, shoppers, businessmen or tourists. A queue is a waiting line (like customers waiting at a supermarket checkout counter); queuing theory is the mathematical theory of waiting lines. More generally, queuing theory is concerned with the mathematical modeling and analysis of systems that provide service to random demands especially in airport transport system. A queuing model is an abstract description of such a system. Typically, a queuing model represents the following:

- The system's physical configuration, by specifying the number and arrangement of the servers, which provide service to the customers.
- The stochastic (that is, probabilistic or statistical) nature of demands, by specifying the variability in arrival process and in service process.

According to Subramanian (2007), three performance metrics for National Airspace System (NAS) could be modelled depending on aggregate econometric models for flight delays, flight cancellation probabilities and passenger delays especially in the United States. Flight delays can be attributed to queuing effects within the air transportation network. As delays in air transportation system worsen, more and more people switch to another mode of transportation. The steady rise in demand for air transportation has demonstrated need for improved air traffic flow management. One of the metrics that has been used to assess the performance of NAS is the actual aggregate delay. Flight delays, in many cases, are caused by application of Traffic Flow Management (TFM) initiatives in response to weather conditions and excessive traffic volume. TFM initiatives such as ground stops, ground delay programs, rerouting, airborne holding and miles-in-trail restrictions, are actions that are needed to control the air traffic demand to mitigate the demand-capacity imbalances due to the reduction in capacity. Consequently, TFM initiatives result in NAS delays. Of all the causes, weather has been identified as the most important causative factor for NAS delays.

Therefore, to guide flow control decisions during day of operations and for post operations analysis, it is useful to create a baseline for NAS performance and establish a model that characterizes the relation between weather and NAS delays. Hence given the demand and expected weather, the model can be used to predict the expected aggregate delay. Flight cancellation probability is defined as the probability that a flight scheduled will be cancelled. Airlines usually cancel flights scheduling when they experience non-availability problems related to crew, maintenance and security personnel, Air Traffic Control (ATC) problems like runway breakdowns etc and weather related problems that reduce airport capacity. Flight delayed or cancelled adversely affect passengers. Loss of productivity (or Passenger Time Value) represents valuation of the loss of passenger time value contributed to Nigeria economy due to bad quality of service. Passenger delay is the actual waiting that passengers experience by disruption in aviation activities, including both flight delay and cancellations.

Delay and cancellation are essentially the same from the passenger perspective. They both impose delays to travel time. These are very common features in Nigerian airline industry of where weather changes, security problems, frequent aircraft breakdowns due to old age and other problems cause incessant flight delays and cancellations. Generally, cancellations generate extremely high passenger delays. In order to estimate passenger delay, transformations must be applied to convert the number of cancellations into delay of relocated passengers on the cancelled flights. Thus the total passenger delay includes not only delays obtained from delayed flights but also delays induced by cancellations (Subramanian, 2007) [6]. According to Salvendy (2001), [7]queuing theory was first known in early 1900s with the work of A. K. Erlang of Copenhagen Telephone Company, who derived several important formulae for teletraffic engineering that today, bear his name. The range of applications has grown to include manufacturing, air traffic control, military logistics, design of theme parks and many other areas that involve service systems whose demands are random.

MATERIALS AND METHODS

Materials: Queue data (material) is needed for modeling air transport system for passengers of (NAIA) Abuja. Data of waiting line for the past ten months of the system were collected because it was available data at the time of study and they were categorized based on the following:

- Case subsystem
- Size of calling population; the passengers
- System capacity

Case Subsystem: The case study is Nnamdi Azikiwe International Airport, Abuja. The subsystem involved for modelling NAIA system were the following four airline transport companies:

- Arik Airline Limited (AA)
- British Airways (BA)
- Ethiopian Airline (EA)
- Aero Contractors (AC)

Both Arik Airline Ltd and Aero Contactors are local air service companies while British Airways and Ethiopian Airline are international air service companies. The local and international companies were selected for comparative analysis and performance purpose. They are also the major airlines in terms of passenger airlifting in NAIA, Abuja.

Size of Calling Population: The size of calling population is infinite because of arrival pattern from large passenger population.

System Capacity: The system capacity was based on the total number of waiting room or passengers and server (number of airplanes).

Table 2.1: Queuing system characteristics

Characteristics	Description
Arrival process	Exponential distribution
Service process	Parallel service for single queue
Number of channels	Multi-channel
System capacity	Infinite
Queue discipline	First come first served (FCFS)

System Characteristics: The system characteristics to be considered are the following:

Arrival process	=	The entry procedure
Service process	=	The system operational procedure
c)Number of channels	=	The systematic way of solving the problem
Queue discipline	=	The pattern for solving waiting line problem

Single Queue with Parallel Servers (Sqps): This is the type of model which deals with the study of a single queue in equilibrium. There is more than one server and each server provides the same type of service or it is to provide identical parallel service. The customers (passengers) wait in a single queue until one of the service channels is ready to take them in for servicing at the rate of one customer at a time per server. Each of these characteristics is described in Table 2.1.

