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Electromagnetic Scattering Computation Methods for Very Small Spheroidal Dust Particles: Theory and Applications

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Abstract: This paper discusses the effect of propagation of electromagnetic waves through a cloud of very small particles. The size of these particles is normally less than 0.2 mm in radius, much smaller than the microwave wavelength; therefore it is possible to avoid laborious computation by using approximations. In this work we have checked the Rayleigh theory (Rayleigh scatterer) against a full solution of Maxwell's main equations through a Point Matching Technique (PMT). The agreement between Rayleigh approximations and PMT for these very small particles is found to be good. These results have been used in quantifying some telecommunication channel impairments.

Key words: Computation methods . electromagnetic scattering . Rayleigh approximations . communication channels . crosspolarization

INTRODUCTION

An electromagnetic wave propagating through a dispersed dust particles experiences attenuation and phase rotation. Dust particles are not spherical in shape, but eccentric; they cause differential attenuation and phase rotation if orthogonal communication channel pass through them. The size of these particles is normally less than 0.2 mm (in radius), much smaller than the microwave wavelength, therefore it is possible to avoid laborious computations (Maxwell's full theory), by using approximations. The simpler approximation is the Rayleigh-scatterer. For dry dust the loss tangent is less than 0.004 for sandy samples [1, 2] and less than 0.03 for clayey samples. Hence, dry dust particles can be considered as a phase rotating medium, with negligible true attenuation.

Two orthogonal polarizations can be used to make the optimum use of the frequency spectrum. If crosspolarization discrimination is high enough, different information channels can be transmitted with the same frequency. But, due to the above mentioned differential attenuation and phase rotation the isolation between these two channels degrades. In this work we will use the Rayleigh-scatterer approximation's results as an input to calculate crosspolarization discrimination.

SCATTERING COMPUTATION METHODS

Rayleigh method approximations: The simplest approximation is the Rayleigh scatterer. This applies in the following limits:

$$\frac{2\pi a}{\lambda} \ll 1 \qquad (1)$$

and

$$\frac{2\pi a}{\lambda}|\varepsilon - 1| \ll 1 \qquad (2)$$

where

a = dust/sand particle's radius,
λ = Transmitting wavelength,
ε = Permittivity of the particle.

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The first simplification introduced by considering this small sizes that the particle may be considered to be placed in a homogeneous (uniform) electric field E_0 and the second simplification ensures that the phase of the internal field is not very much different from the external field. Then, the scattering and absorption cross sections yield the following expressions:

$$\sigma_{sca} = \frac{8}{3}\pi^5 \left(\frac{2}{\lambda}\right)^4 a^6 \left|\frac{\varepsilon-1}{\varepsilon+2}\right|^2 \quad (3)$$

$$\sigma_{abs} = \frac{8\pi^2 a^3}{\lambda} I_m \left(\frac{\varepsilon-1}{\varepsilon+2}\right) \quad (4)$$

From eq. (1), it follows that for most particles in Rayleigh approximation the following yields:

$$\sigma_{abs} \gg \sigma_{sca} \quad (5)$$

Provided $I_m\varepsilon$ is not very small for a given frequency. The total extinction cross section in this approximation is given by:

$$\sigma_{ext} = \sigma_{sca} + \sigma_{abs} \quad (6)$$

When the term $I_m\varepsilon = 0$, $\sigma_{abs} = 0$ and σ_{sca} dominates no matter how small the particle is compared with the wavelength. The lowest value of $I_m\varepsilon$ measured so far is 0.0091 [2] with the corresponding $R_e\varepsilon$ equals to 2.5384. It is found that:

$$I_m \left(\frac{\varepsilon-1}{\varepsilon+2}\right) = 1.32 \times 10^{-2} \quad (7)$$

While

$$\left|\frac{\varepsilon-1}{\varepsilon+2}\right|^2 = 0.1153 \quad (8)$$

Taking a = 0.1mm as the largest particle at 10 GHz we have, even for this permittivity:

$$\frac{\sigma_{sca}}{\sigma_{abs}} = 6.889 \times 10^{-4} \quad (9)$$

This justifies neglecting the scatter contribution to extinction at least up to about 30 GHz.

