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Sensitivity Analysis of One Dimensional Hydrodynamic Fully Coupled Model

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Abstract: Sensitivity analysis illustrates relation between inputs and outputs of a model. In this paper sensitivity analysis of one dimensional flow and sediment coupled model has been done using the first order FAST variance based method. Manning coefficient, n and diffusion coefficient, D_x are two parameters which affect model outputs. This paper attempts to verify the importance of these two parameters on the model output. Sensitivity indices, S_i, were determined for both parameters. According to calculation total sensitivity indices for n and D_x for sediment concentration and flow discharge variances are 38% and 70% of the total variance, respectively. The results show that both parameters have more importance in the model outputs but diffusion coefficient has more effect. Therefore it is necessary to evaluate D_x accurately than n. On the other hand, the coupled model is more sensitive to D_x than n.

Key words: Sensitivity Analysis · FAST Variance Based Method · Sensitivity Index · Hydrodynamic Coupled Model

INTRODUCTION

Sensitivity analysis is the study about the relations between the input and output of a model [1]. On the other hand, sensitivity analysis is the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input [2]. Originally, sensitivity analysis as dealt, like system theory, with the variation respectively the uncertainties of the input quantities. Later on the item input has been extended to uncertainties of model parameters and the general model structure. The variation of input leads to changed output quantities. The relation between varied input and output is measured by different sensitivity measures that are the basis for model validation and optimization [1]. Figure 1 gives an overview on the general procedure of sensitivity analysis. In general, sensitivity analysis is conducted by [3]:

- Defining the model and its input parameters and output variable(s).
- Assessing the influences or relative importance of each input parameters on the output variable.

There are different categories about sensitivity analysis methods. More general classification is as following: (a) derivative based sensitivity indices (b) linear regression and (c) Variance based methods [4]. At the following a brief of advantage and disadvantage of mentioned methods are presented.

About derivative based method, consider a model $Y=f(X_1,X_2,\ldots,X_k)$ with k input factors. The partial derivative of Y with respect to an input factor X_i, $\frac{\partial Y}{\partial Xi}$, measures how sensitive the output is to a perturbation of the input. If factors are uncertain within a known or hypothesized range, then the measure.

$$S_i^{\sigma} = \frac{\sigma i}{\sigma Y}\frac{\partial Y}{\partial Xi} \quad (1)$$

Provides a standardized index where σ_i and σ_Y are standard deviations of the inputs i and output of uncertainty, respectively. This method has limitation in large system of differential equations. In these systems, to evaluate index S_i, each input factor X_i varies in turn by a positive and negative increament around its central value, while keeping all other factors at their central values. This less sophisticated but commonplace approach has more errors if the factor X_i does not vary linearly around Y [4]. Chen and Chen [5], Horritt [6], Oliver and Smetten [7], Podeschin *et al*. [8] and Rocha *et al*. [9] has been mentioned this limitation about this approach.

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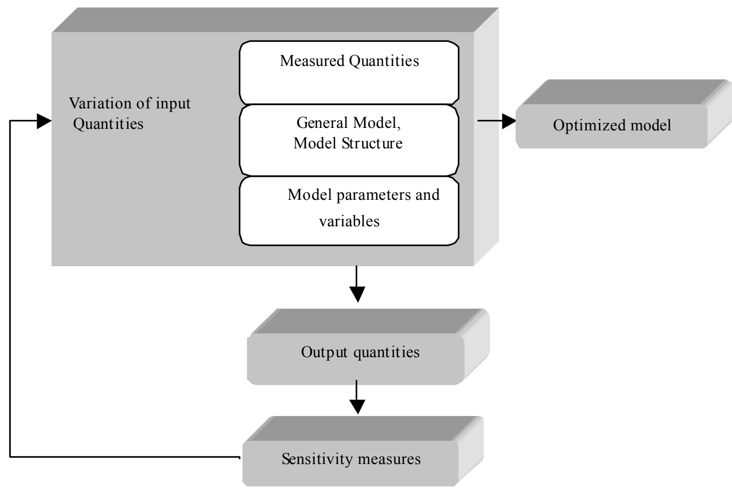

