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Integration of Satellite and Inertial Navigational Systems on the Basis of Non Linear-Filtering Theory

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Abstract: Now for synthesis of the integrated Navigational Systems (NS) the linearized equations of errors of inertial NS, inconvertible only on small time intervals and linearized measuring of satellite NS are used, as a rule. Similar simplifications lead to divergence of assessment process of navigational variables. There is a problem appeared on synthesis of NS integration algorithms invariant to physical model of object motion and to kind of its motion trajectory and also permissive to avoid the simplified representations of SNS measuring signals models. In order to solve this problem in the article it is suggested the complete non-linear model of state vector of autonomous Strapdown Inertial Navigational System (SINS), providing inconvertible estimation of navigational parameters at loss of satellite measuring.

Key words: Integrated navigational systems . mobile object . non-linear filtering . satellite measuring

INTRODUCTION

The solution of problem of navigation of Mobile Objects (MO) with usage of satellite navigational systems is carried out now in two directions: by immediate use of navigational information from SNS MO onboard and by integration of measuring information from SNS with readings of Inertial Navigational System (INS). Since the first approach not yet able to provide the complete solution of the problem of high-precision solution of MO navigational problem, we will consider further only a condition of tight integration of NS. In spite of the fact that its examinations are begun long time ago [1-3], the problem of providing of given accuracy and stability of integrated NS remains still very actual. This circumstance is bound to impossibility of this problem solution on the basis of existing mathematical apparatus supposing the use only of SNS linearized measuring and linear equations of INS errors, inconvertible only on small intervals of time [1, 47]. Thereforethere a problem appears on development of essentially new approach, allowing solving a problem of tight integration of SNS and INS in the most general case.

A MATHEMATICAL MODEL OF STATE VECTOR OF INERTIAL NS

As INS we will consider a model of strapdown INS [3, 4] (SINS) at which synthesis we will use the following right Co-ordinate Systems (CS) [4, 5]:

- Instrumental CS (ICS)J 0xyz, which beginning is disposed in center of mass MC) of object and the axes are directed on orthogonal sensitivity axes of the equipment which are a part of SINS meters,
- Inertial CS (ICS) I $0\eta\xi\zeta$ with origin incenter of Earth,
- Rotating together with Earth Greenwich CS (GrCS) G $0\eta_1\xi_1\zeta_1$,
- Accompanying (ACS) SOXYZ which begining coincides with MO centre of mass, an axis Z coincides with local vertical, an axis Y is parallel to an initial meridian plane (movement starts from), an axis X completes the system to a right one.

We consider as well that at reference time the axes ICS and ACS (and ICS and GrCS as well) coincide and the three accelerometres and three Angular-Rate Sensors (ARS) include into measuring complex SINS.

For synthesis of SINS state vector further use the parameters of Rodrigues-Hamilton. The mutual current orientation of ACS and ICS is described by system of the kinematic equations [4, 5]

$$\dot{\lambda} = \frac{1}{2}\Phi_0(\lambda)\omega_{S_0} \quad (1)$$

Where

$$\Phi_0(\lambda) = \begin{vmatrix} -\lambda_2 & -\lambda_3 & -\lambda_4 \\ \lambda_1 & -\lambda_4 & \lambda_3 \\ \lambda_4 & \lambda_1 & -\lambda_2 \\ -\lambda_3 & \lambda_2 & \lambda_1 \end{vmatrix} \lambda_1(0) =$$

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$$=\lambda_{1_0}\ \lambda_2(0)=\lambda_{2_0}\ \lambda_3(0)=\lambda_{3_0}\ \lambda_4(0)=\lambda_{4_0},$$

$\lambda = |\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4|^T$ Vector of parameters of Rodriga's-Gamilton;

$$\omega_{S_0} = |\omega_X \quad \omega_Y \quad \omega_Z|^T\ \omega_i\ i = X, Y, Z$$

-The projections of absolute angular velocity of ACS to its axis, equal to:

$$\omega_X = \omega_{X_S} + 2\Omega(\lambda_2\lambda_3 + \lambda_1\lambda_4)$$

$$\omega_Y = \omega_{Y_S} + \Omega(2\lambda_1^2 + 2\lambda_3^2 - 1)$$

$$\omega_Z = 2\Omega(\lambda_3\lambda_4 - \lambda_1\lambda_2)$$

ω_{i_S} , i = X,Y-the projections of ACS angular velocity to its axisconditioned by MO movement concerning the Earth; Ω-the rotational velocity of the Earth.