Methods: The following methods are employed in the study. It was predicted based on performance of service level. For systems processing discrete jobs or customers like airports (Salvendy, 2001; Viswanadhan and Narahari, 1992 [7]; Yin and Zang, 1996) categorized and treated different modeling approaches that are suitable for use as adoptable for this work. Modeling was based on stochastic/probabilistic process (general and exponential distribution) [8-13].

- Multi-Server (MS) model development approach.
- Chi-square distribution assumption.

Study Assumption: Queues represent the state of a system such as number of people inside an airport terminal (Trani, 2011). Considering multiple servers with infinite calling population, they based on references to previous related work. The mathematical models for analysing waiting lines have the following assumptions as adopted from Mehri, Djemel and Kammoun (2009).

- Arrivals come from an infinite or very large population.
- Arrivals are Poisson distributed.
- Arrivals are treated on a first in first out (FIFO) basis and do not balk or renege.

The arrival of most queuing models assumes that an arriving passenger is a patient traveller. Patient customer is people or machines that wait in queue until they are served and do not switch between lines. Unfortunately, life and quantitative analysis are complicated by the fact that people have been known to balk or renege. Balking refers to passengers who refuse to join the waiting lines because it is not suitable to their needs or interests. Reneging passengers are those who enter the queue but then become impatient and leave; hence the need for queuing theory and waiting lines analysis.

- Service times follow the negative exponential distribution or are constant
- The average service rate is faster than the average arrival rate

Performance Characteristics of Queuing Systems: Using Little's law, performance of queue system as adopted by Blumenfeld, (2001) can be assessed as follows:

$$L_q = \lambda W_q \quad (2.1)$$
$$L = \lambda W \quad (2.2)$$
$$L = L_q + \lambda/\mu \quad (2.3)$$
$$L = W_q + 1/\mu \quad (2.4)$$
$$L = \sum_{n=0}^{\infty} nP_n \quad (2.5)$$
$$L_q = \sum_{n=s}^{\infty}(n-s)P_n \quad (2.6)$$
$$\rho = \frac{\lambda}{s\mu} \qquad (\rho < 1) \quad (2.7)$$

Equations (2.1) and (2.2) are the mathematical representation of Little's law.

where:

n = Number of passengers in the system
$P_n(t)$ = Probability of exactly (n) passenger in queueing system at time (t)
L_q = Average queue length (average number of passengers in queue)
L = Average system length (average number of passengers in system, including those being served)
W_q = Average waiting time in queue (average time a passenger spends in a queue)
W = Average time in system (average time a passenger spends in queue plus service)
$N(t)$ = Total number of passengers in the system at a particular time
T = Time that a passenger spends in the system
s = Number of servers
λ = Arrival rate (number of passengers arriving per unit time)
$1/\lambda$ = Mean interarrival time
μ = Service rate per unit server (number of passenger served per unit time)
$1/\mu$ = Mean service time
ρ = Traffic intensity

Modelling Based on Multi-server with Infinite Source: Use of following notations is adopted:

- M/M/s/Y/Z and this is a Kendall's Notation and in generalized form is given by:
- A/B/c/K
- A describes the interarrival time distribution
- B is the service time distribution
- c is the number of server
- K is the size of the system capacity (including the node or server)
- Symbols traditionally used for A and B is as follows:
- M for exponential distribution (M stands for Markov)
- D for deterministic distribution
- G for general distribution
- In the study the Kendal notation was adopted as follows.
- A/B/s/Y/Z

where:

A = Stands for arrival distribution
B = Stands for service pattern distribution
s = Stands for number of servers
Y = stands for system capacity
Z = Stands for queueing discipline

In this case considered, $s > 1$

Thus: $\lambda_n = \lambda$ For n = 0,1,2...s

$$\mu_n = \begin{cases} n\mu & \text{for } n = 0,1,2,\ldots s \\ s\mu & \text{for } n = s, s+1, \ldots\ldots \end{cases}$$

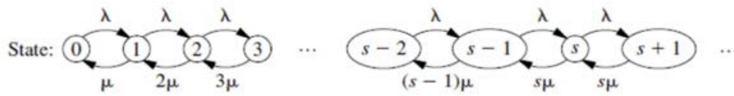

Fig. 2.1: Shows the modelling pattern of multi-service and this is adopted in the study.

Figure 2.1 shows the modelling pattern of multi-service and this is adopted in the study.

The pattern of the system with *n* passenger is given by steady state balancing equation.