Considering a dielectric ellipsoid with principal axes labeled 1, 2 and 3 and the corresponding semi-axis a, b and c, the particle forward ($\theta = 0$) scattering coefficient S(0) for a plane wave incident along a principal axis and linearly polarized parallel to the i'th principal axis is given in the small particle Rayleigh approximation by:

$$S(0) = j\beta^3 \alpha_i \quad (10)$$

where

$$\beta = \frac{2\pi}{\lambda}$$

α is polarizability and is given by eq. (11):

$$\alpha_i = \frac{V_p}{4\pi\left(L_i + \frac{1}{\varepsilon_r - 1}\right)}, i = 1,2,3 \quad (11)$$

where

V_p = Is the particle volume,
L_i = Factors which depend on the shape only [3], they satisfy: $L_1+L_2+L_3 = 1$.
ε_r = Relative permittivity of the particle.

L_i are approximated by McEwan and Bashir [4]
Thus:

$$L_1 = 0.22, L_2 = 0.34, L_3 = 0.44$$

Point-Matching-Technique

The Point-Matching-Technique (PMT) is a solution to Maxwell's two main equations:

$$\nabla \times E + j\omega\mu H = 0 \tag{12}$$

$$\nabla \times H - (\sigma + j\omega\varepsilon)E = 0 \tag{13}$$

where they are exactly satisfied in the boundaries, exterior to the dust/sand particle and the incident plane wave is expanded in a complex Fourier series similar to the treatment used by Morrison *et al*. [5]. This is simply because of symmetry of the semi-axis of the ellipsoid at the origin and this made it easier for solving the unknown coefficients, as opposed to expanding the incident wave in spherical harmonics [6]. The forward scattered amplitudes (θ = 0) for the dual polarizations (vertical as well as horizontal) are given by the following expressions:

$$S_v(0) = \frac{1}{E_v}\sum_{m=-\infty}^{\infty}\sum_{n\geq|m|}^{\infty}(-j)^{n-1}\, x \left[a_{mn}^{v}\frac{m}{\sin\tau}P_n^{|m|}(\cos\tau) + b_{mn}^{v}\frac{dP_n^{|m|}(\cos\tau)}{d\tau}\right] \tag{14}$$

and

$$S_h(0) = \frac{1}{E_h}\sum_{m=-\infty}^{\infty}\sum_{n\geq|m|}^{\infty}(-j)^{n+2}\, x \left[a_{mn}^{h}\frac{dP_n^{|m|}(\cos\tau)}{d\tau} + b_{mn}^{h}\frac{m}{\sin\tau}P_n^{|m|}(\cos\tau)\right] \tag{15}$$

where $P_n^{|m|}$ are Legendre functions of the first kind and τ is the angle of incidence between the direction of propagation and the axis of symmetry. For more details of eqs. (14) and (15), [2, 7].

A computer program is available and is used to check the Rayleigh theory (approximation) results against the Maxwell's full solution through the Point-Matching-Technique (PMT). The results for vertical as well as the horizontal polarization are depicted in Table 1 and 2.

Table 1: Comparison (check) between Rayleigh approximation for ellipsoids against Maxwell's full theory (PMT) f=12.5 GHz, ε=3.8-j0.038

Method	S_v(o,a) scattering amplitudes for vertical polarization	(c) Radians of miner axis (mm)	c/a eccentricity
Rayleigh scatterer	$9.541*10^{-12}-j*3.287*10^{-10}$	0.0031	0.8
	$7.289*10^{-11}-j*2.510*10^{-9}$	0.0063	0.8
	$5.569*10^{-10}-j*1.918*10^{-8}$	0.0125	0.8
	$4.255*10^{-9}-j*1.465*10^{-7}$	0.0250	0.8
	$3.251*10^{-8}-j*1.119*10^{-6}$	0.0500	0.8
	$2.485*10^{-7}-j*8.549*10^{-6}$	0.1000	0.8
	$1.443*10^{-6}-j*4.923*10^{-5}$	0.1500	0.8
	$1.900*10^{-6}-j*6.529*10^{-5}$	0.2000	0.8
P.M.T	$9.465*10^{-12}-j*3.274*10^{-10}$	0.0031	0.8
	$7.252*10^{-11}-j*2.500*10^{-9}$	0.0063	0.8
	$5.524*10^{-10}-j*1.911*10^{-8}$	0.0125	0.8
	$4.221*10^{-9}-j*1.453*10^{-7}$	0.0250	0.8
	$4.398*10^{-8}-j*1.300*10^{-6}$	0.0500	0.8
	$3.364*10^{-7}-j*9.940*10^{-6}$	0.1000	0.8
	$1.432*10^{-6}-j*4.904*10^{-5}$	0.1500	0.8
	$2.579*10^{-6}-j*7.600*10^{-5}$	0.2000	0.8