In its turn, a current orientation of trihedral J of ICS concerning a trihedral I of ICS can be set as follows:

$$\dot{\mu} = \frac{1}{2}\Phi_0(\mu)\omega_J \tag{2}$$

$$\mu_1(0) = \mu_{1_0},\ \mu_2(0) = \mu_{2_0},\ \mu_3(0) = \mu_{3_0},\ \mu_4(0) = \mu_{4_0}$$

$\mu = |\mu_1 \quad \mu_2 \quad \mu_3 \quad \mu_4|^T$, $\omega_J = |\omega_x \quad \omega_y \quad \omega_z|^T$ Vector of absolute angular velocity of instrumental trihedral rotation which can be gained under indications

$$Z_d = |Z_x \quad Z_y \quad Z_z|^T$$

of three orthogonal ARSes, positioned on MO:

$$\omega_J = Z_d - m_d - W_d \tag{3}$$

where

$W_d = |W_x \quad W_y \quad W_z|^T$ Vector of the additive interferences of ARS measuring, described by white Gaussian noise (WGN) with zero average and lower intentions matrix D_d ; $m_d = |m_{d_x} \quad m_{d_y} \quad m_{d_z}|^T$ Vector of mathematical expectation of ARS zero replacement.

Taking into account (3) an angular movement of SINS (2) regarding ICS can be presented in a vector kind as follows:

$$\dot{\mu} = \frac{1}{2}\Phi_0(\mu)(Z_d - m_d - W_d) \tag{4}$$

For final synthesis of MO navigational system state vector it is necessary to present in a closed form the right members of equations initial systems (1), (4). Thus further we will consider that the projections of angular velocity $\omega_{X_S}, \omega_{Y_S}$ of trihedral S are related to projections of MO linear speed V_X, V_Y on ACS axes by linear ratios [4, 5]:

$$V_X = \omega_{Y_S}(r + h) \tag{5}$$

$$V_Y = -\omega_{X_S}(r + h) \tag{6}$$

where r-Radius of the Earth; h-an object altitude above the sea level.

For synthesis of expressions of projections V_X, V_Y we will turn to main equation of inertial navigation [4, 5]:

$$a = \dot{V}_S + (2O_S + \omega_S) \times V_S + g_S \tag{7}$$

where $\times$-a sign of vector product; g_S-gravitational vector; a-the acceleration vector, measured by accelerometers;

$V_S = |V_X \quad V_Y \quad V_Z|^T$ Velocity vector of object concerning the Earth;

$O_S = |\Omega_X \quad \Omega_Y \quad \Omega_Z|^T$ the vector of Earth rotation angular velocity, which projections on the axis ACS having a view:

$$\Omega_X = 2\Omega(\lambda_2\lambda_3 + \lambda_1\lambda_4)$$

$$\Omega_Y = \Omega(2\lambda_1^2 + 2\lambda_3^2 - 1)$$

$$\Omega_Z = 2\Omega(\lambda_3\lambda_4 - \lambda_1\lambda_2) \tag{8}$$

For chosen orientations of ACSaxes of projections of vector

$$g_S = |g_X \quad g_Y \quad g_Z|^T$$

on ACS axes define as:

$$g_X = 4\Omega^2 (r+h)(\lambda_2\lambda_3 + \lambda_1\lambda_4)(\lambda_3\lambda_4 - \lambda_1\lambda_2) \tag{9}$$

$$g_Y = 2\Omega^2 (r+h)(2\lambda_1^2 + 2\lambda_3^2 - 1)(\lambda_3\lambda_4 - \lambda_1\lambda_2)$$

