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The Multistage Adomian Decomposition Method for Solving Chaotic Lü System

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Abstract: In this paper, a numerical scheme based on adaption of standard Adomian Decomposition Method (ADM) is applied to the chaotic Lü system. Then, the standard ADM is converted into a hybrid numeric-analytic method called the multistage ADM (MADM). Numerical comparisons with the standard ADM and the fourth-order Runge-Kutta method (RK4) is made in order to prove that MADM is the reliable method for nonlinear problems.

Key words: Standard Adomian Decomposition Method . Multistage Adomian Decomposition Method . Fourth-order Runge-Kutta . Chaotic Lü system

INTRODUCTION

An enormous research work has been devoted to the study of chaos nonlinear phenomena for the past three decades. Dynamical systems that exhibit chaotic behaviour are very sensitive to the initial conditions. Due to the complexity of the chaotic system; it is difficult to be solved using existing numerical solutions. The present work is motivated by the desire to obtain numerical solutions to the chaotic Lu system using Adomian decomposition method (ADM). In 2002, Lu and Chen constructed the following chaotic system [1]

$$\dot{x} = a(y - x) \tag{1}$$

$$\dot{y} = -xz + cy \tag{2}$$

$$\dot{z} = xy - bz \tag{3}$$

where x,y and z are state variables and a,b,and c are positive parameters. This system exhibits chaotic behaviour when $a = 36$, $b = 3$ and $c = 20$. According to a mathematical sense defined by Vanecek and Celikovské, this system represents the transition between the Lorenz system and the Chen system [1, 2]. System (1-3) is later referred to as the Lu system. Some detailed investigations on the Lu system (1-3) can be found in [3] for numerical study of some dynamical behaviours of the system.

The ADM is proven to be a powerful method in solving various problems [4-7]. ADM has been used in solving nonlinear many chaotic systems like Lorenz, Chen and Rossler [8-11]. However, the implementation of the decomposition method mainly depends upon the calculation of Adomian polynomials for nonlinear operators [12, 13]. Although ADM is said to be powerful method to be used, but as proved by [11], it is not guaranteed to give analytical solutions valid globally in time. This can be overcome by applying ADM over successive time intervals, as first hinted in [4]. This hybrid numeric-analytic procedure of ADM, is called multistage ADM (MADM), has been widely applied to many systems including Lorenz [8], Rossler [11] and for solving nonlinear algebraic equations and boundary value problems [14-19].

In this paper, MADM is introduced to solve the chaotic LU system (1-3). The numerical comparison of MADM with standard ADM and existing method fourth-order Runge-Kutta (RK4) is made.

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SOLUTION METHOD

Following [8], we consider the general system

$$X_i' = \sum_{j=1}^{n} a_{ij} X_j + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq} X_p X_q, \quad i = 1,2,\ldots,n. \tag{4}$$

where the prime denotes differentiation with respect to time. If we denote the linear term as R_{i1} and the non-linear term as R_{i2}, then we can write the above system of equation in the operator form

$$LX_i = R_{i1} + R_{i2} \qquad i = 1,2,\ldots,n \tag{5}$$

where L is the differential operator d(.)/dt. Applying the inverse (integral) operator L^{-1} to (5) we obtain

$$X_i(t) = X_i(t^*) + L^{-1}R_{i1} + L^{-1}R_{i2} \quad i = 1,2,\ldots,n. \tag{6}$$

By assuming the general system in (4) (or equivalent to (5)) is an initial-value problem, its solution is uniquely determined via the information $x_i(t^*)(i = 1,2,\ldots,n)$. According to the ADM [5, 6], the solution $x_i(t)$ is given by the series

$$X_i(t) = \sum_{m=0}^{\infty} X_{im}(t), \qquad i = 1,2,\ldots,n \tag{7}$$

Bearing this in mind, the linear term R_{i1} then becomes

$$R_{i1} = \sum_{j=1}^{n}\sum_{m=0}^{\infty} a_{ij} X_{im} \tag{8}$$

So that $L^{-1}R_{i1}$ is given by

$$L^{-1}R_{i1} = \sum_{j=1}^{n}\sum_{m=0}^{\infty} a_{ij}\int_{t^*}^{t} X_{im}\,dt \quad i = 1,2,\ldots,n. \tag{9}$$

