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Generalized Water Supply Management over Time

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Abstract: A crucial characteristic of realistic water resources management systems is flow variation over time. Also, in many real applications flows, there are some relations, such as equality, among the amounts of flow on some subsets of arcs over time. In this paper, we consider the problem of water resources management on generalized networks where delay values associated with each arc, limited storage of the intermediate nodes and arc capacity variations over time are taken into account. In additional, we assume that the arcs flows for some subset of arcs are linearly dependent on a reference arc over time. For such systems, we first formulate continuous-time and discrete-time models. Then, by using an auxiliary time-expanded network, a solvable model is introduced, which is equivalent to the main model. Finally, a numerical example is used to illustrate the applicability of the approach. We also discuss how adding storage capability at intermediate nodes reduces the overall cost of the optimal solution.

Key words: Water supply systems % Network optimisation % Generalized network flow % Time-expanded network

INTRODUCTION

A regional water supply system can be included several components, such as water sources, demand centres and transfer and storage commodities. Water resource management problems can be formulated using network models. The supply nodes represent surface water storage, ground water storage and rainfalls capacity nodes. The demand nodes describe the evaporation and irrigation, urban and industrial. The arcs correspond to rivers, channels, pipes, ditches and inter-basin transfers. An example of the design case is illustrated in the simple system shown in Fig. 1, which has two reservoirs and demands. In order to obtain a preliminary assessment for resource allocation within the water supply system, we can use minimum cost flow (MCF) algorithms and obtain overall optimization cost. There are several works in literature which deal with MCF algorithms to water supply optimisation problems [1-3].

In general terms, the water supply system analysis must be extended to a dynamic multi-period network to support the functionality of the system components with a finite time horizon [4]. Each node in the multi-period network can be associated with positive or negative values to represent water input and output in the system respectively. Flow transfer cost and bounds representing capacity or functional constraints can be associated with each arc in each period. In other hand, in the standard MCN for the water supply system analysis, the main constraints are only the flow conservatives for each node. That is, the amount of flow on any arc that leaves its tail node equals the amount of flow that arrives at its head node. However in the practical applications, this conservation assumption is violated with gain/loss flow on arc. In such cases, a basic generalized network flow model is considered, in which a positive multiplier, μ_{ij}, is associated to each arc (i, j), so that each unit of the flow leaving node i is changed by μ_{ij} unit when it reaches node j. Moreover, MCN structure is lost when side constraints are added to the problem. One group of such important side constraints are relations between some arcs flow in network. First ALi. *et al*. [5] studied the so called equal flow problem in which given pairs of arcs are required to have identical flow. For this problem, they developed an algorithm, which uses relaxation and decomposition techniques. An integer version of the equal flow problem (where arc flows must be integer) studied by Ali *et al*. is the NP-complete problem. Ahuja *et al*. [6] introduced the

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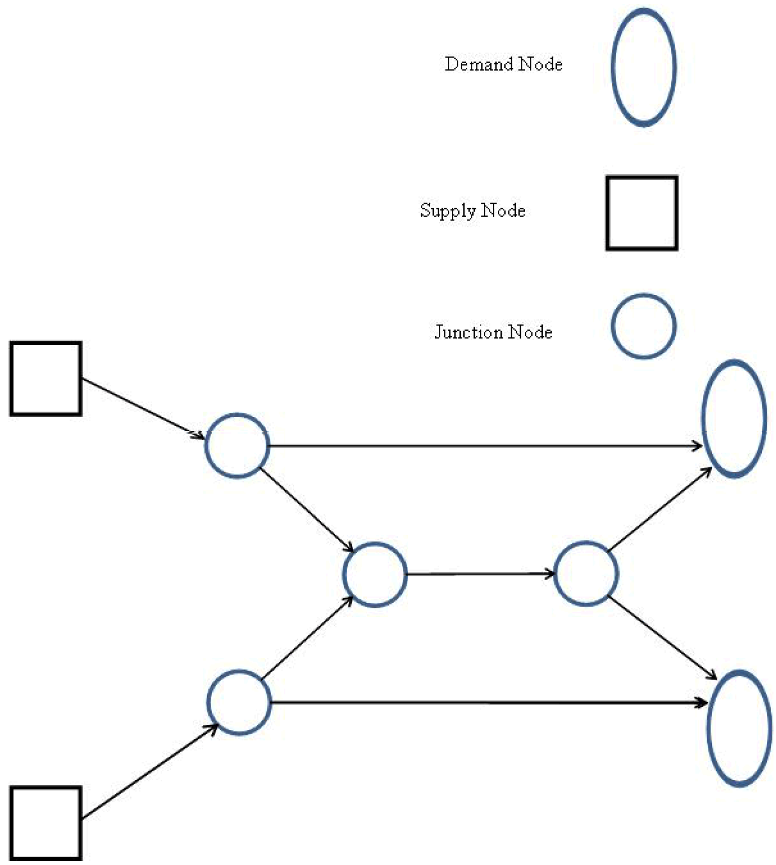