$$g_Z = -\Omega^2 (r+h)\left(2(\lambda_2\lambda_3 + \lambda_1\lambda_4)^2 + (2\lambda_1^2 + 2\lambda_3^2 - 1)^2\right) - g_0(r, h, \varphi(\lambda_1 \div \lambda_4))$$

where g_0-the gravitational acceleration, considered as function [4, 5] of altitude hand latitude

$$\varphi(\lambda_1 \div \lambda_4) = \arcsin 2(\lambda_3\lambda_4 - \lambda_1\lambda_2) + k\pi, \; k = \overline{0,\infty}, \text{[4, 5]}.$$

Thevectorof accelerometers output signals

$$Z_a = |Z_1 \quad Z_2 \quad Z_3|^T$$

can be presented as follows:

$$Z_a = Ca + W_a \tag{10}$$

where

$W_a = |W_{a_x} \quad W_{a_y} \quad W_{a_z}|^T$ a vector of accelerometers disturbances, described by WGN with zero math expectation and matrix of intensities $D_a(t)$;

$$C(\mu, \lambda) = D(\mu)B^T(\lambda)$$

$C(\mu,\lambda)$-a matrix of guiding cosines, determining ICS orientation regarding ACS;

$D(\mu)$-a matrix of second kind turn [4, 5], determining the ICS orientation regarding ICS; $B = D(\lambda)$-a matrix of second kind, determining ACS orientation regarding ICS.

Then, taking into account (10), a vector of accelerations, measured by accelerometers, can be presented as kind follows:

$$a = C^T(Z_a - W_a) \tag{11}$$

Expression (7) in scalar form has a view:

$$a_X = \dot{V}_X - 2\Omega_Z V_Y + (2\Omega_Y + \omega_{Y_S})V_Z + g_X \tag{12}$$

$$a_Y = \dot{V}_Y + 2\Omega_Z V_X - (2\Omega_X + \omega_{X_S})V_Z + g_Y$$

$$a_Z = \dot{V}_Z - (2\Omega_Y + \omega_{Y_S})V_X + (2\Omega_X + \omega_{X_S})V_Y + g_Z$$

where from,substituting in (12) the expressions (5), (6) and (11), we will receive:

$$\dot{V}_X = C_{(1)}(\mu, \lambda)(Z_a - W_a) + 2\Omega_Z V_Y - \left(2\Omega_Y + V_X (r+h)^{-1}\right)V_Z - g_X$$

$$\dot{V}_Y = C_{(2)}(\mu, \lambda)(Z_a - W_a) - 2\Omega_Z V_X + \left(2\Omega_X - V_Y (r+h)^{-1}\right)V_Z - g_Y \tag{13}$$

$$\dot{V}_Z = C_{(3)}(\mu, \lambda)(Z_a - W_a) + \left(2\Omega_Y + V_X (r+h)^{-1}\right)V_X - \left(2\Omega_X - V_Y (r+h)^{-1}\right)V_Y - g_Z$$

where through $C_{(i)}(\mu, \lambda)(i = 1,..,3)$ it is designated i-th row of matrix $C^T(\mu,\lambda)$.

Included into given earlier expressions an object altitude haboveseaelevel is determined by vertical constituent of its velocity

$$\dot{h} = V_Z \tag{14}$$

that allows locking up into uniform system all expressions received.

