

Prediction of Bias-Ply Tire Deflection Based on Overall Unloaded Diameter, Inflation Pressure and Vertical Load

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Abstract: This study was conducted to predict deflection (δ) of bias-ply tire based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W). For this purpose, deflection of four bias-ply tires with different overall unloaded diameters was measured at five levels of inflation pressure and five levels of vertical load. Results of deflection measurement for bias-ply tires No. 1, 2 and 3 were utilized to determine regression model and three-variable linear regression model $\delta_p = 45.67 - 0.020 d - 0.905 P + 3.534 W$ with $R^2 = 0.981$ was obtained. Also, results of deflection measurement for bias-ply tire No. 4 were used to verify model. The paired samples t-test results indicated that the deflection values predicted by model were more than the deflection values measured by test apparatus. To check the discrepancies between the deflection values predicted by model with the deflection values measured by test apparatus, RMSE and MRPD were calculated. The amounts of RMSE and MRPD were 8.0 mm and 25.1%, respectively. Corrigible amounts of RMSE and MRPD confirmed that the three-variable linear regression model may be used to predict deflection of bias-ply tire based on overall unloaded diameter, inflation pressure and vertical load. On the other hand, to calculate actual deflection values or deflection values measured by test apparatus (δ_M) based on deflection values predicted by model (δ_p) the linear regression model $\delta_M = 0.654 \delta_p + 4.554$ with $R^2 = 0.986$ can be strongly recommended.

Key words: Bias-ply tire • Deflection • Overall unloaded diameter • Inflation pressure • Vertical load • Prediction

INTRODUCTION

A rule of thumb which can be used for estimation of tire contact area is shown by equation 1 [1]:

$$A = bL \quad (1)$$

Where:

- A = Contact area (m²)
- b = Section width (m)
- L = Contact length (m)

Wong [2] and Bekker [3] gave an approximate method for calculating contact length as equation 2:

$$L = 2(d\delta - \delta^2)^{0.5} \quad (2)$$

Where:

- d = Overall unloaded diameter (m)
- δ = Deflection (m)

Deflection is a key parameter and many equations have been developed based on it to evaluate the tractive performance of bias-ply and radial-ply tires operating in cohesive-frictional soils. Gross traction, motion resistance, net traction and tractive efficiency are predicted as a function of soil strength, tire load, tire slip, tire size and tire deflection [4]. The most widely used dimensional analysis approach for predicting off-road traction makes use of the following ratios [4-6]:

$$C_n = \frac{Cl \cdot b \cdot d}{W} \quad (3)$$

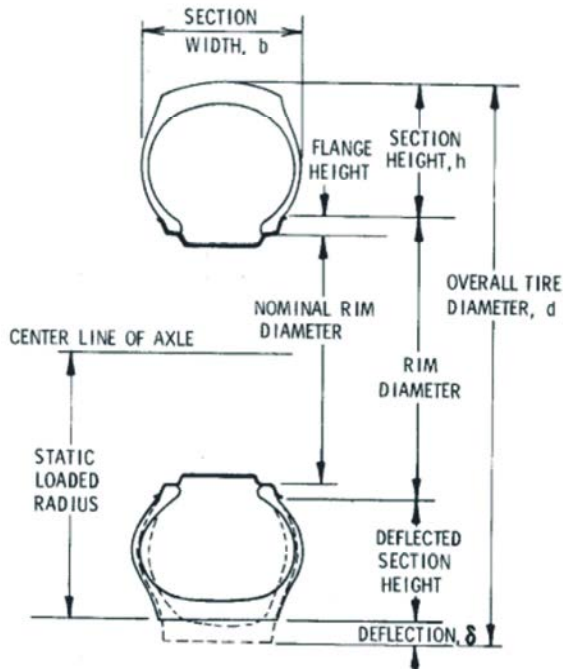


Fig. 1: Tire dimensions, adapted from Brixius [4]

$$WD = \frac{b}{d} \quad (4)$$

$$WD = \frac{b}{d} \quad (5)$$

Where:

- C_n = Wheel numeric (dimensionless)
- CI = Cone index (kPa or kNm^{-2})
- W = Vertical load (kN)
- WD = Section width to overall unloaded diameter ratio (dimensionless)
- DR = Deflection ratio (dimensionless)
- h = Section height (m)

Fig. 1 shows the tire dimensions (b, d, δ and h) used. The tire dimensions can be obtained from tire data book or by measuring the tire [4]. The section width (b) is the first number in a tire size designation (i.e. nominally 18.4 inches for an 18.4-38 tire). The overall unloaded diameter (d) can be obtained from the tire data handbooks available from off-road tire manufacturers. The tire deflection (δ) on a hard surface is equal to $d/2$ minus the measured static loaded radius. The static loaded radius for the tire's rated load and inflation pressure is also standard tire data from the tire data handbooks. It can also be obtained by

measuring the tire. The section height (h) is equal to half the difference between the overall unloaded diameter and the rim diameter. The rim diameter can in turn be estimated by adding 50 mm to the nominal rim diameter, which is the second number in a tire size designation, i.e. 38 inches for an 18.4-38 tire [4, 5].