Mean entering rate of passenger = mean leaving rate of passenger

$$\mu_1 P_1 = \lambda_0 P_0 \quad (2.8)$$

$$\mu_{n-i}P_{n-i} + \mu_{n+i}P_{n+i} = \lambda_n P_n + \mu_n P_n \quad (2.9)$$

$$\mu_{n-i}P_{n-i} + \mu_{n+i}P_{n+i} = (\lambda_n + \mu_n)P_n$$

$$\mu_{n+i}P_{n+i} = (\lambda_n + \mu_n)P_n - \lambda_{n-i}P_{n-i}$$

$$P_{n+1} = \frac{(\lambda_n + \mu_n)P_n}{\mu_{n+1}} - \frac{\lambda_{n-1}P_{n-1}}{\mu_{n+1}}$$

$$P_{n+1} = \frac{\lambda_n P_n}{\mu_{n+1}} + \frac{\mu_n P_n}{\mu_{n+1}} - \frac{\lambda_{n-1}P_{n-1}}{\mu_{n+1}}$$

$$P_{n+1} = \frac{\lambda_n P_n}{\mu_{n+1}} + \frac{1}{\mu_{n+1}}(\mu_n P_n - \lambda_{n-1}P_{n-1})$$

If $\frac{1}{\mu_{n+1}}(\mu_n P_n - \lambda_{n-1}P_{n-1}) \to 0$

$$P_{n+1} \cong \frac{\lambda_n P_n}{\mu_{n+1}} \quad (2.10)$$

From equation (2.3), if $n = 0$, we have: $\lambda_{n-z}...\lambda_0$

$$P_1 = \frac{\lambda_0 P_0}{\mu_1} \quad (2.11)$$

Let $C_n = \frac{\lambda_{n-1}\lambda_{n-2} \cdots \lambda_0}{\mu_n \mu_{n-1} \cdots \mu_1}$ for n = 1,2,...

If $n = 1$

$$C_1 = \frac{\lambda_0}{\mu_1} \quad (2.12)$$

Therefore equation (2.11) becomes:

$$P_1 = C_1 P_0 \quad (2.13)$$

Thus, the steady-state probability would be given by (Hillier and Liebermann, 2001):

$$P_n = C_n P_0 \quad (2.14)$$

$$\sum_{i=0}^{n} P_n = 1 \quad (2.15a)$$

$$P_0 + \sum_{i=i}^{n} P_n = 1 \quad (2.15b)$$

$$\Rightarrow P_0 + \left(\sum_{n=0}^{\infty} C_n\right)P_0 = 1 \quad (2.16)$$

$$P_0 = \frac{1}{1 + \left(\sum_{n=0}^{\infty} C_n\right)} \quad (2.17)$$

$$\bar{\lambda} = \sum_{n=0}^{\infty} \lambda_n P_n \quad (2.18a)$$

Therefore, L and L_q are given by the equation below (Trani, 2011):

$$L = \sum_{n=s}^{\infty} n P_n \quad (2.18b)$$

$$L_q = \sum_{n=s}^{\infty} (n - s)P_n \quad (2.18c)$$

where:

$\bar{\lambda}$ = Average arrival rate over the long run

C_n = Steady service rate

The steady service rate in multi-service is given by the following equations:

$$C_n = \begin{cases} \dfrac{\left(\frac{\lambda}{\mu}\right)^n}{n!} & \text{for } n = 0,1,2,\ldots s \\ \dfrac{(\lambda/\mu)^s}{s!}\dfrac{(\lambda)^{n-s}}{s\mu} = \dfrac{(\lambda)^n}{s!s^{n-s}} & \text{for } n = s, s+1, \ldots.. \end{cases} \quad (2.19)$$

Consequently, if

$$\lambda < s\mu; \ \therefore \rho = \frac{\lambda}{s\mu} \quad (utilisation\ factor) \quad (\rho < 1)$$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} \sum_{n=1}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^{n-s}}{s\mu!}}$$

If n=0 term in the last simulation yields the correct value

of 1, n! = 1

$$P_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} \cdot \left(\frac{1}{1-\lambda/s\mu}\right)} \quad (2.20)$$

Thus, substituting equation (2.19) into equation (2.14) gives idle probability of the system:

$$P_n = \begin{cases} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} P_0 & \text{if } 0 \leq n \leq s \\ \frac{(\lambda)^n}{s! s^{n-s}} P_0 & \text{if } n \geq s \end{cases} \quad (2.21)$$

P_n is probability of n entity in the system. The following equations were adopted from (Trani, 2011).

$$L = \rho P_0 \frac{(\lambda/\mu)^s}{s!(1-\rho)^2} + \lambda/\mu \quad (2.22)$$

$$L_q = \rho P_0 \frac{(\lambda/\mu)^s}{s!(1-\rho)^2} \quad (2.23)$$

$$W_q = \frac{L_q}{\lambda} \quad (2.24)$$

$$W = \frac{L}{\lambda} = W_q + 1/\lambda \quad (2.25)$$

Equations (2.23 and 2.24) could be stated in approximate form as follows (Blumenfeld, 2001):

$$L_q = \frac{\rho^{\sqrt{2(s+1)}}}{1-\rho} \quad (2.26)$$

$$W_q = \frac{\rho^{\sqrt{2(m+1)-1}}}{\mu(1-\rho)} \quad (2.27)$$

However, probability distribution of waiting time is given by Hillier and Liebermann, (2001) as:

$$P(W > t) = e^{-\mu t}\left[1 + \frac{P_0\left(\frac{\lambda}{\mu}\right)^s}{s!(1-\rho)}\left(\frac{1-e^{-\mu t\left(s-1-\frac{\lambda}{\mu}\right)}}{s-1-\frac{\lambda}{\mu}}\right)\right] \quad (2.28a)$$