In final view the expressions of stochastic vector of SINS on bases of three ARS and of three accelerometers have a view:

$$\dot{\lambda} = \frac{1}{2}\Phi_0(\lambda)\left\{(r+h)^{-1}\begin{vmatrix} -V_Y \\ V_X \\ 0 \end{vmatrix} + \begin{vmatrix} 2\Omega(\lambda_2\lambda_3 + \lambda_1\lambda_4) \\ \Omega(2\lambda_1^2 + 2\lambda_3^2 - 1) \\ 2\Omega(\lambda_3\lambda_4 - \lambda_1\lambda_2) \end{vmatrix}\right\}$$

$$\dot{\mu} = \frac{1}{2}\Phi_0(\mu)(Z_d - m_d - W_d)$$

$$\dot{V}_X = C_{(1)}(\mu, \lambda)(Z_a - W_a) + 4\Omega(\lambda_3\lambda_4 - \lambda_1\lambda_2)V_Y - \left(2\Omega(2\lambda_1^2 + 2\lambda_3^2 - 1) + V_X (r+h)^{-1}\right)V_Z - 4\Omega^2(r+h)(\lambda_2\lambda_3 + \lambda_1\lambda_4)(\lambda_3\lambda_4 - \lambda_1\lambda_2),$$

$$\dot{V}_Y = C_{(2)}(\mu,\lambda)(Z_a - W_a) - 4\Omega(\lambda_3\lambda_4 - \lambda_1\lambda_2)V_X + \\ +\left(4\Omega(\lambda_2\lambda_3 + \lambda_1\lambda_4) - V_Y(r+h)^{-1}\right)V_Z - \\ -2\Omega^2(r+h)\left(2\lambda_1^2 + 2\lambda_3^2 - 1\right)\left(\lambda_3\lambda_4 - \lambda_1\lambda_2\right),$$

$$\dot{V}_Z = C_{(3)}(\mu,\lambda)(Z_a - W_a) + \left(2\Omega(2\lambda_1^2 + 2\lambda_3^2 - 1) + V_X(r+h)^{-1}\right)V_X - \\ -\left(4\Omega(\lambda_2\lambda_3 + \lambda_1\lambda_4) - V_Y(r+h)^{-1}\right)V_Y + \\ +\Omega^2(r+h)\left(2(\lambda_2\lambda_3 + \lambda_1\lambda_4)^2 - (2\lambda_1^2 + 2\lambda_3^2 - 1)^2\right) + \\ +g_0\left(r,h,\varphi(\lambda_1 \div \lambda_4)\right), \dot{h} = V_Z$$

Or in canonical view– in Lanzheven's vector form:

$$\dot{Y} = F(Y,t) + F_0(Y,t)\xi \quad (15)$$

where

$$Y = \left|\lambda^T \quad \mu^T \quad V_X \quad V_Y \quad V_Z \quad h\right|^T$$

$$\xi = \left|W_d^T \quad W_a^T\right|^T$$

The principal features of expressions (15) are their general character, first of all and, secondly, the possibility of utilization on its bases of nonlinear filtration methods providing the navigational estimations optimality at integration of SINS and SNS.

In order to use this possibility it is necessary to receive an equation of vector Y observer, following [7] (i.e. an analytical model of signal, carrying an information on vector Y components).

MATHEMATICAL MODEL OF AUTONOMOUS OBSERVER OF SINS CONDITION STOCHASTIC VECTOR

In order to solve this problem we consider a possibility of SINS complexion with Doppler's sensors of velocity (DSV).

ForusageofDSVinformationinalgorithmswesuppose further that the vector of outlet signals

$$Z_D = \left|Z_{Dx}\, Z_{D_y}\, Z_{Dz}\right|^T$$

of DSV which axes are orthogonal and directed per ICS axes have a view:

$$Z_D = V + W_D + U_D \quad (16)$$

where

$$V = \left|V_x\, V_y\, V_Z\right|^T$$

a vector of normalized velocity of MO in INS;
W_D-markovian noise vector on DSV outlet;
U_D-WGN with zero average and matrix of intensities D_U.

In general case the vector W_D is being described by stochastic equation:

$$\dot{W}_D = f_D(W_D,t) + f_{D_0}(W_D,t)\xi_D \quad (17)$$

where f_D, f_{D_0} -the known vector and matrix functions,

ξ_D-WGN with zero average and matrix of intensities D_D.