To further simplify the prediction equations, Brixius [4] combined above three dimensionless ratios into a single product termed the mobility number, which is given by equation 6 [5-7]:

$$B_n = \frac{CI \cdot b \cdot d}{W} \left(\frac{1 + 5 \frac{\delta}{h}}{1 + 3 \frac{b}{d}} \right) \quad (6)$$

Where:

B_n = Mobility number (dimensionless)

The empirical model developed by Brixius [4] is widely used for prediction of off-road tire performance. It has also been adopted in ASAE standard D497.4 [8] for predicting tractor performance. In this model, soil condition is represented by the cone index value, which is the average force per unit area required to force a cone-shaped probe vertically into the soil at a steady rate. The average before-traffic cone index for the top 150 mm layer of soil is used in the prediction equations [5, 7]. ASAE standards S313.3 [9] and EP542 [10] describe the soil cone penetrometer and procedures for its use. An average of several cone index values obtained at a test site often yields a representative measure of soil strength [11].

As deflections for a given tire size, inflation pressure and vertical load are significantly different between bias-ply and radial-ply tires [4], this study was conducted to predict deflection (δ) of bias-ply tire based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W).

MATERIALS AND METHODS

Tire Deflection Test Apparatus: A tire deflection test apparatus (Fig. 2) was designed and constructed to measure deflection of tires with different sizes at diverse levels of inflation pressure and vertical load. As deflection on a hard surface is equal to $d/2$ minus the measured static loaded radius [4, 5], the static loaded radius was obtained by measuring as shown in Fig. 3.



Fig. 2: Tire deflection test apparatus



Fig. 3: Measuring static loaded radius

Experimental Procedure: Deflection of four bias-ply tires with different overall unloaded diameters was measured at five levels of inflation pressure and five levels of vertical load. The dimensions of four bias-ply tires are given in Table 1. Results of deflection measurement for bias-ply tires No. 1, 2 and 3 (Tables 2, 3 and 4) were utilized to determine regression model and results of deflection measurement for bias-ply tire No. 4 (Table 5) were used to verify model.

Regression Model: A typical three-variable linear regression model is shown in equation 7:

$$Y = C_0 + C_1X_1 + C_2X_2 + C_3X_3 \quad (7)$$

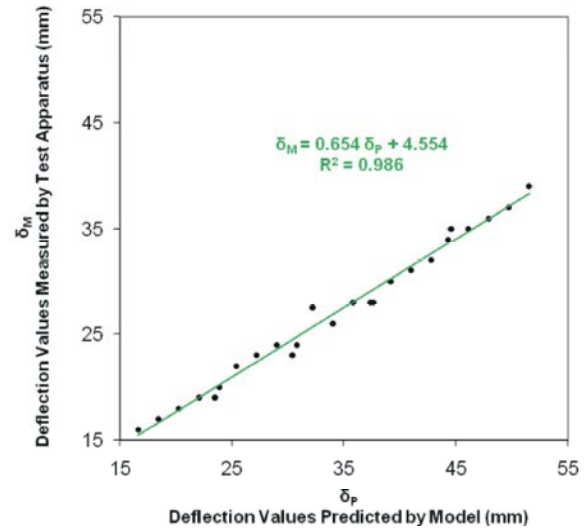


Fig. 4: Curve of deflection values measured by test apparatus (δ_M) based on deflection values predicted by three-variable linear regression model (δ_p) for bias-ply tire No. 4

Where:

Y = Dependent variable, for example deflection of bias-ply tire

X_1, X_2, X_3 = Independent variables, for example overall unloaded diameter, inflation pressure and vertical load, respectively

C_0, C_1, C_2, C_3 = Regression coefficients

In order to predict deflection of bias-ply tire from overall unloaded diameter, inflation pressure and vertical load, a three-variable linear regression model was suggested and all the data were subjected to regression analysis using the Microsoft Excel 2007.