The non-linear term R_{i2} is decomposed as [5, 6]

$$R_{i2} = \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq}\sum_{m=0}^{\infty} A_{im,p,q} \tag{10}$$

Where the $A_{im,p,q}$'s are the so-called Adomian polynomials. In this case, it is given by the formula [5, 6]

$$A_{im,p,q} = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[M\left(\sum_{k=0}^{\infty}\lambda^k X_{kp}\sum_{k=0}^{\infty}\lambda^k X_{kq}\right)\right]_{\lambda=0} \tag{11}$$

Where M(x,y) = xy for each m = 0,1,2,… Moreover $L^{-1}R_{i2}$ is given by

$$L^{-1}R_{i2} = \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq}\sum_{m=0}^{\infty}\int_{t^*}^{t} A_{im,p,q}\,dt \tag{12}$$

Putting (7), (9) and (12) into (6) we have then for each i = 1,2,…,n

$$\sum_{m=0}^{\infty} X_{im}(t) = X_i(t^*) + \sum_{j=1}^{n}\sum_{m=0}^{\infty} a_{ij}\int_{t^*}^{t} X_{im}\,dt + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq}\sum_{m=0}^{\infty}\int_{t^*}^{t} A_{im,p,q}\,dt \tag{13}$$

Consequently, we have for each i = 1,2,…,n

$$X_{i0} = X_i(t^*) \tag{14}$$

$$X_{i1} = \sum_{j=1}^{n} a_{ij}\int_{t^*}^{t} X_{i0}\,dt + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq}\int_{t^*}^{t} A_{i0,p,q}\,dt \tag{15}$$

$$X_{i2} = \sum_{j=1}^{n} a_{ij}\int_{t^*}^{t} X_{i1}\,dt + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq}\int_{t^*}^{t} A_{i1,p,q}\,dt \tag{16}$$

$$X_{i,m+1} = \sum_{j=1}^{n} a_{ij}\int_{t^*}^{t} X_{im}\,dt + \sum_{p=1}^{n}\sum_{q=1}^{n} a_{ipq}\int_{t^*}^{t} A_{iim,p,q}\,dt \tag{17}$$

Upon calculating the polynomials (11) and integrating, one then has for all $t \geq t^*$

$$X_i(t) = \sum_{m=0}^{\infty} d_{im}\frac{(t-t^*)^m}{m!}, \qquad ,i = 1,2,\dots,n \tag{18}$$

Where the coefficients d_{im} are given by

$$d_{i0} = X_i(t) \tag{19}$$

$$d_{im} = \sum_{j=1}^{n} a_{ij} d_{j(m-1)} + (m-1)!\sum_{p=1}^{n}\sum_{q=1}^{n}\sum_{k=0}^{m-1} a_{ipq}\frac{d_{qk}}{k!}\frac{d_{p(m-k-1)}}{k!(m-k-1)}, \qquad m \geq 1 \tag{20}$$

RESULTS AND DISCUSSION

There are 3 methods used in this paper to solve the LU system, which are ADM, MADM and the well-established RK4 as the reference. The MADM algorithm discussed is coded in the computer algebra package Maple together with the Maple built-in fourth-order Runge-Kutta. The Maple environment variable Digits controlling the number of significant digits is set to 16 in all calculations done in this paper. In this paper, the system (1)-(3) is chaotic for the parameter a = 36, b = 3 and c = 20 the initial conditions are x(0) = −1, y(0) = 2 and z(0) = 6. The simulation done in this paper is between t = 0 to 5. The solutions for 15-terms ADM on LU system are obtained as following:

$$\begin{aligned} x = {} & -1.0 + 108.0t - 1116.000000\,t^2 + 14904.00000\,t^3 - 99663.00000\,t^4 + 633329.4000\,t^5 \\ & -2953565.400\,t^6 + 12353885.16\,t^7 - 57191750.94\,t^8 + 348616602.2\,t^9 - 3132645470\,t^{10} \\ & + 27260231260\,t^{11} - 224688814200\,t^{12} + 1637561100000\,t^{13} - 10722147350000\,t^{14} \end{aligned}$$