As soon as a projection of relative linear velocity of MO on ICS axes V_x, V_y and V_z determine on projections of velocity V_X, V_Y and V_Z on ACS axis as:

$$V = C(\mu,\lambda)V_S \quad (18)$$

the n the expression of observer of SINS condition vector will have a view:

$$Z_D = C(\mu,\lambda)V_S + W_D + U_D$$

or in more generalized view (canonical):

$$Z_D = H(Y,t) + U_D \quad (19)$$

where

$$H(Y,t) = C(\mu,\lambda)V_S + W_D$$

(Thus the vector WD should be included into composition of all SINS state vector Y and vectorξ_D-into composition of its vector of noise: $\xi^{(1)} = \left|W_d^T \quad W_a^T \quad \xi_D^T\right|^T$). Representation of SdNS motion equations in form "object-observer" uncloses the principal possibility of optimum estimation of MO movements' parameters.

SYNTHESIS OF ALGORITHMS OF NON-LINEAR FILTERING OF AUTONOMOUS SDNS NAVIGATIONAL PARAMETERS

In the theory of non-linear filtering for deriving of processes estimations of kind (15) utilize the different approximate (suboptimal) methods most demanded of which is the non-linear (Gaussian) filter of Kalman-Byyusi ensuring the required compromise between an estimation precision and volume of computing expenditures [7, 8].

The equations (15, 19) in form "object-observer" easily allow, following [7], to record the non-linear filter for considered NS as:

$$K\left(\hat{Y},t\right) = R\frac{\partial H^{T}\left(\hat{Y},t\right)}{\partial \hat{Y}} D_{U}^{-1},$$

$$\begin{aligned}\dot{R}\left(\hat{Y},t\right) = \frac{\partial F\left(\hat{Y},t\right)}{\partial \hat{Y}} R\left(\hat{Y},t\right) + R\left(\hat{Y},t\right)\frac{\partial F^{T}\left(\hat{Y},t\right)}{\partial \hat{Y}} + \\ + F_{0}\left(\hat{Y},t\right) D_{\xi} F_{0}^{T}\left(\hat{Y},t\right) - K\left(\hat{Y},t\right) D_{U} K^{T}\left(\hat{Y},t\right),\end{aligned} \quad (21)$$

$$\hat{Y}_{0} = M\left(Y_{0}\right),\ R_{0} = M\left\{\left(Y_{0} - \hat{Y}_{0}\right)\left(Y_{0} - \hat{Y}_{0}\right)^{T}\right\},$$

where $\hat{Y}$ -a vector of current estimate of NS state vector Y (t).

The gained estimates of SdNS navigational parameters allow ensuring inconvertible estimation of MO navigational parameters even at absence of satellite measuring. At the same time a presence of last ones can essentially increase a precision of estimation, so in this connection we will consider a possibility of integration of satellite and autonomous measuring in more detail.

MATHEMATICAL MODEL OF SIGNALS OF SATELLITE MEASURING

In reference condition an informational signal of code measuring (pseudo-distance) can be recorded (taking into account an algorithmic compensation errors stipulated by transition of radio signal through ionosphere and troposphere, the errors of clocks of the receiver and satellite) as:

$$Z_{R} = \sqrt{(\xi_{c} - \xi)^{2} + (\eta_{c} - \eta)^{2} + (\zeta_{c} - \zeta)^{2}} + W_{Z_{R}} \quad (22)$$

where ξ_c, η_c, ζ_c-satellite coordinates in Greenwich CS, calculated onboard of satellite and transmitted to an object in navigational notification.

$\xi\eta\zeta$-an object current coordinates in the Greenwich CS;

W_{Z_R}-WGN with zero average and known variance $D_{Z_R}(t)$.