Statistical Analysis: A paired samples t-test was used to compare the deflection values predicted by model with the deflection values measured by test apparatus. Also, to check the discrepancies between the deflection values predicted by model with the deflection values measured by test apparatus, root mean squared error (RSME) and mean relative percentage deviation (MRPD) were calculated using the equations 8 and 9, respectively [12-19]:

Table 1: Dimensions of the four bias-ply tires used in this study

Tire No.	Tire size designation	Overall unloaded diameter d (mm)
1	5.50-13	585
2	6.50-14	690
3	6.00-16	725
4	7.50-16	770

Table 2: Overall unloaded diameter, inflation pressure, vertical load and deflection for bias-ply tire No. 11

Tire No.	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
1	585	30	5.8690	24.0
			7.8250	33.0
			9.7810	41.0
			11.738	48.0
		32	13.694	60.0
			5.8690	24.0
			7.8250	31.0
			9.7810	40.0
		34	11.738	47.0
			13.694	53.0
			5.8690	23.0
			7.8250	30.5
		36	9.7810	38.0
			11.738	45.5
			13.694	51.0
			5.8690	23.0
		38	7.8250	29.0
			9.7810	35.0
			11.738	41.0
			13.694	49.0
			5.8690	20.0
			7.8250	29.0
			9.7810	36.0
			11.738	42.0
	13.694	50.0		

Table 3: Overall unloaded diameter, inflation pressure, vertical load and deflection for bias-ply tire No. 2

Tire No.	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
2	690	30	5.8690	24.0
			7.8250	31.0
			9.7810	38.0
			11.738	45.0
		32	13.694	52.0
			5.8690	24.0
			7.8250	30.0
			9.7810	37.0
		34	11.738	43.0
			13.694	49.5
			5.8690	22.0
			7.8250	28.0
		36	9.7810	35.0
			11.738	41.0
			13.694	47.0
			5.8690	20.0
		38	7.8250	27.0
			9.7810	32.0
			11.738	39.0
			13.694	46.0
			5.8690	20.0
			7.8250	27.0
			9.7810	31.0
			11.738	37.0
	13.694	42.0		

Table 4: Overall unloaded diameter, inflation pressure, vertical load and deflection for bias-ply tire No. 3

Tire No.	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
3	725	30	5.8690	23.0
			7.8250	32.0
			9.7810	40.0
			11.738	47.0
			13.694	54.0
		32	5.8690	23.0
			7.8250	31.0
			9.7810	37.0
			11.738	46.0
			13.694	52.0
		34	5.8690	23.0
			7.8250	28.0
			9.7810	35.0
			11.738	43.0
			13.694	47.5
		36	5.8690	19.0
			7.8250	27.0
			9.7810	33.0
			11.738	41.0
			13.694	47.0
38	5.8690	17.0		
	7.8250	24.0		
	9.7810	29.0		
	11.738	37.0		
	13.694	45.0		

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (\delta_{Mi} - \delta_{Pi})^2}{n}} \quad (8)$$

Where:

RMSE = Root mean squared error (mm)

δ_{Mi} = Deflection measured by tire deflection test apparatus (mm)

δ_{Pi} = Deflection predicted by three-variable linear regression model (mm)

$$MRPD = \frac{100 \times \sum_{i=1}^n \frac{|\delta_{Mi} - \delta_{Pi}|}{\delta_{Mi}}}{n} \quad (9)$$

Where:

MRPD = Mean relative percentage deviation, %

RESULTS AND DISCUSSION

Three-variable linear regression model, p-value of independent variables and coefficient of determination (R^2) of the model are shown in Table 6. In this model

deflection of bias-ply tire can be predicted as a function of overall unloaded diameter (d), inflation pressure (P) and vertical load (W). The p-value of independent variables (d, P and W) and R^2 of the model were 6.35E-10, 7.57E-24, 2.30E-61 and 0.981, respectively. Based on the statistical results, the three-variable linear regression model was initially accepted, which is given by equation 10:

$$\delta_p = 45.67 - 0.020 d - 0.905 P + 3.534 W \quad (10)$$

Deflection of bias-ply tire No. 4 was then predicted at five levels of inflation pressure and five levels of vertical load using the three-variable linear regression model. The deflection values predicted by model were compared with the deflection values measured by test apparatus and are shown in Table 7. The paired samples t-test results indicated that the deflection values predicted by model were more than the deflection values measured by test apparatus. The average deflection difference between two methods was 7.22 mm (95% confidence interval for difference in means: 5.71 mm and 8.72 mm; p-value = 1.0000). The standard deviation of the deflection difference was 3.65 mm (Table 8). To check the discrepancies between the deflection values predicted by model with the deflection values measured by test apparatus, RMSE and MRPD were calculated.

