Middle-East Journal of Scientific Research 13 (5): 628-635, 2013
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2013.13.5.2951

Prediction of Radial-Ply Tire Deflection Based on Overall Unloaded Diameter, Inflation Pressure and Vertical Load Using Linear Regression Model

*Majid Rashidi, Mohammad-Ali Sheikhi, Morteza Arkian,
Seyyed Mohammad Emadi and Shahram Razavi*

Department of Agricultural Machinery, Takestan Branch,
Islamic Azad University, Takestan, Iran

Abstract: As deflection is a key parameter and many equations have been developed based on it to evaluate the tractive performance of radial-ply tires, this study was conducted to predict deflection (δ) of radial-ply tire based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W). For this purpose, deflection of four radial-ply tires with different overall unloaded diameter were measured at five levels of inflation pressure and five levels of vertical load. Results of deflection measurement for radial-ply tires No. 1, 2 and 3 were utilized to determine regression model, and three-variable linear regression model δ_P = 77.43 - 0.078 d - 0.758 P + 3.519 W with R^2 = 0.985 was obtained. Also, results of deflection measurement for radial-ply tire No. 4 were used to verify model. The paired samples t-test results indicated that the deflection values predicted by model were less than the deflection values measured by test apparatus. To check the discrepancies between the deflection values predicted by model with the deflection values measured by test apparatus, RMSE and MRPD were calculated. The amounts of RMSE and MRPD were 2 mm and 5.5%, respectively. Slight amounts of RMSE and MRPD confirmed that the three-variable linear regression model may be used to predict deflection of radial-ply tire based on overall unloaded diameter, inflation pressure and vertical load. On the other hand, to calculate actual deflection values or deflection values measured by test apparatus (δ_M) based on deflection values predicted by model (δ_P) the linear regression model δ_M = 0.896 δ_P + 5.316 with R = 0.995 can be strongly recommended.

Key words: Radial-ply tire · Deflection · Overall unloaded diameter · Inflation pressure · Vertical load · Prediction

INTRODUCTION

A flexible tire has a smaller contact area on hard surface than it dose on soft ground. A rule of thumb which can be used for estimation of tire contact area is shown by equation 1 [1]:

$$A = bL \quad (1)$$

where:

A = Contact area (m^2)
b = Section width (m)
L = Contact length (m)

Wong [2] and Bekker [3] gave an approximate method for calculating contact length as given below in equation 2:

$$L = 2(d\delta - \delta^2)^{0.5} \quad (2)$$

where:

d = Overall unloaded diameter (m)
δ = Deflection (m)

Deflection is a key parameter and many equations have been developed based on it to evaluate the tractive performance of bias-ply and radial-ply tires operating in cohesive-frictional soils. Gross traction, motion

Corresponding Author: Majid Rashidi, Ph.D., Department of Agricultural Machinery, Takestan Branch, Islamic Azad University, Takestan, Iran.

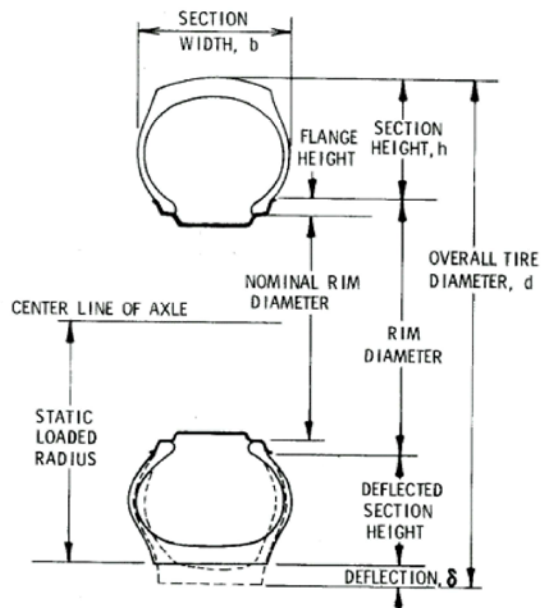

Fig. 1: Tire dimensions, adapted from Brixius [4]

resistance, net traction and tractive efficiency are predicted as a function of soil strength, tire load, tire slip, tire size and tire deflection [4]. The most widely used dimensional analysis approach for predicting off-road traction makes use of the following ratios [4-6]:

$$C_n = \frac{CI \cdot b \cdot d}{W} \quad (3)$$

$$WD = \frac{b}{d} \quad (4)$$

$$DR = \frac{\delta}{h} \quad (5)$$

where:

C_n = Wheel numeric (dimensionless)
CI = Cone index (kPa or kNm^{-2})
W = Vertical load (kN)
WD = Section width to overall unloaded diameter ratio (dimensionless)
DR = Deflection ratio (dimensionless)
h = Section height (m)

Fig. 1 shows the tire dimensions (b, d, δ and h) used. The tire dimensions can be obtained from tire data book or by measuring the tire [4]. The section width (b) is the first number in a tire size designation (i.e., nominally 18.4 inches for an 18.4-38 tire). The overall unloaded diameter (d) can be obtained from the tire data handbooks available from off-road tire manufacturers. The tire deflection (δ) on a hard surface is equal to d/2 minus the measured static loaded radius. The static loaded radius for the tire's rated load and inflation pressure is also standard tire data from the tire data handbooks. It can also be obtained by measuring the tire. The section height (h) is equal to half the difference between the overall unloaded diameter and the rim diameter. The rim diameter can in turn be estimated by adding 50 mm to the nominal rim diameter, which is the second number in a tire size designation, i.e. 38 inches for an 18.4-38 tire [4, 5].

To further simplify the prediction equations, Brixius [4] combined above three dimensionless ratios into a single product termed the mobility number, which is given by equation 6 [5-7]:

$$B_n = \frac{CI \cdot b \cdot d}{W}\left(\frac{1+5\frac{\delta}{h}}{1+3\frac{b}{d}}\right) \quad (6)$$

where:

B_n = Mobility number (dimensionless)

The empirical model developed by Brixius [4] is widely used for prediction of off-road tire performance. It has also been adopted in ASAE standard D497.4 [8] for predicting tractor performance. In this model, soil condition is represented by the cone index value, which is the average force per unit area required to force a cone-shaped probe vertically into the soil at a steady rate. The average before-traffic cone index for the top 150 mm layer of soil is used in the prediction equations that follow [5, 7]. ASAE standards S313.3 [9] and EP542 [10] describe the soil cone penetrometer and procedures for its use. An average of several cone index values obtained at a test site often yields a representative measure of soil strength [11].

As deflections for a given tire size, inflation pressure and vertical load are significantly different between bias-ply and radial-ply tires [4], this study was conducted to predict deflection (δ) of radial-ply tire based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W).

Fig. 2: Tire deflection test apparatus

Fig. 3: Measuring static loaded radius

MATERIALS AND METHODS

Tire Deflection Test Apparatus: A tire deflection test apparatus (Fig. 2) was designed and constructed to measure deflection of tires with different sizes at diverse levels of inflation pressure and vertical load. As deflection on a hard surface is equal to d/2 minus the measured static loaded radius [4, 5], the static loaded radius was obtained by measuring as shown in Fig. 3.

Experimental Procedure: For this purpose, deflection of four radial-ply tires with different overall unloaded diameters were measured at five levels of inflation pressure and five levels of vertical load. The dimensions of four radial-ply tires are given in Table 1. Results of deflection measurement for radial-ply tires No. 1, 2 and 3 (Tables 2, 3 and 4) were utilized to determine regression model and results of deflection measurement for radial-ply tire No. 4 (Table 5) were used to verify model.

Table 1: Dimensions of the four radial-ply tires used in this study

Tire No.	Tire size designation	Overall unloaded diameter d (mm)
1	R13-165/65	535
2	R14-185/65	580
3	R15-185/65	610
4	R16-216/60	650

Regression Model: A typical three-variable linear regression model is shown in equation 7:

$$Y = C_0 + C_1X_1 + C_2X_2 + C_3X_3 \quad (7)$$

where:

Y = Dependent variable, for example deflection of radial-ply tire

X_1, X_2, X_3 = Independent variables, for example overall unloaded diameter, inflation pressure and vertical load, respectively

C_0, C_1, C_2, C_3 = Regression coefficients

In order to predict deflection of radial-ply tire from overall unloaded diameter, inflation pressure and vertical load, a three-variable linear regression model was suggested and all the data were subjected to regression analysis using the Microsoft Excel 2007.