Fig. 1: An example of the simple system shown

so called simple equal flow problem in which one subset of arcs is required to carry the same amount of flow. This model is an extension of Ali's model to a subset of arcs, instead of some certain pairs; and for this problem, they propose two different algorithmic approaches. As an extension of Ahuja's model, Calvete [7] considered a more extended version of the equal flow problem in his model in which there are several subsets of arcs requiring the flow of arcs in some given sets of arcs to take on the same value. He developed a simplex primal algorithm that exploits the network structure of the problem and requires only slight modifications of the network simplex algorithm. Salehifathabadi and Raayatpanah [8] argued that the problem of simple linear flow relations in minimum cost network flow has been solved for only standard network flows.

In this paper, we consider a dynamic multi-period water supply system with a time horizon on generalized networks, where the flow values on some arcs are linearly dependent on the flow value on some other reference arcs. In addition, the effect of limited storage in intermediate nodes is taken into account. However, in reality, flows are delayed while transmitted along each arc and the output flows do not leave a node at the same time that flows are arriving at the same node. Needless to say, such assumptions affect the overall performance analysis of a given network. Another realistic characteristic of realistic networks is their variation over time. In fact, important characteristics of real-world networks such as arc costs and capacities are often subject to fluctuations over time.

The rest of this paper is organized as follows. In Section 2, the system model for the water supply system is presented. Section 3 extends the problem to the discrete-time network model. Simulation results are presented in Section 4. Finally, Section 5 concludes the paper.

Problem Definition and Modelling: The multi-period water supply system is represented by a directed graph $G = (V, A, D)$, where V is the set of nodes, $|V| = n$, A is the set of directed arcs, $|V| = m$ and D is the integer time horizon over which network behavior at the destination is observed. The time horizon D is chosen such that all source flow units have arrived at the destination by the end of this period. Each directed arc $e \in A$ is associated with four integer parameters: transit time d_e, transit cost c_e, a positive multiplier μ_e, that denotes gain/loss flow on arc e and a positive integer capacity at time p denoted by $u_e(p)$. As d_e denotes the transit time over arc e, if $x_e(p)$ is the rate of flow entering arc e at time p, the flow $x_e(p)$ arrives at tail (e) at time $p+d_e$. Throughout the paper, we assume that transit time of packets over each arc is an integer value. The integer restriction on the delay does impose some loss of generality because the integer delay solution might not be as accurate as a solution based on continuous delay values. However, we can obtain an integer solution as close as desired to an optimal continuous solution by scaling the time axis properly (i.e., by multiplying all time indexes such as d_e by an integer value M to obtain a higher accuracy value of Md_e, for a proper choice of M, for each arc e).

Each node $i \in N$ is associated with a number $b''(i)$ representing its supply/demand. If $b''(i)>0$, node i is a supply node; if $b''(i)<0$, node i is a demand node; and if $b''(i) = 0$, node i is a transshipment node. Let SN, DN and MN be sets of source, demand, intermediate nodes, respectively.