$$\begin{aligned} y = {} & 2.0 + 46.00\,t + 126.0000000\,t^2 + 3830.333333\,t^3 - 11700.58333\,t^4 + 141068.5000\,t^5 \\ & -551421.0639\,t^6 - 355392.8282\,t^7 + 29962399.58\,t^8 - 521562694.8\,t^9 + 5196869637\,t^{10} \\ & -47636040140\,t^{11} + 366652694300\,t^{12} - 2532162870000\,t^{13} + 15044012470000\,t^{14} \end{aligned}$$

$$\begin{aligned} z = {} & 6.0 - 20.00\,t + 115.0000000\,t^2 + 755.000000\,t^3 - 3503.833332\,t^4 + 156306.0167\,t^5 \\ & -1264706.125\,t^6 + 14342080.01\,t^7 - 105862688.4\,t^8 + 737880888.2\,t^9 - 3943767312\,t^{10} \\ & +12755709360\,t^{11} + 38293419600\,t^{12} - 1278826847000\,t^{13} + 15286095180000\,t^{14} \end{aligned}$$

Table 1: Determination of accuracy of RK4 for Chaotic Lu system

t	$\Delta = \lvert RK4_{0.01} - RK4_{0.001} \rvert$ Δx	Δy	Δz
0.5	0.0014340	0.001185	0.0003733
1.0	0.0009131	0.001317	0.0016940
1.5	0.0035760	0.003748	0.0007725
2.0	0.0010330	0.004104	0.0142100
2.5	0.0212000	0.017110	0.0182000
3.0	0.0313200	0.025750	0.0050150
3.5	0.0189000	0.022650	0.0036750
4.0	0.2626000	0.167600	0.1903000
4.5	0.1756000	0.285300	0.0939600
5.0	1.1660000	0.635400	2.5710000

t	$\Delta = \lvert RK4_{0.001} - RK4_{0.0001} \rvert$ Δx	Δy	Δz
0.5	1.005E-07	7.881E-08	4.693E-09
1.0	3.371E-08	6.750E-08	1.256E-07
1.5	2.338E-07	2.362E-07	1.797E-08
2.0	1.034E-07	2.283E-07	8.886E-07
2.5	1.300E-06	1.048E-06	1.121E-06
3.0	1.941E-06	1.589E-06	3.214E-07
3.5	1.146E-06	1.376E-06	2.293E-07
4.0	1.563E-05	9.836E-06	1.147E-05
4.5	1.094E-05	1.735E-05	5.097E-06
5.0	5.726E-05	1.724E-05	0.0001507

Table 2: Differences between 15-terms ADM and 15-terms MADM with RK4 solutions

t	$\Delta = \lvert ADM - RK4_{0.001} \rvert$ Δx	Δy	Δz
0.5	4.986E-08	6.795E-08	7.897E-08
1.0	9.285E-12	1.284E-13	1.406E-13
1.5	2.838E-15	3.943E-15	4.219E-15
2.0	1.631E-17	2.271E-17	2.402E-17
2.5	3.763E-18	5.248E-18	5.506E-18
3.0	4.879E-19	6.810E-19	7.110E-19
3.5	4.252E-20	5.940E-20	6.178E-20
4.0	2.772E-21	3.874E-21	4.018E-21
4.5	1.448E-22	2.024E-22	2.095E-22
5.0	6.350E-22	8.882E-22	9.175E-22

t	$\Delta = \lvert MADM - RK4_{0.001} \rvert$ Δx	Δy	Δz
0.5	1.005E-07	7.882E-08	4.693E-09
1.0	3.371E-08	6.751E-08	1.256E-07
1.5	2.338E-07	2.362E-07	1.797E-08
2.0	1.034E-07	2.283E-07	8.887E-07
2.5	1.301E-06	1.048E-06	1.121E-06
3.0	1.941E-06	1.589E-06	3.214E-07
3.5	1.146E-06	1.377E-06	2.293E-07
4.0	1.564E-05	9.837E-06	1.147E-05
4.5	1.094E-05	1.735E-05	5.098E-06
5.0	5.727E-05	1.725E-05	0.0001507