Fig. 1: General procedure for sensitivity analysis

In linear regression method, the linear regression coefficients between input and output provide natural sensitivity indices. In the case of numerical models, this can be achieved by constructing a Monte Carlo sample of the model inputs and regressing the corresponding outputs, Y against the inputs X_i using a multiple regression analysis model of the form:

$$Y = b_o + \Sigma b_i X_i \quad i = 1,2,\ldots, k \tag{2}$$

Which b_i are fixed regression coefficients. This equation can be written as following form:

$$\widetilde{Y} = \beta_0 + \Sigma \beta_i \widetilde{X}_i \quad i=1,2,\ldots, k \tag{3}$$

Which is standardized format of equation (2). In this equation, β_i are standardized regression coefficients (SRCs) [10]. The values of âi can be estimated by evaluating y at each point in a Monte Carlo sample of the input variables Xi and then applying regression analysis to the sample of points [4]. For linear models,

For linear models $\beta_i^1 = S_i^\sigma$ and if the model is nonlinear, SRCs are still a reflection of the contribution of the variance of each input factor to the overall output variance and are more attractive than local derivatives as they offer a measure of the effect of each given factor on Y, which is averaged over a sample of possible values, as opposed to being computed at the fixed point. SRCs are, therefore, a global sensitivity measure, their limitation being in their applicability to nonlinear models [4]. Yeh and Tung [11], Manache and Melching [12], Siebera and Uhlenbrook [13] have been mentioned that SRCs method can be used to sensitivity analysis if the measure of the following coefficient:

$$R_\gamma^2 = 1 - \sum_{i=1}^{m} \frac{(y_i - \hat{y}_i)^2}{(y_i - \mu_i)^2} \tag{4}$$

Be 0.7 or higher where m is number of simulation, y_i is simulation result for model realization i and $\hat{y}_i$ are values of y which are provided by the regression model for input vector x_i. It is notable that x_i is amount of factor X_i in its range [4].

Variance based methods for sensitivity analysis were first employed by chemists in the early 1970s Cukier *et al*. [2]. Cukier *et al*. not only proposed conditional variances for a sensitivity analysis based on first-order effects, but also were already aware of the need to treat higher-order terms and of the underlying variance decomposition theorems. Their method, known as FAST (Fourier Amplitude Sensitivity Test). Interesting features of variance-based methods are [2]:

- Model independence: the sensitivity measure is model-free;
- Capacity to capture the influence of the full range of variation of each input factor;
- Appreciation of interaction effects among input factors;
- Capacity to tackle groups of input factors: uncertain factors might pertain to different logical types and it might be desirable to decompose the uncertainty according to these types.

In FAST, the variance V(Y) of Y is decomposed using spectral analysis, so that:

$$V = V_1 + V_2 + \ldots + V_k + r \tag{5}$$

where V_i is that part of the variance of Y that can be attributed to X_i alone and r is a residual. The ratio:

$$S_i = \frac{v_i}{v} \tag{6}$$

can be taken as a measure of the sensitivity of Y with respect to X_i. For linear models. Saltelli *et al*. showed how S_i^{σ} is an effective measure for linear models, β_i is an effective measure for moderately nonlinear models, for which an effective linear regression model can be built and S_i is the model-free extension that is effective even for strongly nonlinear models [4]. Saltelli *et al*. presented a known algebraic result as following [2]:

$$V(Y) = E_{xi}\,[V|X_i)] + V[(E_{x\sim i}\,(Y|X_i)] \tag{7}$$

where $V[(E_{x\sim i}(Y|X_i)$ is referred to as the variance of conditional expectation of Y given X_i, the subscript $X_{\sim i}$ denotes the vector of all factors other than X_i and $Ex_i[V(Y|X_i)]$ is mean expectation value of conditional variance over all possible values of factor X_i. The FAST sensitivity index is simply as following [2, 4]:

$$S_i = \frac{V[(Ex_{\sim i}(Y|X_i)]}{V(Y)} = \frac{V(Y|X_i)}{V(Y)} = \frac{V_i}{V} \tag{8}$$