For the arc $e = (i,j)$ we write head(e) := j and tail(e) := i. For a node $i \in V$, the terms d_i^+ and d_i^- denote the set of arcs leaving node i (tail(e) = i) and entering node i (head(e) = i), respectively.

The linear, separable cost c_e denotes the cost of sending unit flow $x_e(p)$ over arc $e \in A$. Therefore, the total cost is given by $\sum_{e\in A} c_e \int_0^D z_e(x)dx$. The capacity $u_e(p)$ is an upper bound on the rate of flows entering arc e at time p:

$$u_e(p) \geq x_e(p)$$

It should be noted that the conservation constraints for the case of arcs with delay will be included by integration of the classical flow conservation constraints over time. As storage of flow at intermediate nodes is allowed in our model, the flow entering a node can be stored at that node for a given time before it is sent out. Assuming b_i denotes amount of storage for $i \in MN$, $p \in [0, D]$, storing constraints at each node can be written as follows:

$$\int_0^p \Big(\sum_{e\in d_i^+} x_e^{(k)}(x) - \sum_{e\in d_i^-} x_e^{(k)}(x-d_e)\Big)dx \le s_i^{(k)} \tag{1}$$

Consequently, for $p=D$ we have the equality:

$$\int_0^D \Big(\sum_{e\in d_i^+} x_e(x) - \sum_{e\in d_i^-} x_e(x-d_e)\Big)dx = s_i \forall i \in N,$$

where

$$s_i = \begin{cases} b''(i) & i \in SN \\ -b''(i) & i \in DN \\ b_i & i \in MN \end{cases}$$

Assume that $R \subseteq A$ is a specified set of arcs in which the amount of flow on its arcs linearly depends on the amount of flow on the arc (p,q) in the set $p \in [0, D]$ in each period, this arc is called the reference arc, i.e.,

$$R = \{(i,j) \mid x_{ij}(p) = a_{ij} x_{pq}(p) + l_{ij}, (i,j) \in A, p \in [o, D)\},$$

where α_{ij} and a_{ij} are constants related to each arc $e = (i,j)$ in the set R. It is assumed $\alpha_{ij} \neq 0$. The simple equal flow problem is considered as a special case in a period, when the flow on arcs are equal i.e., $\alpha_{ij} = 1$ and $\lambda_{ij} = 0$, $\forall (i, j) \in A$.

Therefore, the generalized minimum cost flow with simple linear flow relations can be formulated as follows:

$$\min \sum_{e\in A} c_e \int_0^D x_e(x)dx \tag{2}$$

$$s,t \tag{3}$$

$$\int_0^p \Big(\sum_{e\in d_i^+} x_e(x) - \sum_{e\in d_i^-} x_e(x-d_e)\Big)dx \le s_i \ \forall i \in N, \forall p \in [0, D] \tag{4}$$

$$\int_0^D \Big(\sum_{e\in d_i^+} x_e(x) - \sum_{e\in d_i^-} x_e(x-d_e)\Big)dx = s_i \forall i \in N, \tag{5}$$

$$x_e(p) = a_e x_{pq}(p) + l_e, \forall e \in R, p \in [o, D] \tag{6}$$

$$0 \le x_e(p) \le u_e(p) \forall e \in A, \forall p \in [0, D] \tag{7}$$

$$x_e(p) = 0 \forall e \in A, \forall p > D \tag{8}$$

We will present a corresponding discrete-time network model G for the problem.