Statistical Analysis: A paired samples t-test was used to compare the deflection values predicted by model with the deflection values measured by test apparatus. Also, to check the discrepancies between the deflection values predicted by model with the deflection values measured by test apparatus, root mean squared error (RSME) and mean relative percentage deviation (MRPD) were calculated using the equations 8 and 9, respectively [12-19]:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\delta_{Mi} - \delta_{Pi})^2}{n}} \quad (8)$$

where:

RMSE = Root mean squared error (mm)

δ_{Mi} = Deflection measured by tire deflection test apparatus (mm)

δ_{Pi} = Deflection predicted by three-variable linear regression model (mm)

Table 2: Overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 1

Tire No.	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
1	535	30	5.8690	31.0
			7.8250	39.0
			9.7810	47.5
			11.738	55.0
			13.694	62.0
		32	5.8690	28.5
			7.8250	38.0
			9.7810	47.0
			11.738	53.0
			13.694	60.0
		34	5.8690	29.0
			7.8250	36.5
			9.7810	44.5
			11.738	51.5
			13.694	58.0
		36	5.8690	27.5
			7.8250	36.0
			9.7810	43.0
			11.738	49.0
			13.694	55.0
		38	5.8690	26.5
			7.8250	35.0
			9.7810	42.5
			11.738	49.0
			13.694	55.0

Table 3: Overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 2

Tire No.	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
2	580	30	5.8690	29.5
			7.8250	38.0
			9.7810	44.5
			11.738	50.5
			13.694	58.0
		32	5.8690	28.5
			7.8250	35.5
			9.7810	43.0
			11.738	48.0
			13.694	55.0
		34	5.8690	28.0
			7.8250	35.0
			9.7810	41.5
			11.738	47.5
			13.694	54.0
		36	5.8690	26.5
			7.8250	33.0
			9.7810	44.5
			11.738	46.0
			13.694	51.5
		38	5.8690	26.0
			7.8250	31.5
			9.7810	40.5
			11.738	43.5
			13.694	50.5

Table 4: Overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 3

Tire No.	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
3	610	30	5.8690	26.0
			7.8250	35.0
			9.7810	42.0
			11.738	48.0
			13.694	54.5
		32	5.8690	28.0
			7.8250	35.0
			9.7810	40.5
			11.738	47.5
			13.694	53.5
		34	5.8690	22.5
			7.8250	31.5
			9.7810	37.0
			11.738	45.0
			13.694	52.0
		36	5.8690	22.0
			7.8250	30.5
			9.7810	36.0
			11.738	42.5
			13.694	49.5
		38	5.8690	21.0
			7.8250	26.5
			9.7810	34.5
			11.738	41.5
			13.694	47.5

Table 5: Overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 4

Tire No.	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
4	650	30	5.8690	26.0
			7.8250	33.5
			9.7810	40.0
			11.738	46.0
			13.694	52.0
		32	5.8690	25.0
			7.8250	32.5
			9.7810	38.0
			11.738	44.0
			13.694	50.5
		34	5.8690	24.0
			7.8250	31.5
			9.7810	37.5
			11.738	42.5
			13.694	50.0
		36	5.8690	23.0
			7.8250	30.5
			9.7810	35.0
			11.738	42.0
			13.694	48.5
		38	5.8690	23.0
			7.8250	29.0
			9.7810	34.5
			11.738	40.5
			13.694	46.0

$$MRPD = \frac{100 \times \sum_{i=1}^{n} \frac{|\delta_{Mi} - \delta_{Pi}|}{\delta_{Mi}}}{n} \quad (9)$$

where:

MRPD = Mean relative percentage deviation, %

RESULTS AND DISCUSSION

Three-variable linear regression model, p-value of independent variables and coefficient of determination (R^2) of the model are shown in Table 6. In this model deflection of radial-ply tire can be predicted as a function of overall unloaded diameter (d), inflation pressure (P) and vertical load (W). The p-value of independent variables (d, P and W) and R^2 of the model were 4.61E-25, 2.69E-22, 1.64E-64 and 0.985, respectively. Based on the statistical results, the three-variable linear regression model was initially accepted, which is given by equation 10:

$$\delta_P = 77.43 - 0.078\ d - 0.758\ P + 3.519\ W \quad (10)$$

Deflection of radial-ply tire No. 4 was then predicted at five levels of inflation pressure and five levels of vertical load using the three-variable linear regression model. The deflection values predicted by model were compared with the deflection values measured by test apparatus, and are shown in Table 7. The paired samples t-test results indicated that the deflection values predicted by model were less than the deflection values measured by test apparatus. The average deflection difference between two methods was -1.62 mm (95% confidence interval for difference in means: -2.14 mm and -1.11 mm; p-value = 1.0000). The standard deviation of the deflection difference was 1.24 mm (Table 8). To check the discrepancies between the deflection values predicted by model with the deflection values measured by test apparatus, RMSE and MRPD were calculated. The amounts of RMSE and MRPD were only 2 mm and 5.5% respectively. Slight amounts of RMSE and MRPD