In classic FAST, only the main effect terms V_i are computed and the success of a given analysis is empirically evaluated by the sum of these terms. Liepman and Stephanopoulos mentioned that this analysis is successful the sum of S_i is high, as rule of the thumb greater than 0.6. Two factor will have interact if their effect on Y cannot be expressed as a sum of their single effect. Interactions represent important features of models and are more difficult to detect than first order effect. By condensing the notation of the variances, i.e. $V(f_i)=V_i$, $V(f_{ij})=V_{ij}$ and so on, the extended FAST can be written as following which is so-called ANOVA-HDMR decomposition [2]:

$$V(Y)=\Sigma_i V_i + \sum_{\substack{i,j\\ i<j}} V_{ij} + \sum_{\substack{i,jl\\ i,j\\ j<l}} V_{ijk} + \ldots + V_{123\sim k} \tag{9}$$

Where

$$V_i = V[E_{x\sim i}(Y|X_i)] \tag{10}$$

$$V_{ij} = V[E_{x\sim ij}\,(Y|X_i,X_j)]-V_i-V_j \tag{11}$$

$$V_{ijl} = V[Ex_{\sim ijl}\,(Y|X_i,X_j,X_l)]-V_{ij}-V_{jl}-V_{il}-V_i-V_j-V_l \tag{12}$$

And so on. Dividing both sides of equation (9) by V(Y), the following equation can be derived [2]:

$$\Sigma_i S_i + \sum_{\substack{i,j\\ i<j}} S_{ij} + \sum_{\substack{i,jl\\ i,j\\ j<l}} S_{ijk} + \ldots + S_{123\sim k} = 1 \tag{13}$$

The total effect index accounts for the total contribution to the output variation due to factor X_i, i.e. its first-order effect plus all higher-order effects due to interactions. A more practical approach is to estimate the k total sensitivity indices, S_{Ti}, where [4]:

$$S_{Ti} = 1 - \frac{V[E(Y|X_{\sim i} = x^*_{\sim i})]}{V(Y)} \tag{14}$$

The general procedure to get sensitivity measures for sample-based sensitivity analysis methods is given in the following:

- Definition of probability distributions functions for the input quantities.
- Generation of samples from the defined probability distributions.
- Evaluation of the model using the generated sample.
- Analysis of the output variance.
- Sensitivity analysis of the output variance in relation to the variation of the input quantities.

Another way to model study is scatter plat of model output Y versus factor X_i. Input/output scatter plots are in general a very simple and informative way of running a sensitivity analysis, since they can provide an immediate visual depiction of the relative importance of the factors. Most sensitivity analysis measures developed by practitioners aim to preserve the rich information provided by scatter plots in condensed format. On the other word, what identifies an important factor is the existence of shape or pattern in the points, while a uniform cloud of points is a symptom (though not a proof) of a non-influential factor [2].

As it clear, variance based method of sensitivity analysis is more informative that the others. In this paper, the variance based method is applied to assessment of sensitivity level of input parameters of fully coupled 1D model of flow and sediment transport.

MATERIALS AND METHODS

Sediment transport in fluvial systems is the most important phenomena. On the other hand, sediment transport and water flow are two simultaneous processes which its coupled simulation has great importance. Fuladipanah *et al*. developed 1D flow and sediment transport fully coupled which was based on mass and momentum conservation principles [15].