Similarly the informational signal of Doppler measuring $D_{Z_V}(t)$ can be presented as follows:

$$\begin{aligned}Z_{V} = \begin{bmatrix}(\xi_{c} - \xi)(V_{\xi c} - V_{\xi}) + (\eta_{c} - \eta)(V_{\eta c} - V_{\eta}) \\ +(\zeta_{c} - \zeta)(V_{\zeta c} - V_{\zeta})\end{bmatrix} \\ \times\left(\sqrt{(\xi_{c} - \xi)^{2} + (\eta_{c} - \eta)^{2} + (\zeta_{c} - \zeta)^{2}}\right)^{-1} + W_{Z_{V}}\end{aligned} \quad (23)$$

where $V_{\xi c}$, $V_{\eta c}$, $V_{\zeta c}$-the projections of satellite velocity onto axis of GrCS,

V_ξ, V_η, V_ζ-the projections of object velocity onto axis of GrCS,

W_{Z_V}-a WGN with zero average and known dispersion $D_{Z_V}(t)$.

Signals of coded and Doppler measuring bear theinformation both: on MO current coordinates and on its velocity, i.e. can be used as signals of observation of object state vector. But for this purpose it is necessary to present them in corresponding ACS. In case being considered we have for object coordinates:

$$\xi = 2(r + h)\cdot\left(\lambda_{2}\lambda_{4} + \lambda_{1}\lambda_{3}\right),$$

$$\eta = 2(r + h)\cdot\left(\lambda_{3}\lambda_{4} - \lambda_{1}\lambda_{2}\right),$$

$$\zeta = (r + h)\cdot(2\lambda_{1}^{2} + 2\lambda_{4}^{2} - 1)$$

At definition of connection of object velocity vector in Greenwich CS V_G with velocity vector V_S in ACS it is necessary to consider that this connection is defined not only by matrix $B(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ of ACS orientation regarding to ICS shown earlier, but it requires also an additional use of matrix of turning point G of Greenwich CS regarding ICS:

$$G = \begin{vmatrix}\cos\Omega t & 0 & -\sin\Omega t \\ 0 & 1 & 0 \\ \sin\Omega t & 0 & \cos\Omega t\end{vmatrix}$$

In this case a representation of vector $V_G$has the appearance follow:

$$V_{G} = G(\Omega t)B^{T}(\lambda_{1},\ \lambda_{2},\ \lambda_{3},\ \lambda_{4})V_{S} = GB^{T}(\lambda)V_{S} \quad (24)$$

Hence, it is finally possible to present the informational models of signals of code and Doppler measuring as follows:

$$Z_{R} = W_{Z_{R}} + \sqrt{(\xi_{c} - 2(r + h)\ (\lambda_{2}\lambda_{4} + \lambda_{1}\lambda_{3}))^{2} + (\eta_{c} - 2(r + h)(\lambda_{3}\lambda_{4} - \lambda_{1}\lambda_{2}))^{2} + (\zeta_{c} - (r + h)(2\lambda_{1}^{2} + 2\lambda_{4}^{2} - 1))^{2}} = H_{R}(\lambda, h) + W_{Z_{R}}$$

$$Z_V = [(\xi_c - 2(r+h)(\lambda_2\lambda_4 + \lambda_1\lambda_3))(V_{\xi c} - [GB^T(\lambda)]_{(1)} V_S) + \\ +(\eta_c - 2(r+h)(\lambda_3\lambda_4 - \lambda_1\lambda_2))(V_{\eta c} - [GB^T(\lambda)]_{(2)} V_S) + (\zeta_c - (r+h)(2\lambda_1^2 + 2\lambda_4^2 - 1))(V_{\zeta c} - [GB^T(\lambda)]_{(3)} V_S)] \\ \times(\sqrt{(\xi_c - 2(r+h)(\lambda_2\lambda_4 + \lambda_1\lambda_3))^2 + (\eta_c - 2(r+h)(\lambda_3\lambda_4 - \lambda_1\lambda_2))^2 + (\zeta_c - (r+h)(2\lambda_1^2 + 2\lambda_4^2 - 1))^2})^{-1} + W_{Z_V} = H_V(\lambda, h, V_S) + W_{Z_V} \quad (25)$$

where $[GB^T(\lambda)]_{(i)}$-i-th row of matrix $GB^T(\lambda)$.

