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Modeling of Time Dependent Creep Deformations in Reinforced Materials Using Fluid Mechanics Theory

[1]*Vahid Monfared* and [2]*Saeed Daneshmand*

[1]Department of Mechanical Engineering, Zanjan Branch,
Islamic Azad University, Zanjan, Iran
[2]Department of Mechanical Engineering, Majlesi Branch,
Islamic Azad University, Isfahan, Iran

Abstract: A new concept for obtaining shear stress based on fluid mechanics theory is presented for stress analysis of the creep deformations and behavior of fibrous thermoset amorphous polymeric composites subjected to an applied tensile axial load. Shear stress is determined by simulation of plastic polymeric matrix flow (or metal matrix at high temperatures) and fluid flow, that is, creep of thermoset polymer matrix is simulated by fluid flow. Viscosity of crept amorphous polymeric matrix such as resin epoxy with respect to various temperatures is determined by exponential and polynomial functions approximately. Moreover shear stress is obtained by exponential, polynomial and logarithmic functions with the mentioned approach. A relationship is presented between shear stress in deformed amorphous polymeric matrix (Solids or metal matrix at high temperatures) and shear stress in fluids.

Key words: Viscosity・Fluid mechanics・Thermoset・Composites・Creep・Amorphous

INTRODUCTION

Many researchers have investigated the creep behavior by theoretical, experimental and numerical methods. Also, a few researches have been carried out for determination of shear stress in crept matrix by shear stress theory using mechanics of fluid methods. For example shear stress was determined by shear lag theory or other formulations by previous researchers in crept matrix and elastic fiber.

In the last years, comprehensive investigations have been conducted to determine the creep properties of short fiber composites [1-3]. Viscous flow of aligned composites subjected to tensile creep in the direction of the fibers has been investigated [4]. An analysis for predicting the stress distribution in unidirectional discontinuous fiber composites based on the shear lag theory and the load transfer at fiber ends has been performed [5].

Other kinds of some deformations have been also investigated in SMA, PZT, rails and so on using theoretical and finite element methods [6, 7]. Herschel-Bulkley model is also known as the yield Power law model, as it is a hybrid of the Bingham Plastic and the Power law models [8]. The Power law model does not consider the yield point while the Herschel-Bulkley model takes the yield point in to account [9-11].

In addition, creep life prediction of thermally exposed Rene 80 super alloy and creep constitutive model and component lifetime estimation (the case of niobium-modified 9Cr-1Mo steel weldments) have been studied respectively [12, 13]. Moreover, analytical solutions for obtaining creep stresses in thick-walled spherical pressure vessels and cylinders have been performed [14, 15].

In recent years, theoretical analyses and different methods have been analytically proposed with purpose of determining proper solutions and algorithms for analyzing nonlinear differential and ordinary equations. Moreover, traveling wave solutions, direct solutions, integral and iteration methods and such approaches have been presented for the solution of the nonlinear equations by various researchers [16-26].

Corresponding Author: Vahid Monfared, Department of Mechanical Engineering, Zanjan Branch, Islamic Azad University, Zanjan, Iran.

Therefore new concept has been presented that used from viscosity of crept thermoset amorphous polymeric matrix (or metal matrix at high temperatures) such as epoxy resin in steady state creep for determination of shear stress by mechanics of fluid theory. In this way, shear stress in crept matrix is determined by shear strain rates and viscosity of amorphous crept polymer matrix in second stage of creep similar to Newtonian fluid. In addition, viscosity of amorphous crept polymeric epoxy matrix with respect to various temperatures has been estimated by exponential and polynomial functions approximately.

This research is based on simulation between high viscous fluids and amorphous polymeric crept matrix (Solids or metal matrix at high temperatures). For example, in fluids, high viscous materials are similar to pitch, peanut butter, molten glass and chocolate, etc. Shear strain rates are determined by the assumed and satisfied axial and radial displacement rates in proper boundary and incompressibility conditions.

Eventually, shear stress in crept matrix is determined by substitution of axial and radial displacement rates into shear stress formulation using fluid mechanics theory. Additionally, these new axial and radial displacement rates (velocities) are determined and satisfy boundary and incompressibility conditions.