The model which simulates flow and sediment transport under unsteady condition involves three following equations [15]:

$$\frac{\partial A}{\partial t}+(S\text{-}1)\frac{\partial (CA)}{\partial t}+\frac{\partial (AU)}{\partial x}+(S\text{-}1)\frac{\partial (ACU)}{\partial x}=0 \qquad (15)$$

$$\frac{\partial (UA)}{\partial t}+(S\text{-}1)\frac{\partial (UCA)}{\partial t}+\frac{\partial (AU^2)}{\partial x}+(S\text{-}1)\frac{\partial (ACU^2)}{\partial x}+ gA\frac{\partial y}{\partial x}+g(S\text{-}1)CA\frac{\partial y}{\partial x}=gA(S_f\text{-}S_o)(1+(S\text{-}1)C) \qquad (16)$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}=D_x\frac{\partial^2 C}{\partial x^2} \qquad (17)$$

where x and t are space and time varibales, respectively, A is cross section area, S is spesific gravity of sediment particle, U is mean velosity, C is sediemt concentration by weight, g is gravity acceleration, y is flow depth, S_f is energy line slope, S_o is bed slope and D_x is diffusion coefficient. As it clear, there are three variabes which should be calculate each time step: y, U and C. Manning roughness coefficient and diffusion coefficient are two constants which should be determined during calibration period. Accordign to the simplifications to derived one dimensional equaiotns, correct estimation of model parameters has more importance. On the other hand, equations 15 to 17 are partial non-linear equations which don't have analytical solution. Numerical techniques include discretization errors. Therefore, it is necessary to evaluate correct values for model parameters. Manning and diffusion coefficients are physical properties of flow and sediment which their acurate estimations increases the model output. What parameter requires more accurance is determined using sensitivity analysis. In order to explore the sensitivity of response of the equations set to variation in the input parameters, the input parameters were assigned the distribution in Table 1. The mentioned range has delibaratly been shosen to illustrate the potential for non-linear response. Figures 2 and 3 illustrate the output of model for sediment concentration and flow discharge for a range of value of n and D_x, respectively. The outputs are determined using implicit finite difference scheme [15]. Global sensitivity analysis was based on Monte Carlo sampling of the input paramentes and caculating ouputs. The equations set response can be visualized in scatter plots which plot flow discharge and sediment concentration for each member of Monte Carlo sample as a function of the two paramenters.

Table 1: Distribution of parameters for the coupled model

Parameters	Distribution type	Range
n	Log-normal	n~(0.012,0.025)
D_x	Log-normal	D_x~(0.001,0.014)

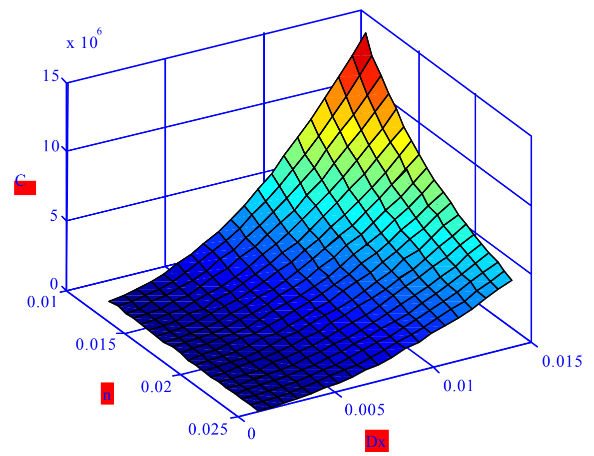

Fig. 2: Variation of sediment concentration, C(by volume), as a function of n and Dx

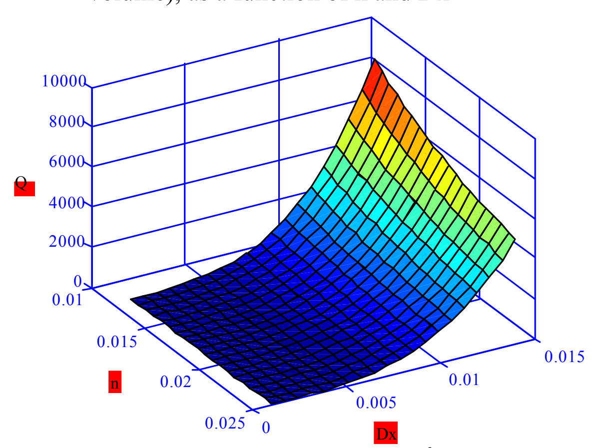

Fig. 3: Variation of flow discharge, Q(m^3/s), as a function of n and Dx

Scatter-plots are the simplest form of analysis and can reveal non-linear relationships [2]. Figure 4 at a galence indicates noticable sensitivity to D_x. Table 2 shows the result of the first order FAST based sensitivity analysis indices, S_i.