The most important feature of satellite observations which is essentially impeding a high-precision estimation of continuous navigational parameters of object-especially for high-speed MO, is their discrete character (in GLONASS time interval between the lines-navigational communications makes 2 with [1]). The similar problem related already to case of continuous-the discrete filtering and can be, as known [7], solved on the basis of sharing use of two kinds of estimates: a continuous-on interval between satellite observations and a discrete-at the moment of reception of navigational conferring. (At subsequent synthesis of the last we will use-in accordance with offered in [7], Gaussian approximation of a posteriori density of probability of state vector).

THE SOLUTION OF NAVIGATIONAL PROBLEM ON COMPLEXION MEASURING OF INTEGRATED NS

So, we will consider a scheme of estimation of integrated NS state vector which complexion observer forms both: on SdNS autonomous measuring and on satellite measuring. Thus the equations of satellite measuring (25) for the purpose of simplification of further build-ups we will represent in vector view:

$$Z = \begin{vmatrix} Z_R \\ Z_V \end{vmatrix} = \begin{vmatrix} H_R(\lambda, h) \\ H_V(\lambda, h, V_S) \end{vmatrix} + \begin{vmatrix} W_{Z_R} \\ W_{Z_V} \end{vmatrix} = H_C(Y, t) + \zeta \quad (26)$$

where ζ-a white Gaussian vector a noise with zero average and matrix of intensity

$$D_{CHC} = \begin{vmatrix} D_{Z_R} & 0 \\ 0 & D_{Z_V} \end{vmatrix}$$

Since these observations are discrete, then the satellite measuring equations (26) is necessary to present in the view follow:

$$Z_K = H_C(Y, K) + \zeta_K \quad (27)$$

where k =1,2, …-a number of step of navigational conferring reception.

The generalscheme of NS integration in this case will be the following: on time interval between the satellite measuring for estimation of navigational parameters the continuous non-linear filter (20,21) is used and while the processing of satellite navigational information it will be used the discrete Gaussian filter [7, 9]. Thus, it is necessary to bear in mind that the continuous filter is used only on time intervals $[t_{K-1}, t_K]$, k = 1,2, …, between the discrete satellite measuring therefore the initial conditions $\hat{Y}(t_{K-1})$, $R(t_{K-1})$ of equations of continuous filtering of kind (20,21) on the interval $[t_{K-1}, t_K]$ are formed as a result of discrete estimation

$$\hat{Y}_{K-1} = \hat{Y}(t_{K-1} + 0), \; R_{K-1} = R(t_{K-1} + 0)$$

ofvector Y int$_{K-1}$instant:

$$\hat{Y}(t_{K-1}) = \hat{Y}_{K-1} = \hat{Y}(t_{K-1} + 0)$$

$$R(t_{K-1}) = R_{K-1} = R(t_{K-1} + 0)$$

The result of integration $\hat{Y}(t_K)$, $R(t_K)$ of equations (20,21) at terminal of time interval $[t_{K-1}, t_K]$ is an initial condition

$$\hat{Y}(t_K - 0) = \hat{Y}_{K0}, \; R(t_K - 0) = R_{K0}$$

for performance of algorithm of discrete estimation in t_K instant:

$$\hat{Y}(t_K - 0) = \hat{Y}_{K0} = \hat{Y}(t_K) \;\; R(t_K - 0) = R_{K0} = R(t_K)$$

Similar connection of initial and terminal conditions of algorithms of discrete and continuous estimation is one of the main requirements of condition of tight integration of autonomous SdNS and SNS.