If, $s - 1 - \frac{\lambda}{\mu} = 0$, then we have:

$$P(W > t) = e^{-\mu t}\left[1 + \frac{P_0\left(\frac{\lambda}{\mu}\right)^s}{s!(1-\rho)}(\mu t)\right] \quad (2.28b)$$

The equations 2.19 - 2.28 were adopted equation for the modelling of MS approach in which arrival and departure rate of passengers with system service's level of performance are determined. The MS approach uses multi-channel or task in solving the problem at hand.

Statistical Testing of the Modeling: The arrival and departure assumes Poisson distribution as follows (Asmussen, 1987):

$$P(s) = \frac{e^{-\mu}\mu^n}{n!} \quad (2.29)$$

where:

$P(s)$ = Probability of sample
μ = Mean value
N = Expected value

The statistical testing is conducted on the modelling using Chi-square distributional assumption as given in equation (2.30):

$T > \chi^2_{\alpha,k-p-1} = T > \chi^2_{\alpha,2}$ Reject, otherwise accept it. (2.30)

$E(s) = P(s)N$ And this is expected sampling data (2.31)

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} \quad (2.32)$$

Where:

T = Statistical testing
k = Number of rows
p = Number of columns
N = Total number of data
χ^2 = chi-square distribution
O_i = Observed frequency
E_i = Expected frequency

RESULTS AND DISCUSSION

Presentation of Results: Simulation of the mathematical model developed for solving passenger queuing problems at NAIA, Abuja, Nigeria, using Multi-Server approach adopted flight passenger historical data obtained from four major air transport companies using the facility that included British Airways, Ethiopian Airline (international routes), Arik Airline Ltd and Aero Contractors (domestic routes). Table 3.1 presents data collected on passengers for year 2013 from British Airways for Abuja to London route. It had expected average number of 188passengers per day. Table 3.2 is data for year 2013 from Ethiopian Airline for Abuja to Addis Ababa with expected average number of 120passengers per day [9].

Table 3.3 is that on passengers for year 2013 from Arik Airline for Abuja to Lagos route with daily expected number of passengers at average of 1573. Table 3.4 shows data collected on passengers for the year 2013 from Aero Contractor with expected number of passengers per day at an average of 1034. Table 3.5 presents the average of passengers in the system for the year 2013 [10].

Table 3.1: British airways passengers in 2013

Month	Passengers	Passengers/day
January	5174	172
February	4484	149
March	4633	154
April	6125	204
May	5593	186
June	6266	209
July	6441	215
October	5719	191
November	5665	189
December	6320	211
Average	5620	188

Table 3.2: Ethiopian Airline Passengers in 2013

Month	Passengers	Passengers/day
January	2170	72
February	2165	72
March	2872	96
April	3644	121
May	3495	117
June	4085	136
July	3910	130
October	4016	134
November	3815	127
December	5966	199
Average	3614	120

Table 3.3: Arik Airline Ltd Passengers in 2013

Month	Passengers	Passengers/day
January	25015	834
February	39292	1310
March	42498	1417
April	42739	1425
May	42876	1429
June	44307	1477
July	53332	1778
October	61334	2045
November	61467	2049
December	59027	1968
Average	47189	1573

Table 3.4: Aero Contractor Passengers in 2013

Month	Passengers	Passengers/day
January	18821	627
February	25919	864
March	27753	925
April	28941	965
May	34994	1166
June	31779	1059
July	34093	1136
October	36074	1202
November	37017	1234
December	34894	1163
Average	31029	1034

Table 3.5: Average Number of Passenger in the System in 2013

Airline Company	Passengers	Passengers/day	Arrival/Hour
British Airways	5620	188	8
Ethiopian Airline	3614	120	5
Arik Airline	47189	1573	66
Aero Contractor	31029	1034	43
Average	21863	729	31

Simulation of Queuing Modelling: The Matlab Graphical User Interface (GUI) program that was developed for the study is as shown in Figure 3.1. Installation of the software was done using the following procedure:

- Installation of MATLAB 2009 version or above into personal computer system
- Loading the file called queue.fig onto screen
- Loading the basic parameter for analysis
- Pressing of calculate button to observe the simulated result
- Pressing of quit button after being satisfied with the result

GUI's calculate button displays the result of the Multi-service mathematical modeling program as developed in the previous section result on pressing the submission button [11].

Required Basic Parameters: The required basic parameters for this study are as given in Table 4.6 which gives values of the boundary conditions needed to make the model work.

Arrival

Departure Rate of Passengers with Service Factor of 0.5: The arrival rate and departure rate of passengers were modelled using the MS modelling.

Use of Ms Model: Arrival rate of passengers into NAIA, Abuja is modelled using Multi-Server (MS) Model at service factor of 0.5 per month to obtain the values shown in Table 3.7.