Viscosity of Solids: On the basis that all solids such as granite flow in response to small shear stress, known as amorphous solids, such as glass and many polymers (or metal matrix at high temperatures), may be considered to have viscosity shown in Figures 1, 2. This has led some to the view that solids are simply liquids with a very high viscosity, typically greater than 10^{12} Pa.s. This position is often adopted by supporters of the widely held misconception that glass flow can be observed in old buildings.

Granite has a measured viscosity at standard temperature and pressure of about 4.5×10^{19} Pa.s. The below figure shows shear strain rate, shown in Figure 1.

For presentation of the new work, models, relations must be introduced initially. It is used from a micromechanical model to reduce calculations with preserving the physical conditions of composite structure. In this model is assumed that fibers are parallel, regular and unidirectional.

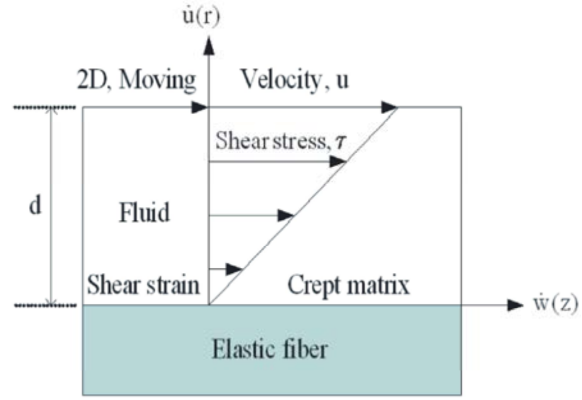

Fig. 1: Shear strain rate and stress in crept matrix and elastic fiber.

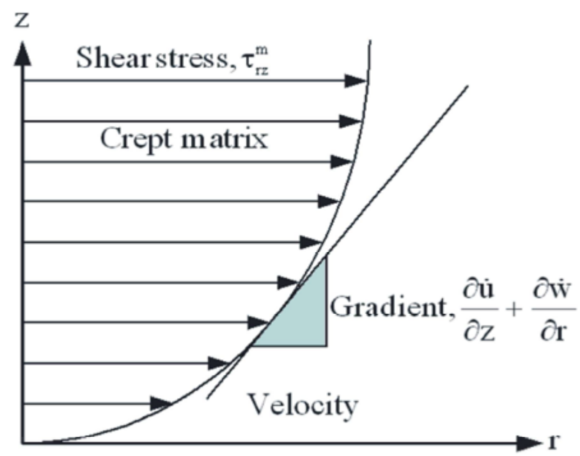

Fig. 2: Shear stress in fluid or crept polymeric matrix in steady state creep (small and large deformations).

Boundary Conditions: In order to determine the proper and exact shear strain, shear stress, radial and axial displacement rates of the composite, it is essential to use proper boundary conditions to the mathematical and analytical model (such as $\dot{u}(d)=\dot{u}, \dot{u}(0)=\dot{w}(0)=0, \tau_{\tau=0}=\tau_0, \tau_{\tau=d}=0$ and so on). Obviously, the iso-displacement condition is suitable for the present problem. It should be mentioned that correct and suitable boundary conditions are led to correct behavior of unknowns.

MATERIAL AND METHOD

This method is based on two displacement functions $\dot{u}(r,z)$ and $\dot{w}(r,z)$ which are radial and axial displacement rates respectively. After obtaining $\dot{u}(r,z)$, the coupled displacement rate $\dot{w}(r,z)$ is determined by incompressibility conditions. That is:

$$\dot{u}(r,z) \cong \varnothing(r,z) \tag{1a}$$

$$\varnothing(r,z) \cong \varnothing(r_0,z_0) + (r-r_0)\varnothing_x(r_0,z_0) + (z-z_0)\varnothing_y(r_0,z_0) + \frac{1}{2}[(r-r_0)^2\varnothing_{xx}(r_0,z_0) + 2(r-r_0)(z-z_0)\varnothing_{xy}(r_0,z_0) + (z-z_0)^2\varnothing_{yy}(r_0,z_0)] \tag{1b}$$