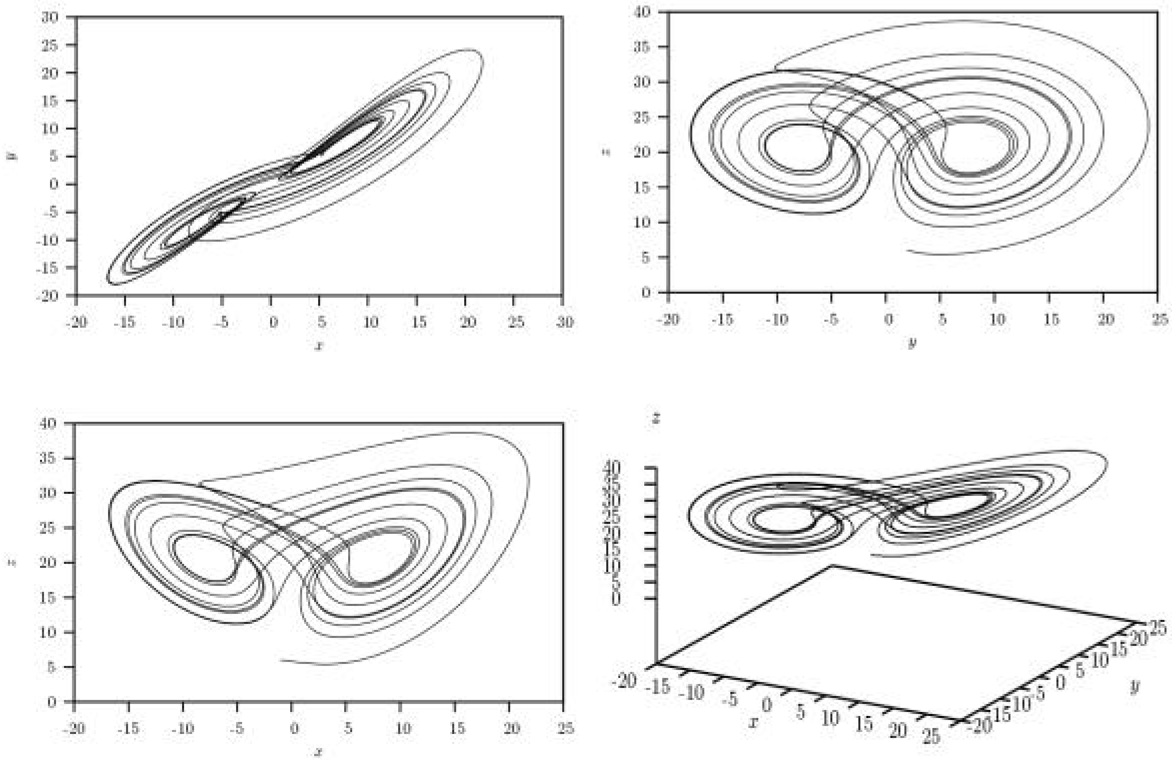

Fig. 1: Phase portraits of 15-term MADM

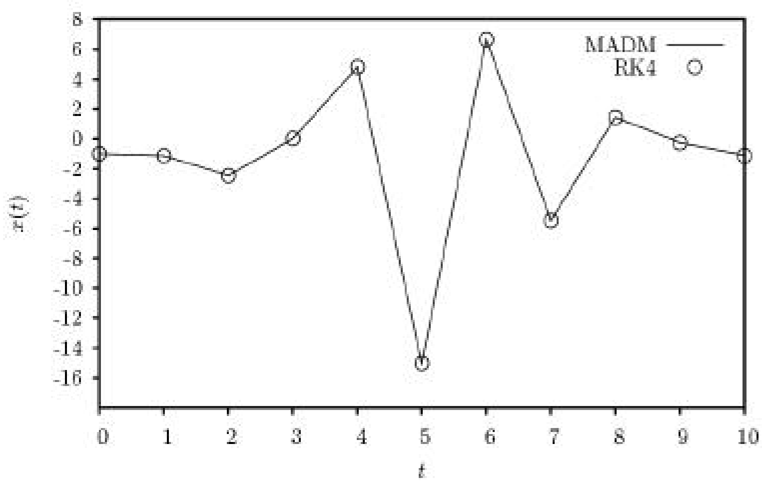

Fig. 2: Comparing 15-term MADM with RK4 for x on $\Delta t=0.001$

As there is no exact solution for the chaotic system, the accuracy of the proposed method is compared to the numerical solution by Runge-Kutta method. The step size of $\Delta t=0.001$ is chosen from the differences of the solution of RK4 at different step size; $\Delta t=0.01$, $\Delta t=0.001$ and $\Delta t=0.0001$ as presented in Table 1. The step size of $\Delta t=0.001$ is selected in this entire work as it gives small error and computationally costly as the time taken is reasonable In this paper, we fix the number of terms to be 15 and step size to be $\Delta t=0.001$. The differences of solutions of ADM, MADM and RK4 are given in Table 2. As we can see the ADM solution is far away from the Runge-Kutta even at t=1 while MADM agree with RK4 solution very well. The 15-term