Statistical Testing of Arrival and Departure Rate at 0.5 Service Factor: It is observed that arrival/departure rate of the MS model is the same except for server capacity in which the MS model is 243 at all utilisation factors as shown in Tables 3.7 and 3.8 respectively. Table 3.9 is the basic requirement for testing the model at service factor 0.5. As compared with the work of Ademoh and Anosike (2014) it is observed that using Birth and Death Rate (BDR) model requires more servers (aircraft) in

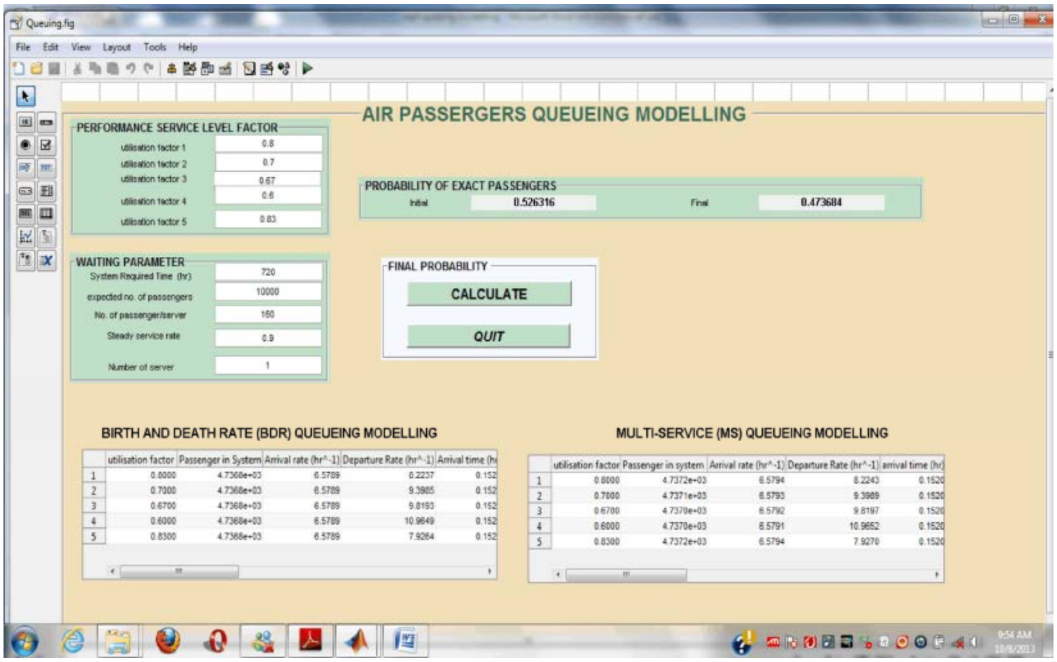

Fig. 3.1: Graphical user interface for modelling NAIA system

Table 3.6: Boundary Condition

Parameters	Quantity
System utilisation factor	0.2, 0.25, 0.4, 0.6, 0.9
Time	30days
Estimated Passengers/month	21863
Estimated Passengers/day	729
Server (Aircraft)	2
Server capacity	160
Service factor	0.5, 0.9

Table 3.7: Arrival and Departure Rate Using MS Model at 0.5 Service Factor

Utilisation Factor	0.2	0.25	0.4	0.6	0.9
Arrival rate/hr	10	10	10	10	10
Departure rate/hr	25	20	13	8	6
Arrival time (min)	5.93	5.93	5.93	5.93	5.93
Service time/passenger (min)	2.37	2.96	4.74	7.12	10.67
Server capacity	243	243	243	243	243

Table 3.9: Requirement of Service Factor 0.5

Utilisation factor	Arrival rate/hr	Departure rate/hr	P(Arrival)	P(Departure)
0.2	10	25	0.125	0.00241
0.25	10	20	0.125	0.0286
0.4	10	13	0.125	0.106
0.6	10	8	0.125	0.0304
0.9	10	6	0.125	0.0087
Average	10	14		

comparison to this work with the MS model at the specified period of time. For instance, in modelling of 160 passengers per server, it was observed that BDR required 3 servers while the MS requires 2 servers. BDR model gave classical improvement of service level because twice the number of passengers of MS model would be served at specified period of time. The requirement of the model at service factor 0.5 was analysed as presented in Table 3.9. Expected arrival/departure at service factor of 0.5 is presented in Table 3.10 [12].

The arrival rate,

$$\chi^2 = \frac{(10-6)^2}{6} + \frac{(10-6)^2}{6} + \frac{(10-6)^2}{6} + \frac{(10-6)^2}{6} + \frac{(10-6)^2}{6} = 13$$

The departure rate,

$$\chi^2 = \frac{(20-2)^2}{2} + \frac{(13-8)^2}{8} + \frac{(8-2)^2}{2} = 183$$

The null hypothesis criterion was used to analyse the model result at service factor of 0.5. The result is presented in Table 3.11. Result in table 3.11 shows that both the arrival and departure rate should be within the range of 0-13 passengers per hour from tested criterion, otherwise it will be rejected.