Axial displacement rate $\dot{w}(r,z)$ is obtained by incompressibility condition $(\Delta V = 0,\ \dot{u}_r + \dot{u}/r + \dot{w}_z = 0)$, It then follows that,

$$\dot{w}(r,z) = -\int \frac{\partial\varnothing(r,z)}{\partial r}dz - \int \frac{\partial\varnothing(r,z)}{r}dz \tag{2a}$$

$$\frac{\partial\dot{u}(r,z)}{\partial r} + \frac{\dot{u}(r,z)}{r} - \frac{\partial}{\partial z}\left[\int \frac{\dot{u}(r,z)}{r} + \frac{\partial\dot{u}(r,z)}{\partial r}dz\right] = 0 \tag{2b}$$

$$\dot{\varepsilon}_r = \frac{\partial\dot{u}}{\partial r} = \frac{\partial\varnothing(r,z)}{\partial r} \tag{3}$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}}{r} = \frac{\varnothing(r,z)}{\partial r} \tag{4}$$

$$\dot{\varepsilon}_z = \frac{\partial\dot{w}}{\partial z} = \frac{\partial}{\partial z}\left[-\int \frac{\partial\varnothing(r,z)}{\partial r}dz - \int \frac{\varnothing(r,z)}{r}dz\right] \tag{5}$$

And for shear strains have,

$$\dot{\gamma}_{rz} = \frac{\partial\dot{u}}{\partial z} + \frac{\partial\dot{w}}{\partial r} = \frac{\partial\varnothing(r,z)}{\partial z} + \frac{\partial}{\partial r}\left[-\int \frac{\partial\varnothing(r,z)}{\partial r}dz - \int \frac{\varnothing(r,z)}{r}dz\right] \tag{6}$$

$$\dot{\varepsilon}_{rz} = \frac{1}{2}\dot{\gamma}_{rz} = \frac{1}{2}\left(\frac{\partial\dot{u}}{\partial z} + \frac{\partial\dot{w}}{\partial r}\right) = \frac{1}{2}\left(\frac{\partial\varnothing(r,z)}{\partial z} + \frac{\partial}{\partial r}\left[-\int \frac{\partial\varnothing(r,z)}{\partial r}dz - \int \frac{\varnothing(r,z)}{r}dz\right]\right) \tag{7}$$

Also shear stress in matrix is presented in the below various forms

$$\tau_{rz} = \mu\dot{\gamma}_{rz} \tag{8}$$

$$\tau_{rz} = \mu_{plastic}\dot{\gamma}_{rz} + \tau_{yield} \tag{9}$$

$$\tau_{rz} = k\dot{\gamma}^m \tag{10}$$

$$\tau_{rz} = k\dot{\gamma}^m + \tau_{yield} \tag{11}$$

where μ, $\mu_{plastic}$, k, $\dot{\gamma}$, τ_{yield} and m are dynamic viscosity, plastic viscosity, consistency index, shear strain rates, yield stress and power law exponent, respectively. A thermosetting plastic, also known as a thermoset, is polymer material that irreversibly cures. The cure may be done through heat (generally above 200°C (392°F)), through a chemical reaction (two-part epoxy, for example), or irradiation such as electron beam processing. Viscous flow in amorphous materials (e.g. in glasses and polymers) is a thermally activated process,

$$\mu = A.e^{Q/RT} \tag{12}$$

where Q is activation energy, T is temperature, R is the molar gas constant and A is approximately a constant.
The viscosity of amorphous materials is quite exactly described by a two exponential equation,

$$\mu = A_1.T.\left[1 + A_2.e^{B_1/RT}\right].\left[1 + C.e^{B_2/RT}\right] \tag{13}$$

with constants A_1, A_2, B_1, C and B_2 related to thermodynamic parameters of joining bonds of an amorphous material.

In many materials at ordinary temperatures, rate-dependent inelastic deformation is insigni?cant when the stress is below a yield stress. A simple model describing this effect is the Bingham model,

$$\dot{\varepsilon}_e = \dot{\varepsilon}^i \begin{cases} 0 & |\sigma| < \sigma_y \\ \left(1 - \dfrac{\sigma_y}{|\sigma|}\right)\dfrac{\sigma}{\mu} & |\sigma| \geq \sigma_y \end{cases} \tag{14}$$

In which μ is a viscosity and the yield stress σ_y may depend on strain. The Bingham model is the simplest model of viscoplasticity ($1cP = 0.001\ Pa.s$). In what follows, shear stress will be determined by substituting $\dot{u}(r,z)$ and $\dot{w}(r,z)$ into shear stress formulation using fluid mechanics theory.