RESULT AND DISSCOUSION

Sensitivity analysis is an essential aspect of analytical and numerical models. This becomes more important about more complex and multidisciplinary ones. Sensitivity analysis helps to decision makers to analysis plausible variations in the model inputs, legitimately. The results can be used to engineering design decisions

Table 2: The first order FAST sensitivity analysis results

	S_i		S_{Ti}	
Parameter	Sediment concentration	Flow discharge	Sediment concentration	Flow discharge
D_x	0.23	0.45	0.69	0.54
N	0.15	0.25	0.37	0.31
Total	0.38	0.70	-	-

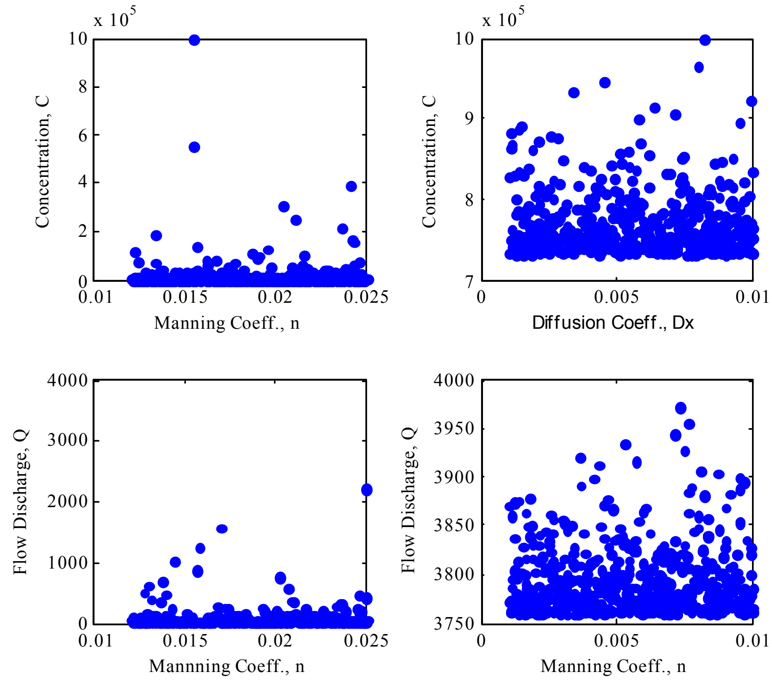

Fig. 4: Scatter plots for the model output versus manning and diffusion coefficients

more effectively by identifying the parameters that exert the greatest influence on system performance. There are different methods to do sensitivity analysis which variance based method is more applicable, because it has not limitation of other methods which are mentioned in previous sections. In this paper, the first order FAST based sensitivity indices were used to do sensitivity analysis about one dimensional flow and sediment fully coupled model which was developed based on mass and momentum conservation. The model has two input parameters which should determined during calibration period: Manning coefficient, n and Diffusion coefficient, D_x. This paper attempts to verify the importance of these two parameters on the model outputs. Table 2 shows that total sensitivity indices for n and D_x for sediment concentration and flow discharge variances are 38% and 70% of the total variance, respectively. Based on Table 2, it is clear that n and D_x have more effects on model iutputs. But among these two parameters, D_x has more importance and therefore, it is necessary to indentify accuratly during calibration period. This effect is clear in Figure 4, because corresponding variation of model sediment concentration output has vast range than the flow discharge output. Then, estimation of correct value of D_x is more important than manning coefficient value.

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