Table 2: Comparison (check) between Rayleigh approximation for ellipsoids against Maxwell's full theory (PMT) f=12.5 GHz, ε=3.8-j0.038

Method	S_v(o,a) Scattering Amplitudes for Horizontal Polarization	(c) Radians of Miner axis (mm)	c/a eccentricity
Rayleigh scatterer	$1.287*10^{-11}$-j*$3.812*10^{-10}$	0.0031	0.8
	$9.835*10^{-11}$-j*$2.912*10^{9}$	0.0063	0.8
	$7.514*10^{-10}$-j*$2.225*10^{8}$	0.0125	0.8
	$5.741*10^{-9}$-j*$1.700*10^{-7}$	0.0250	0.8
	$4.386*10^{-8}$-j*$1.299*10^{-6}$	0.0500	0.8
	$3.352*10^{-7}$-j*$9.928*10^{-6}$	0.1000	0.8
	$1.922*10^{-6}$-j*$5.686*10^{-5}$	0.1500	0.8
	$2.579*10^{-6}$-j*$7.600*10^{-5}$	0.2000	0.8
P.M.T	$1.277*10^{-11}$-j*$3.798*10^{-10}$	0.0031	0.8
	$9.756*10^{-11}$-j*$2.900*10^{9}$	0.0063	0.8
	$7.454*10^{-10}$-j*$2.216*10^{8}$	0.0125	0.8
	$5.695*10^{-9}$-j*$1.694*10^{-7}$	0.0250	0.8
	$4.398*10^{-8}$-j*$1.300*10^{-6}$	0.0500	0.8
	$3.364*10^{-7}$-j*$9.940*10^{-6}$	0.1000	0.8
	$1.923*10^{-6}$-j*$5.664*10^{-5}$	0.1500	0.8
	$2.579*10^{-6}$-j*$7.600*10^{-5}$	0.2000	0.8

Table 3: Polarization dependent numerical attenuation and phase shift (f=12.5 GHz, ε =3.8-j0.038)

Av (N/km)	Ah (N/km)	Bv (rad/km)	Bh (rad/km)
0.02944590	0.03972002	-0.50722500	-0.5882390
0.02680911	0.03617336	-0.46159190	-0.5355202
0.02624104	0.03540586	-0.45187936	-0.5242083
0.02504383	0.03379004	-0.43113057	-0.5002880
0.02391941	0.03227024	-0.41165522	-0.4778732
0.02283444	0.03080122	-0.39277987	-0.4561374
0.02193625	0.02977557	-0.37689934	-0.4387249

Table 4: Corresponding XPD results (f=12.5 GHz, ε =3.8-j0.038)

r (m)	XPD (dB)
3.62E-06	27.77543
7.35E-06	28.57137
1.46E-05	28.76145
2.92E-05	29.1515
5.83E-05	29.52943
0.000117	29.91365
0.000233	30.12524

APPLICATION OF RESULTS

The frequency spectrum is nowadays congested. Therefore, if we could transceive two communication channels (with different info's), then the outcome is that: as if we "doubled" the frequency spectrum. This kind of transmission is called dual-polarization. In free-space the isolation between these two channels is excellent; but if the transmission medium is filled with any precipitation particles (rain, snow, dust…etc.) then the isolation will be degraded. In communication language this is called cross-polarization discrimination (XPD).

The calculated complex scattering amplitudes in Table 1 and 2 are used as I/P to calculate the differential attenuation and differential phase shift depicted in Table 3.

Using Bashir's treatment [8], the results in Table 4 are used to deduce the corresponding XPD at Ku band depicted in Fig. 1.

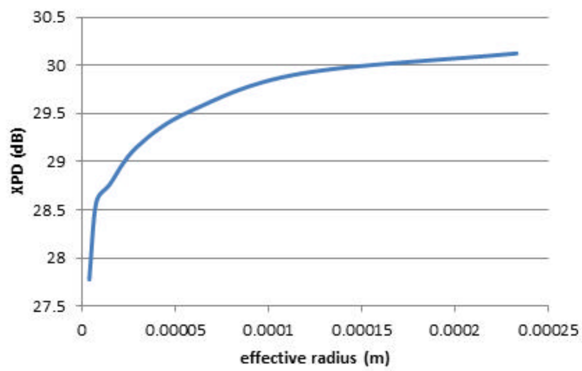

Fig. 1: Crosspolarization discrimination as a function of dust rad

CONCLUSION

The agreement between the two methods for vertical and horizontal polarizations is very good: the relative error, for both polarizations, is less than 1% for the real parts and less than 0.4% for the imaginary parts.

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