RESULTS AND DISCUSSIONS

Viscosity of crept amorphous polymeric epoxy matrix with respect to various temperatures is determined by polynomial function approximately as blow (Figures 3-8).

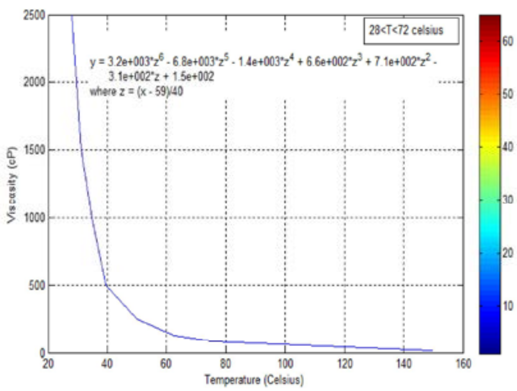

Fig. 3: Nonlinear behavior of pure epoxy viscosity at various temperatures (28<T<72).

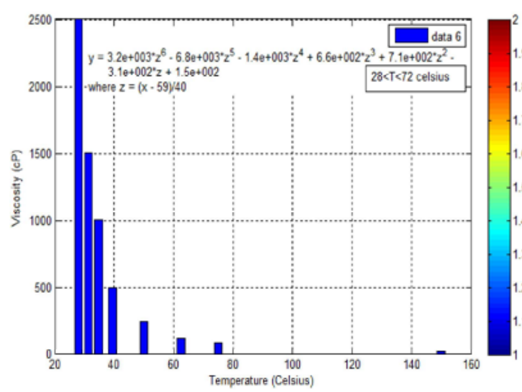

Fig. 5: Behavior of pure epoxy viscosity at various temperatures (28<T<72) in bar form and curve.

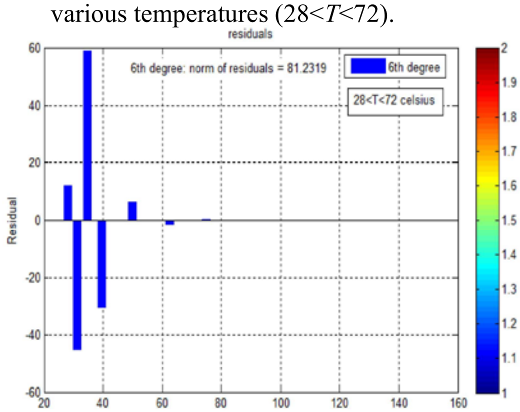

Fig. 4: Residuals of pure epoxy viscosity at various temperatures (28<T<72) in bar form and curve.

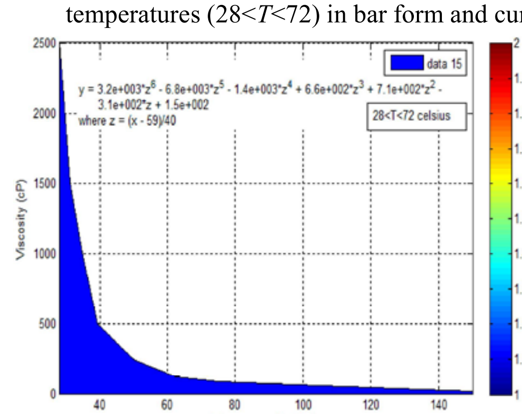

Fig. 6: Behavior of pure epoxy viscosity at various temperatures (28<T<72) in surface form and curve.

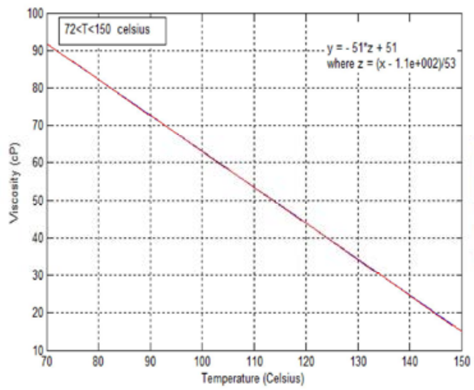

Fig. 7: Linear behavior of pure epoxy viscosity at various temperatures (28<T<150).