Table 3.10: Expected arrival and departure at Service Factor 0.5

Utilisation factor	Arrival rate/hr	Departure rate/hr	E(Arrival)	E(Departure)
0.2	10	25	6	0
0.25	10	20	6	2
0.4	10	13	6	8
0.6	10	8	6	2
0.9	10	6	6	0
Average	10	14		

Table 3.11: Criterion of Null Hypothesis at Service Factor 0.5

Criteria (α)	0.05	0.025	0.01	0.005
Significance level (α)	5%	2.5%	1%	0.5%
Confidence interval (1- α)	0.950	0.975	0.990	0.995
χ^2-Test (arrival)	13	13	13	13
χ^2-Test (departure)	183	183	183	183
χ^2 α_3(chi- from Table)	7.81	9.35	11.34	12.84
Null Hypothesis (arrival)	Reject	Reject	Reject	Accept
Null Hypothesis (departure)	Reject	Reject	Reject	Reject

Table 4.12: Arrival and Departure Rate Using MS Model at 0.9 Service Factor

Utilisation Factor	0.2	0.25	0.4	0.6	0.9
Arrival rate/hr	14	14	14	14	14
Departure rate/hr	36	29	18	12	8
Arrival time (min)	4.17	4.17	4.17	4.17	4.17
Service time/passenger (min)	1.67	2.09	3.34	5.00	7.51
Server capacity	345	345	345	345	345

Table 3.13: Requirement of Service Factor 0.9

Utilisation factor	Arrival rate/hr	Departure rate/hr	P(Arrival)	P(Departure)
0.2	14	36	0.095	0.00081
0.25	14	29	0.095	0.02
0.4	14	18	0.095	0.075
0.6	14	12	0.095	0.012
0.9	14	8	0.095	0.00071
Average	14	21		

Table 3.14: Expected arrival and departure at Service Factor 0.9

Utilisation factor	Arrival\` rate/hr	Departure rate/hr	E(Arrival)	E(Departure)
0.2	14	36	7	0
0.25	14	29	7	2
0.4	14	18	7	8
0.6	14	12	7	1
0.9	14	8	7	0
Average	14	21		

Table 3.15: Criterion of Null Hypothesis at Service Factor 0.9

Criteria (α)	0.05	0.025	0.01	0.005
Significance level (α)	5%	2.5%	1%	0.5%
Confidence interval (1- α)	0.950	0.975	0.990	0.995
χ^2 -Test (arrival)	35	35	35	35
χ^2-Test (departure)	498	498	498	498
χ^2 α_3(chi- from Table)	7.81	9.35	11.34	12.84
Null Hypothesis (arrival)	Reject	Reject	Reject	Reject
Null Hypothesis (departure)	Reject	Reject	Reject	Reject

Use of Ms Model: The modelling result of arrival rate of passengers in the NAIA, Abuja system by Multi-Server (MS) Model at service factor of 0.9 per month is shown in Table 3.12.

Statistical Testing of Arrival and Departure Rate at 0.9 Service Factor: The model result at service factor of 0.9 are the same except for server capacity in which the MS model was 345 at all utilisation factors as shown in Tables 3.11 and 3.12 respectively. Table 3.13 is the basic requirement for testing the model at service factor 0.9. Modelling of 160 passengers per server on the MS model required 2 servers. Again BDR model (Ademoh and Anosike, 2014) gave classical improvement of service level because double (4servers) of the passenger of MS model would be served at specified period of time. Expected arrival/departure at service factor of 0.9 is as in Table 3.14.

The arrival rate,

$$\chi^2 = \frac{(14-7)^2}{7} + \frac{(14-7)^2}{7} + \frac{(14-7)^2}{7} + \frac{(14-7)^2}{7} + \frac{(14-7)^2}{7} = 35$$

The departure rate, $\chi^2 = \frac{(29-2)^2}{2} + \frac{(18-8)^2}{8} + \frac{(12-1)^2}{1} = 498$

Result in Table 3.15 shows that both arrival and departure rate should be within the range of 0-13 passengers per hour even at service factor of 0.9 after testing the modelling using chi-distributional assumption. Considering the result stated in Table 3.5, only the two international airlines were able to meet this standard. Arriving passengers of the two local airlines do not satisfy this condition because there more travellers within nation and over-utilisation of server would experience. For better service of the system average arrival rate of 31 passengers per hour would be rejected if the domestic airlines are to be using only 2 servers (aircrafts) for their services.

Graphs of Arrival and Departure Rate: The effects of the choice of service factor for the models were plotted in graphical forms to show their significance. Figures 3.2 and 3.3 show the effect of service factor on both arrival and departure rate respectively based on expected passengers in the NAIA system.