At synthesis of discrete non-linear filter it is necessary to consider that in contrast to a case of continuous filtering the discrete estimation is carried out at expanded vector of observation:

$$Z_D = Z_K^{(1)} = H(Y,K) + U_D,\ Z_K = H_c(Y,K) + \zeta_K$$

this is further in order to simplify an entry will be represented as:

$$Z_?^{???} = \begin{vmatrix} Z_K^{(1)} \\ Z_K \end{vmatrix} = \begin{vmatrix} H(Y,K) \\ H_c(Y,K) \end{vmatrix} + \begin{vmatrix} U_D \\ \zeta_K \end{vmatrix} = H^{???}(Y,K) + ?_K^{???}$$

where $\zeta_K^{???}$ -WGN with zero average and intensity matrix

$$D_{???} = \begin{vmatrix} D_U & 0 \\ 0 & D_{CHC} \end{vmatrix}$$

The algorithm of discrete estimation at similar expanded observer according to [7] looks like:

$$\hat{Y}(t_K+0) = \hat{Y}_{K0}\int + R(t_K+0)\frac{\partial H^{???T}\left(\hat{Y}_{K0},K\right)}{\partial \hat{Y}} D_{???}^{-1}\left[Z_K^{???} - H^{???}\left(\hat{Y}_{K0},K\right)\right], \qquad (28)$$

$$R^{-1}(t_K+0) = R_{K0}^{-1} + \frac{\partial H^{??TT}\left(\hat{Y}_{K0},K\right)}{\partial \hat{Y}} D_{??T}^{-1} \frac{\partial H^{??T}\left(\hat{Y}_{K0},K\right)}{\partial \hat{Y}}$$

Hence, the offered scheme of sharing usage of algorithms of estimation (20, 21) and (28) allows in essence and in most general case, without any simplifying assumptions, to solve the problem of tight integration of autonomous SdNS and SNS.

EXAMPLE

For illustration of possibility of effective use of the offered algorithm of integration of SINS with SNS the numerical modeling of estimation equations (20), (21), (28) has been carried out.

Modeling operation was carried out on time interval $t \in [0;10000]$c with step Δt=0,01c by method of Runge-Kutta of fourth order [10] at following chosen initial conditions:

$$\lambda_0 = \frac{\pi}{5}, \varphi_0 = \frac{\pi}{4}, \alpha_0 = \frac{\pi}{3}, \beta_0 = -\frac{\pi}{5}, \gamma_0 = \frac{\pi}{4}$$

At modeling operation a character of object CM movement relative to Earth surface has been defined by time functions:

$$\dot{V}_X = 200\exp(-0{,}1t) + 10\cos(0{,}25t)$$

$$\dot{V}_Y = 100\exp(-t) + 5\sin(0.25t)$$

$$\dot{V}_Z = \begin{cases} 300\exp(-0.08t) + 3\sin(0.25t), & \text{if } 0 < t \le 25 \\ \left(-284\exp(-0.07(t-25)) + 3\sin(0.25t)\right), & \text{if } 25 < t \le 100 \\ 3\sin(0.25t), & \text{if } 100 < t \le 1000 \end{cases}$$

As a model of noises the additive Gaussian uncorrelated vector-noise with zero mathematical expectation and intensity has been used for: accelerometers-$(10^{-5}\ m/s^3)^2$, DSV-$(0.5m/s)^2$, SAR-$(10^{-4}\ 1/c)^2$, coded measuring-$(15m)^2$, Doppler measuring-$(0.5m/s)^2$. Modeling of satellite signals loss was carried out twice: on 110th second on time interval 15sec. and on 3100th sec. on time interval 35sec.

Upon the termination of modeling time interval the peak errors of vector Y components have made: on vertical constituent of object velocity vector V_Z-7 %, on projection V_X-5,8%, on projection V_Y-4,7%, per orientation angles-0,4 %, on longitude-17 m, on latitude-12 m that testifies to possibility of effective practical use of offered algorithm.

CONCLUSIONS

It is developed a complete non-linear model of state vector of autonomous SINS, containing in structure a measuring DSV complex and providing inconvertible estimation of navigational parameters at loss of satellite measuring.

In presence of SNS measuring the offered model allows constructing a general nonlinear observer of complete vector of navigational parameters on the basis of complexation of independent and satellite measuring.

Synthesis of complex observer, in turn, provides a possibility of rigorous application of nonlinear filtering theory and development on its basis of procedure of common solution of navigational problem of integrated NS, ensuring a high-precision parameter estimation of object motion at presence of satellite measuring and inconvertible estimation-at their loss.

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