MADM have been illustrated into phase portraits xy, yz, xz and xyz in Fig. 1. Also, the solutions of 15-term MADM for x, y and z have been plotted into graphs in Fig. 2-4 respectively for comparing MADM with RK4.

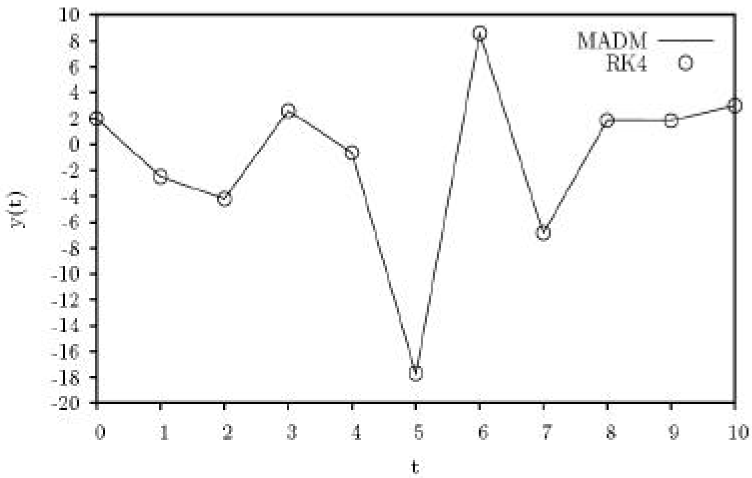

Fig. 3: Comparing 15-term MADM with RK4 for y on Δt=0.001

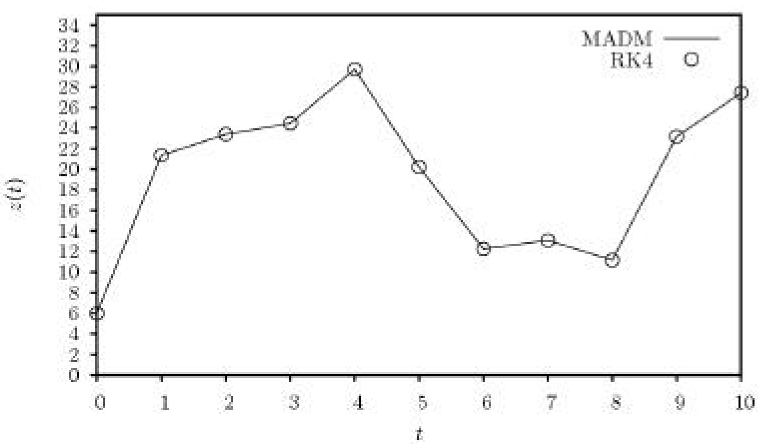

Fig. 4: Comparing 15-term MADM with RK4 for z on Δt=0.001

CONCLUSION

The chaotic Lü system is solved accurately by MADM. The method has the advantage of giving an analytical form of the solution within each time interval which is not possible in purely numerical techniques like RK4. The present technique offers an explicit time-marching algorithm that works accurately over such a bigger time step than the RK4. The results presented in this paper suggest that MADM is also readily applicable to the chaotic systems involving more complex dynamical behaviours.

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