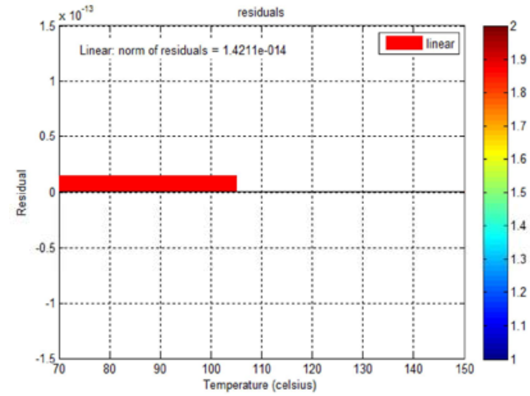

Fig. 8: Residuals of pure epoxy viscosity at various temperatures (28<T<150) in bar form and curve.

$$\mu[cP] \cong 7.6 \times 10^{-7} T^{-6} - 0.00033T^5 + 0.058T^4 - 5.2T^3 + 260T^2 - 6600T + 71000,$$
$$28 < T < 72 \tag{15}$$

$$\mu[cP] \cong = 0.96T + 160 \quad \textit{for} \quad 72 < T < 150 \; \textit{celsius} \tag{16}$$

So, the exponential form in mentioned temperatures is,

$$\mu[cP] \cong \exp(-0.0391T + 8.0264), \quad \textit{for} \quad 30 < T < 170 \; \textit{celsius} \tag{17}$$

Finally, shear stress in crept polymeric epoxy matrix is determined by substitution of Equation (6) into Equations (8-11) with considering Equations (12-14, 1-5). So, these new concept can be used to determine shear stress in crept polymer matrix.

Mentioned approach is a bridge and relation between the solid structures and mechanics of fluid, well. Also results of the exponential and polynomial functions for predicting viscosity of amorphous materials are correct approximately. General form of shear stress is given as below,

$$\tau_{rz}\big|_{crept\ spoxy\ polymeric\ matrix} = \left[\mu[cP]\right] \times \left[\frac{\partial \dot{u}}{\partial z} + \frac{\partial \dot{w}}{\partial r}\right] + \tau_{yield}\big|_{spoxy\ polymer} \tag{18}$$

For example, one of the mentioned forms is given as,

$$\tau_{rz}\big|_{crept\ spoxy\ polymeric\ matrix} = \left[exp\left(-0.0391T + 8.0264\right)\right] \times$$
$$\left[(-kra^2e^{ar} - 3ake^{ar} + \frac{c}{r^2} ln(br) + \frac{d}{r^2} - 8Gr - 3h)z\right] + \tau_{yield}\big|_{spoxy\ polymer} \tag{19}$$

In which, constants k, c, d, g, h are obtained by proper boundary conditions. As well as, a and b are arbitrary values. It should be mentioned that combination of exponential, logarithmic and polynomial functions has been assumed for obtaining radial and axial displacement rates $\dot{u}(r,z)$ and $\dot{w}(r,z)$. Equation (19) is arising from the mentioned functions.
Above figures show the results obviously.

CONCLUSIONS

In this paper, first, shear stress in crept epoxy polymeric matrix (or metal matrix at high temperatures) was determined by viscosity of amorphous polymeric and assumed axial and radial displacement rates utilizing mechanics of fluid theory.

Second, new axial and radial displacement rates (velocities) were obtained in crept matrix which these displacement rates were satisfied by boundary and incompressibility conditions.

Third, viscosity of crept amorphous epoxy polymeric matrix with respect to different temperatures was determined by polynomial and exponential functions approximately. Simulation between high viscous fluids such as pitch and crept amorphous crept epoxy polymer matrix was logical and correct. So, we can use high viscous fluids such as pitch instead of crept polymer matrix (or metal matrix at high temperatures) in creep by smart engineering vision.

Finally, the mentioned formulations can be used to determine shear stress, displacement rates (velocities) and viscosity of amorphous polymeric materials in crept polymeric matrix (or metal matrix at high temperatures) in creep phenomenon (in small or large deformations) for fibrous composites correctly.

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