Table 6: Three-variable linear regression model, p-value of independent variables and coefficient of determination (R^2)

Model	p-value d	p-value P	p-value W	R^2
δ = 77.43 - 0.078 d - 0.758 P + 3.519 W	4.61E-25	2.69E-22	1.64E-64	0.985

Table 7: Overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 4 used in evaluating three-variable linear regression model

Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm) Measured by test apparatus	Deflection δ (mm) Predicted by model
650	30	5.8690	26.0	24.6
		7.8250	33.5	31.5
		9.7810	40.0	38.4
		11.738	46.0	45.3
		13.694	52.0	52.2
	32	5.8690	25.0	23.1
		7.8250	32.5	30.0
		9.7810	38.0	36.9
		11.738	44.0	43.8
		13.694	50.5	50.7
	34	5.8690	24.0	21.6
		7.8250	31.5	28.5
		9.7810	37.5	35.4
		11.738	42.5	42.3
		13.694	50.0	49.1
	36	5.8690	23.0	20.1
		7.8250	30.5	27.0
		9.7810	35.0	33.9
		11.738	42.0	40.7
		13.694	48.5	47.6
	38	5.8690	23.0	18.6
		7.8250	29.0	25.5
		9.7810	34.5	32.3
		11.738	40.5	39.2
		13.694	46.0	46.1

Table 8: Paired samples t-test analyses on comparing deflection determination methods

Determination methods	Average difference (mm)	Standard deviation of difference (mm)	p-value	95% confidence intervals for the difference in means (mm)
Test apparatus vs. model	-1.62	1.24	1.0000	-2.14, -1.11

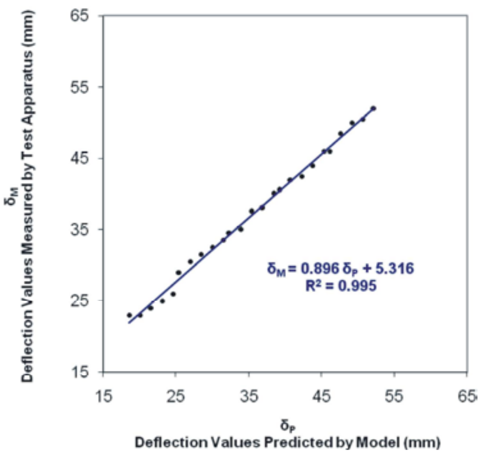

Fig. 4: Curve of deflection values measured by test apparatus (δ_M) based on deflection values predicted by three-variable linear regression model (δ_P) for radial-ply tire No. 4

confirmed that the three-variable linear regression model $\delta_P = 77.43 - 0.078\ d - 0.758\ P + 3.519\ W$ with $R^2 = 0.985$ may be used to predict deflection of radial-ply tire based on overall unloaded diameter, inflation pressure and vertical load. On the other hand, as it is indicated in Fig. 4, our attempts to relate deflection values predicted by three-variable linear regression model (δ_P) to deflection values measured by test apparatus (δ_M) using a linear equation resulted in very good agreements (R^2=0.995) as equation 11:

$$\delta_M = 0.896\ \delta_P + 5.316 \quad (11)$$

It means that actual or measured deflection (δ_M) can be computed in two steps. At first step predicted deflection (δ_P) can be calculated based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W) using the three-variable linear regression model, i.e. equation 10. Second step is calculating actual or measured deflection (δ_M) based on predicted deflection (δ_P) using the linear model, i.e. equation 11.

CONCLUSIONS

It can be concluded that actual or measured deflection (δ_M) of radial-ply tire can be computed in two easy steps. At first step, predicted deflection (δ_P) can be calculated based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W) using the three-variable linear regression model $\delta_P = 77.43 - 0.078\ d - 0.758\ P + 3.519\ W$ with $R^2 = 0.985$. Second step is calculating actual or measured deflection (δ_M) based on predicted deflection (δ_P) using the linear equation $\delta_M = 0.896\ \delta_P + 5.316$ with R^2=0.995.

ACKNOWLEDGMENTS

The authors ask God's favor for their late friend and student, Engineer Hadi Khalkhali, who designed and constructed the tire deflection test apparatus.

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