In Figure 3.2, maximum server capacity per day was 312 passengers MS Model. The service factor 0.5 had maximum arrival rate of passenger of 240 per day and service factor 0.9 had 312 passengers on arrival per day. In Figure 3.3, the departing passenger was estimated with different service factors in which using service factor of

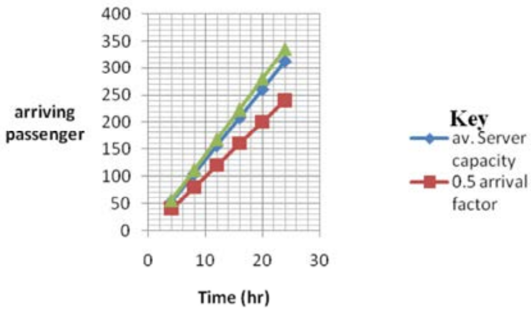

Fig. 3.2: Effect of service factor on arrival rate

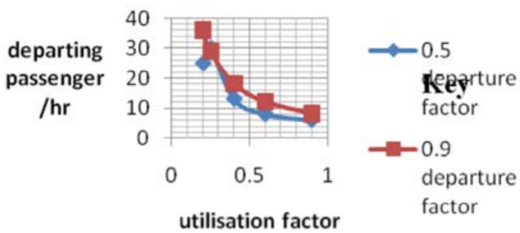

Fig. 3.3: Effect of Utilisation Factor on Departure Rate

0.5 with utilisation factor of 0.2 had maximum passenger leaving the system with 25 per hour. Using service factor of 0.9 Model, number of passengers increases to 36 per hour with utilisation factor 0.2. In Tables 3.9 and 2.13 it was observed that the average departing per hour was 10 passengers on using service factor 0.5 and 21 passengers with service factor 0.9.

Expected of Passengers in System: Table 3.16 shows expected passengers into aviation system. Result is based on arrival and departure rate in section 3using Multi-Server (MS) Model at a service factor of 0.5 per month. Results were estimated between initial probability of 0.5263 and final probability of 0.4737f expected monthly passengers of 21863 from initial evaluation.

Performance of Service Level of the System: The passengers entering and leaving the system was analysed based on aforementioned model and the system was tested for based on performance of the service time as presented in Tables 3.17 - 3.20.

Required Server per Day: The model was based on 2 servers per day. The result was estimated based on arrival/departure rate of 13 passengers as in Tables 3.10 (0.5 service factor) and 3.15 (0.9 service factor) and with this possibility a server has capacity of 156 passengers. The modelling result of 156 passengers was adopted for presenting the require server in Table 3.20 presents

Table 3.16: Expected Passengers Using MS Model at 0.9 Service Factor

Utilisation Factor	0.2	0.25	0.4	0.6	0.9
Capacity/day	320	320	320	320	320
Expected capacity/day	345	345	345	345	345
Reserved passengers	-25	-25	-25	-25	-25
Waited Passenger/month	10355	10355	10355	10355	10355
Expected passenger/month	10356	10356	10356	10356	10357

Table 3.17: Service Level's Performance using MS Model at 0.5 Factor

Utilisation Factor	0.2	0.25	0.4	0.6	0.9
Service time/trip (hr)	8.90	11.12	17.80	26.69	40.04
Delay time/trip (hr)	15.10	12.87	6.20	-2.70	-16.04
Total service time/trip (hr)	24	24	24	24	24
Percent delay (%)	62.92	53.62	25.8	0	0
Capacity/trip	320	320	320	320	320
Waited Passenger/month	10355	10355	10355	10355	10355
Completion of Service in days	22.77	22.77	22.77	22.77	22.77

Table 3.18: Service Level's Performance using MS Model at 0.9 Factor

Utilisation Factor	0.2	0.25	0.4	0.6	0.9
Service time/trip (hr)	8.90	11.12	11.12	11.12	11.12
Delay time/trip (hr)	15.10	12.88	6.20	-2.70	-16.05
Total service time/trip (hr)	24	24	17.32	8.42	-4.93
Percent delay (%)	62.92	53.67	35.80	0	0
Capacity/trip	320	320	320	320	320
Waited Passenger/month	10356	10356	10355	10355	10354
Completion of Service in days	32.36	32.36	32.60	32.60	32.60

Table 3.20: Average Number of Passenger in the System

Airline Industry	Passenger/day	Required server
British Airways	188	2
Ethiopian Airline	120	1
Arik Airline	1573	10
Aero Contractor	1034	7
Average	729	5

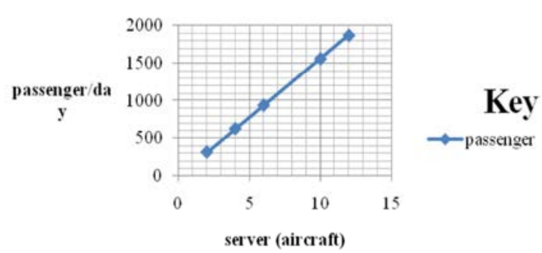

Fig. 3.4: Expected passengers per Server

required server for each airline operator. From data collected, average number of passengers required per server (aircraft) was 160 but on applying the MS model, require number of passengers per server was 156.This happens to be same value with work of Ademoh and Anosike (2014) that used BDR model. Figure 3.4 shows effect of servers on passenger on board [11].