Discrete-Time Formulation: In the continuous time model, p can take any value in $[0, D]$. However, in discrete time models, network is observed only at time instances

p = 0, 1, 2,, D. By considering the same network parameters as before, our model in discrete time can be formulated as follows:

$$Min \sum_{e \in A} \sum_{p=0}^{D-1} c_e \bar{x}_e(p) \tag{9}$$

s.t

$$\sum_{e \in d_i^+} \sum_{p=0}^{J} \bar{x}_e(p) - \sum_{e \in d_i^-} \sum_{p=0}^{J} \bar{x}_e(p-d_e) \le s_i \forall i \in N, \forall q \in \{0,1,...,D-1\}, \tag{10}$$

$$\sum_{e \in d_i^+} \sum_{p=0}^{D} \bar{x}_e(p) - \sum_{e \in d_i^-} \sum_{p=0}^{D} \bar{x}_e(p-d_e) = s_i \forall i \in N, \tag{11}$$

$$x_e(p) = a_e x_{pq}(p) + l_e, \forall e \in R, \forall p \in \{0,1,...,D-1\}, \tag{12}$$

$$\bar{x}_e(p) \le \bar{u}_e(p) \forall e \in A, \forall p \in \{0,1,...,D-1\}, \tag{13}$$

$$\bar{x}_e(p) = 0 \forall p > D-1, \forall e \in A \tag{14}$$

where for the discrete-time version of G, the rate of flow sent into arc e during the interval [p, p+1], denoted by $\bar{x}_e^{(k)}(p)$, is equal to $\bar{x}_e^{(k)}(p) = \int_p^{p+1} x_e^{(k)}(x)dx$ for each p , {0, 1, 2,, D}. In addition, the discrete-time capacity $\bar{u}_e(p)$ is defined as follows:

$$\bar{u}_e(p) = \int_p^{p+1} u_e(x)dx$$

In order to simplify the problem, the usual approach for deriving practical algorithms for a continuous-time coded network problem is to reduce it to a discrete time one. As mentioned earlier, the approximation error can be reduced by choosing a smaller discrete time step at the cost of additional complexity.

Lemma 1: By transforming the continuous variables into their corresponding discrete variables (as given above), the corresponding discrete time model in (3) is obtained that satisfy the corresponding constraints.

Proof: Suppose $x_e(p)$ satisfy the constraints in model (2) for the arc capacities $u_e(p)$, e , A, p , [0, D]. For every integral time step p , {0,1,2,...,D–1} and time horizon D, $\bar{x}_e(p)$ can be bounded as follows:

$$\bar{x}_e(p) = \int_p^{p+1} x_e(x)dx \le \int_p^{p+1} u_e(x)dx = \bar{u}_e(p)$$

It is easy to verify that flow conservation constraints still hold. Let 2 , {0,1,2,...,D–1}, then for each i , N, we get:

$$\sum_{e \in d_i^+} \sum_{p=0}^{J} \bar{x}_e(p) - \sum_{e \in d_i^-} \sum_{p=0}^{J} \bar{x}_e(p-d_e) =$$
$$\sum_{e \in d_i^+} \sum_{p=0}^{J} \int_p^{p+1} x_e(x)dx - \sum_{e \in d_i^-} \sum_{p=0}^{J} \int_p^{p+1} x_e(x-d_e)dx =$$
$$\sum_{e \in d_i^+} \int_0^J x_e(x)dx - \sum_{e \in d_i^-} \int_0^J x_e(x-d_e)dx =$$
$$\int_0^J \Big(\sum_{e \in d_i^+} x_e(x) - \sum_{e \in d_i^-} x_e(x-d_e) \Big) dx \le s_i$$

and for 2=D, the above equation will be satisfied as an equality. Therefore, regarding the objective function we get:

$$\sum_{e \in A} \sum_{p=0}^{D-1} c_e \bar{x}_e(p) = \sum_{e \in A} \sum_{p=0}^{D-1} c_e \int_p^{p+1} x_e(x)dx = \sum_{e \in A} c_e \int_0^D x_e(x)dx.$$

Conversely, let $\bar{x}_e(p)$ satisfy the constraints in model (3) for the arc capacities $\bar{u}_e(p)$. We define $x_e(q) = \bar{x}_e(p)$ and capacity $u_e(q) = \bar{u}_e(p)$ for $q \in [p, p+1]$. It is obvious that $x_e(p)$ satisfy the constraints in model (2) for the arc capacities $u_e(p)$. P

In summary, the above Lemma shows that every continuous-time problem can be transformed to a corresponding discrete-time one.