Table 5: Overall unloaded diameter, inflation pressure, vertical load and deflection for bias-ply tire No. 4

Tire No.	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
4	770	30	5.8690	20.0
			7.8250	24.0
			9.7810	28.0
			11.738	35.0
			13.694	39.0
			5.8690	19.0
		32	7.8250	24.0
			9.7810	28.0
			11.738	32.0
			13.694	37.0
			5.8690	18.0
			7.8250	23.0
		34	9.7810	26.0
			11.738	31.0
			13.694	36.0
			5.8690	17.0
			7.8250	22.0
			9.7810	27.5
		36	11.738	30.0
			13.694	35.0
			5.8690	16.0
			7.8250	19.0
			9.7810	23.0
			11.738	28.0
38	13.694	34.0		
	5.8690	16.0		
	7.8250	19.0		
	9.7810	23.0		
	11.738	28.0		
	13.694	34.0		

Table 6: Three-variable linear regression model, p-value of independent variables and coefficient of determination (R^2)

Model	p-value			R^2
	d	P	W	
$\delta = 45.67 - 0.020 d - 0.905 P + 3.534 W$	6.35E-10	7.57E-24	2.30E-61	0.981

Table 7: Overall unloaded diameter, inflation pressure, vertical load and deflection for bias-ply tire No. 4 used in evaluating three-variable linear regression model

Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)	
			Measured by test apparatus	Predicted by model
770	30	5.8690	20.0	23.9
		7.8250	24.0	30.8
		9.7810	28.0	37.7
		11.738	35.0	44.6
		13.694	39.0	51.5
		5.8690	19.0	22.1
	32	7.8250	24.0	29.0
		9.7810	28.0	35.9
		11.738	32.0	42.8
		13.694	37.0	49.7
		5.8690	18.0	20.2
		7.8250	23.0	27.2
	34	9.7810	26.0	34.1
		11.738	31.0	41.0
		13.694	36.0	47.9
		5.8690	17.0	18.4
		7.8250	22.0	25.3
		9.7810	27.5	32.3
	36	11.738	30.0	39.2
		13.694	35.0	46.1
		5.8690	16.0	16.6
		7.8250	19.0	23.5
		9.7810	23.0	30.4
		11.738	28.0	37.4
38	13.694	34.0	44.3	

Table 8: Paired samples t-test analyses on comparing deflection determination methods

Determination methods	Average difference (mm)	Standard deviation of difference (mm)	p-value	95% confidence intervals for the difference in means (mm)
Test apparatus vs. model	7.22	3.65	1.0000	5.71, 8.72

The amounts of RMSE and MRPD were only 8.0 mm and 25.1%, respectively. Corrigible amounts of RMSE and MRPD confirmed that the three-variable linear regression model $\delta_p = 45.67 - 0.020 d - 0.905 P + 3.534 W$ with $R^2 = 0.981$ may be used to predict deflection of bias-ply tire based on overall unloaded diameter, inflation pressure and vertical load. On the other hand, as it is indicated in Fig. 4, our attempts to relate deflection values predicted by three-variable linear regression model (δ_p) to deflection values measured by test apparatus (δ_M) using a linear equation resulted in very good agreements ($R^2 = 0.986$) as equation 11:

$$\delta_M = 0.654 \delta_p + 4.554 \quad (11)$$

It means that actual or measured deflection (δ_M) can be computed in two steps. At first step predicted deflection (δ_p) can be calculated based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W) using the three-variable linear regression model, i.e. equation 10. Second step is calculating actual or measured deflection (δ_M) based on predicted deflection (δ_p) using the linear model, i.e. equation 11.

CONCLUSIONS

It can be concluded that actual or measured deflection (δ_M) of bias-ply tire can be computed in two easy steps. At first step, predicted deflection (δ_p) can be calculated based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W) using the three-variable linear regression model $\delta_p = 45.67 - 0.020 d - 0.905 P + 3.534 W$ with $R^2 = 0.981$. Second step is calculating actual or measured deflection (δ_M) based on predicted deflection (δ_p) using the linear equation $\delta_M = 0.654 \delta_p + 4.554$ with $R^2 = 0.986$.

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