RESULTS AND DISCUSSION

Tables 3.1 - 3.4 are the collected data of passengers from four different airline operators for varied routes. International operators included British Airways and Ethiopian Airline. Domestic operators included Arik Airline and Aero Contractor. In table 3.5 British Airways had average daily passengers of 188. Ethiopian airline had daily average of 120passengers. Arik Airline had daily average of 1573 passengers while Aero contractor had average passengers of 1034 per day. Expected daily passengers into the system were 723 with arrival rate of 31 passengers per hour. Table 3.6 specified boundary condition for modelling waiting line of NAIA with monthly passengers of 21863 that would be ready for service every month. Result showed that most airline operators had two aircrafts or servers per day based on service factors of 0.5; 0.9 and different utilisation factors of 0.2, 0.25, 0.4, 0.6 and 0.9 respectively. In Table 3.10, hypothesis of 31 passengers arriving per hour was tested. The acceptable hypothesis is passengers arriving within ranges of 1 to 13 per hour. The arriving passengers at 5% significance satisfied the condition at service factor of 0.5. Also, both arrival and departure rate of passengers were rejected at service factor of 0.9 because all passengers were above 13. Meeting this condition required each airline to be operating with more than 2 aircrafts per day. [17].

In Figure 3.2, the MS model showed that arriving passengers required service factor of 0.9 per hour. This factor would meet the demands ready for service per day. This was because the 310 passengers on the waiting line required service factor of 0.9 that met the demand of 340 passengers per day. On the other hand, service factor of 0.5 met demand of just 240 passengers per day. Thus, 70 passengers would be delayed per day waiting for service. In the graph, airlines with above 1000 passengers/day required more than 5 aircrafts for its operation. The service factor of 0.9 in Figure 3.3 with utilisation factor of 0.25, 0.4 and 0.6 were very effective on average to provide service for 21 passengers per hour. In section 3.7, 21863 passengers needed the service based on data collected but on service requirement of 0.5 using MS models only 7287 met the demand. It is similar to result with BDR (Ademoh and Anosike, 2014). Using service factor of 0.9, 10355 passengers would met the demand based on these two models. The service level was somewhat delayed with the service factor of 0.5 and 0.9 with utilisation factor of 0.4, 0.6 and 0.9 based on the MS model. However, Ademoh and Anosike (2014) showed that service level was improved with BDR modeling because there was no delay using service factor of 0.5 and 0.9 with utilisation factor of 0.4, 0.6 and 0.9. These imply that if airline operators in the system are to meet current demand the international operators requires single aircraft per day while the local requires up to 5 aircrafts per day as shown in table 3.20.

CONCLUSION

The Multi-service mathematical modeling developed in this work as simulated with flight passenger data obtained from four airlines has shown that NAIA is underserved by aircraft facilities. This is the main reason for waiting line of passengers who queue stressfully in anticipation of service. The model if properly adopted can help in assessing the performance of the systems by minimizing the waiting line of air transport services system of the airport. The study showed that in order to meet the current demand of passengers in NAIA, Abuja there is the need for each of the domestic air transport companies using the facility to operate with 6 aircrafts for daily service to arrest waiting lines. Each of the operators of internal routes requires at least one additional aircraft for daily operation. Delay and waiting line will be minimized if domestic carriers operate at least 5 aircrafts per day than the current average daily usage of 2 aircrafts to meet the monthly demands of 21,863 passengers. With this, services at NAIA will become reliable and available with better passenger satisfaction at 5% significance level based on service factor of 0.9 with utilisation factor of 0.4, 0.6 and 0.9.

The service factor of 0.5 using MS model met demand of 7,287 passengers per month just as it did in the work that used BDR (Ademoh and Anosike). Using service factor of 0.9, the demand of 10,355 passengers were met on the basis of the two models. At the current level of operation, it was estimated that 67% of the service was delayed to meet 21,863 status using service factor of 0.5 and 53% of the service was delayed using service factor 0.9. However, the system would be reliable without delay if it was underutilised by passengers so as to achieve better quality of service in the industry. International airlines had better service because they were operated within daily capacity of 312 passengers. The modeling showed that the existing aircrafts in NAIA Abuja are over utilised at over 50% of their operational capacities to cope with the demands of passengers.

This affected the reliability and availability of the system. As observed by Ademoh and anosike (2014); these observations could however be worse if the month of September whose historical data wasn't available was included in the study as it fell within peak flight periods of Muslim hajj operations, overseas holiday tours, school end of session vacation journeys and other events. Because of tax evasion airline companies could have deliberately hid their passenger data for this month which does not favour the country's realizable income from NAIA. It was a tedious and herculean task to achieve the level of data collected for this study as airline operators were hardly willing to divulge information about their operation. Moreover, none of them was found to have a standard method of record keeping. This work can be extended to solve similar problems in other Nigerian airports (both local and international) for better service delivery.

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