It should be noted that the model given in (3) although is a discrete time model, is still not a static model to be solved in polynomial time. Hence, our proposed algorithm is based on reduction of this problem to a static time-expanded problem that can be solved in polynomial time with respect to the time-expanded network.

Time-Expanded Network: We now present a method for solving model (3). In order to solve the above formulated problem, we propose an approach based on the time-expanded network proposed. In this paper, our goal is to obtain the optimal solution by considering the discrete time model in which delay times will take integer values and all are bounded in the time horizon D.

The time-expanded version of network G is a digraph $G^D = (V^D, A^D)$, wherein there is a copy of the nodes for each time step in the time horizon $\{0,1,,D\}$. The formal definitions are as follows:

$$V^D = \left\{ v_p \mid v \in V, p = 0,1,.....,D \right\}$$

$$A' = \begin{Bmatrix} e_p = (v_p, u_{p+d_e}) \mid e = (u,v) \in A; p = 0,1,.....,D - \\ d_e; v_p, u_{p+d_e} \in V^D \end{Bmatrix}$$

$$A'' = \left\{ (v_p, v_{p+1}) \mid v_p, v_{p+1} \in V^D; p = 0,1,.....,D-1; v \in MN \right\}$$

$$A^D = A' \bigcup A''$$

Note that for a network G^D, $|V^D| = D|V|$ and:

$$\left|A^D\right| = \sum_{e \in A} (D - d_e + 1) + D|V| = (|V| + |A|)D + |A| - \sum_{e \in A} (d_e)$$

Since the maximum amount of storage at node v is denoted by b_v, at each time instant at most b_v units of information can be stored in that node. Consequently, in the equivalent time-expanded network, the same amount of information is transformed from v_p to v_{p+1}. Therefore, the capacity of the arc connecting nodes v_p and v_{p+1} in the time-expanded network will be equal to b_v.

Consequently, finding an optimal sub-graph in a network with delays can be solved by finding a sub-graph optimal in the time-expanded graph.

$$Min \sum_{e_p \in A^D} \sum_{p=0}^{D-d_e} c_e x_{e_p} \tag{15}$$

s.t

$$\sum_{e \in d_{i_p}^+} x_{e_p} - \sum_{e \in d_{i_p}^-} x_{e_p} = s_i, \forall i_p \in N^D, \tag{16}$$

$$x_{e_p} \le u_{e_p} \forall e_p \in A^D, \; p \in \{0,1,...,D\} \tag{17}$$

$$x_{e_p} = a_e x_{pq_p} + l_e, \forall e \in R, \forall p \in \{0,1,...,D\}, \tag{18}$$

$$x_{e_p} \ge 0 \forall e_p \in A^D, \; p \in \{0,1,...,D\}, \tag{19}$$

where x_{e_p} and u_{e_p} indicate the rate of flow on arc $e_p = (u_p, v_{p+d_e})$ and arc capacities, respectively. It is easy to verify that models (3) and (3.1) will be equivalent by setting $z_{e_p} = z_e(p),\; x_{e_p}^{(k)} = x_e^{(k)}(p) \text{ and } u_{e_p} = u_e(p)$ for each $e_p \in A^D, \; p \in \{0,1,...,D-d_e\}$.

Simulation Results: In this section, the computational results of simulations undertaken are presented in order to evaluate the performance of the proposed technique. we assess the proposed method on random networks. Firstly, we describe how to generate the random test instances of the problem and then the computational experiments and associated results are presented. The random directed graphs were generated using to the methodology proposed by Erdos and Renyi [9]. In this case, it was assumed that there is a arc from node i to node j with probability 0.5. Then, for each arc, a uniform random number was generated on an interval, say [a, b], in order to represent the arc cost. In experiments conducted in this case, we selected [a, b] equal to [1, 2]. The capacity of each arc and storage of each node in the network is one unit. Moreover, arc delays were assigned randomly and the source and demand nodes were randomly chosen as well.

Considering network variations, the simulations are performed for the following three scenarios as well. In the first case, no arc delay and storage were assumed. In fact, the system was considered in a period. In the second case, storage and arc costs was included, however assumed that their values do not change over time. In the third case, arc costs were assumed to be constant but storage vary over time (randomly chosen from a uniform distribution in the interval three to ten). In second and third cases, time horizon was changed to evaluate of performance system over time. The results are shown in Table I. The results shown that (i) For small values of time horizon, sending data through low-delay paths may lead to higher cost results. In addition, if time horizons are assigned to very small values, it is possible to encounter scenarios in which finding an optimal sub-graph is impossible. (ii) By increasing the time horizon, the proposed algorithm has more flexibility in choosing lower cost solutions. (iii) Increasing the storage of nodes can reduce the overall cost, as intermediate nodes have the flexibility to store incoming packets in their storage when their lower-cost output arcs are occupied. Such nodes can then send out the stored packet at a later time instant at which the given lower-cost arcs become available again.

Table 1: Overall network cost for the cases 1, 2 and 3

	First case		Second case		Third case		
Original network size	D=1	D=4	D=6	D=8	D=4	D=6	D=8
12 nodes 35 arcs	37.67	79.107	169.515	237.321	94.9284	135.612	379.7136
20 nodes 90 arcs	64.86	181.608	243.225	369.702	163.4472	340.515	480.6126
28 nodes 138 arcs	96.25	269.5	409.0625	529.375	377.3	449.9688	487.025

CONCLUSION

In this paper, we addressed the problem of water supply system management in generalized networks with arc delays, storing capability at intermediate nodes and side constraints. The side constraints represent linear dependency of flow on arcs in some subset of arcs to the flow on a reference arc in the network. Having formulated the problem of continuous delay network, we described the relationship between continuous and discrete models in generalized networks. The derived discrete model was then converted to an time-expanded network which can be solved through well-known algorithms. As shown by the simulation results, inclusion of node storing in our analysis my lead to significant cost reduction. This is due to the fact that with the aid of storing, intermediate nodes can adopt a more opportunistic transmission scheme over time.

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REFERENCES

1. Kuczera, G., 1992. Network linear programming codes for water-supply networks modeling. ASCE J Water Resour Plan Manage, 45(10): 365-382.
2. Podda, A., G.M. Sechi, P. Susnik and P. Zuddas, 1994. Network algorithms for water resource systems optimization. In: Proceeding of the Hydrosoft, 94: 1:81-88.
3. Sechi, G.M. and P. Zuddas, 1995. A large scale water resources network optimization algorithm. In In: Proceedings of the international conference on optimization ICOTA'95, 1: 378-385, Chengdu, China.
4. Labadie, J.W., 2004. Optimal operation of multireservoir systems: State-of-the-art review. Journal of water resources planning and Management, 130(2): 93-111.
5. Ali, A.I., J.L. Kennington and B. Shetti, 1988. The equal flow problem. European Journal of Operational Research, 36: 107-115.
6. Ahuja, R.K., J.B. Orlin, G.M. Sechi and P. Zuddas, 1999. Algorithms for the simple equal flow problem. Management Science, 45(10): 1440-1455.
7. Calvete. H.I., 2003. Network simplex algorithm for the general equal flow problem. European Journal of Operational Research, 150(3): 585-600.
8. Salehifathabadi, H. and M.A. Raayatpanah, 2007. Minimum cost network flow problem with additional linear relation between arcs'flow. Amirkabir Journal, 2: 91-98.
9. Bollobas. B., 2001. Random Graphs, 2nd ed. Cambridge University Press, second edition.