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Hypothesis Testing on the Parameters of Exponential, Pareto and Uniform Distributions Based on Extreme Ranked Set Sampling

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Abstract: The problem of hypothesis testing of the location parameter of the two-parameter exponential distribution and scale parameter of the pareto and uniform distribution are considered. We construct the test statistics based on Ranked Set Sampling (RSS) and Extreme RSS (ERSS) and compared their powers to the power of Uniformly Most Powerful (UMP) test by numerical computation. The results show that the test based on ERSS is more powerful than the one based on RSS and UMP test.

Key words: Exponential distribution . extreme ranked set sampling . hypothesis testing . pareto distribution . uniform distribution . ranked set sampling

INTRODUCTION

In sampling situations where the units drown from a population are difficult or expensive to quantify but can be easily ranked, the Ranked Set Sampling (RSS) procedure, that was first suggested by McIntire [1] and the Extreme Ranked Set Sampling (ERSS) procedure, that was first suggested by Samawi *et al*. [2], provide efficient methods to estimate the population parameters.

Some researchers used RSS to obtain the tests, which were most powerful than the usual Uniformly Most Powerful (UMP) tests based on Simple Random Samples (SRS). Among them, Abu-Dayyeh and Muttlak [3] and Muttlak and Abu-Dayyeh [4] constructed new tests using RSS method for testing parameters of exponential, uniform and normal distributions. They showed that these tests outperform substantially against the usual UMP and Likelihood Ratio (LR) tests. Wang and Tseng [5] introduced RSS-based tests for testing population mean of normal and exponential distribution, which were more powerful than UMP and UMP Unbiased (UMPU) tests. Tseng and Wu [6] constructed the similar tests based on Median RSS (MRSS) and showed that the resulting tests were most powerful than the tests based on RSS.

For testing scale parameters of the exponential and rectangular distributions, some test procedure involving the MRSS data were introduced by Hossain and Muttlak [7]. They showed that for the scale parameter of exponential distribution, the MRSS-based tests were more powerful than RSS-based tests and for rectangular distribution, the RSS-based tests were much better than the MRSS-based tests.

In this paper we introduce some test statistics based on ERSS and RSS procedures for testing location parameter of the two-parameter exponential distribution and scale parameter of the pareto and uniform distributions, which are more powerful than the UMP test based on SRS. To this end, in Section 2, we introduce the tests based on ERSS and RSS methods for testing location parameter of the two-parameter exponential distribution and compare them with UMP test. In Sections 3 and 4, we introduce new tests based on ERSS and RSS methods for testing scale parameter of the pareto and uniform distributions and compare them with UMP tests. A conclusion is given in Section 5.

Testing the location parameter of two-parameter exponential distribution

Let X_1, X_2,…, X_n be a random sample from the two-parameter exponential distribution with probability density function (p.d.f.)

$$f(x) = \frac{1}{\sigma} e^{-\frac{1}{\sigma}(x-\mu)} I_{(\mu,+\infty)}(x),\ \mu \in (-\infty,+\infty), \sigma > 0$$

We want to test

$$H_0 : \mu = \mu_0 \quad \text{vs} \quad H_0 : \mu \neq \mu_0 \tag{2.1}$$

It is well known that the UMP test of size α for testing (2.1) is given by

$$\varphi_{UMPT1}(\mathbf{x}) = \begin{cases} 1 & \min\{x_i\} < \mu_0 \text{ or } \min\{x_i\} > c_{SRS}(n,\alpha) \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

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where $x = (x_1,\ldots,x_n)$. Without lost of generality we may take $\mu_0 = 1$ and $\sigma = 1$. Then

$$c_{SRS}(n,\alpha) = 1 - \frac{1}{n}\ln(\alpha)$$

The power function of this test is given by

$$\pi_{\varphi_{UMPT1}}(\mu) = P_\mu(\min\{x_i\} < 1) + P_\mu(\min\{x_i\} > 1 - \frac{1}{n}\ln(\alpha))$$

$$= \begin{cases} 1-(1-\alpha)e^{n(\mu-1)} & \mu < 1 \\ \alpha e^{n(\mu-1)} & 1 < \mu < 1 - \frac{1}{n}\ln(\alpha) \\ 1 & \mu > 1 - \frac{1}{n}\ln(\alpha) \end{cases} \quad (2.3)$$

To obtain the test based on RSS, let $X_{11}, X_{12},\ldots X_{1n}$; $X_{21}, X_{22},\ldots X_{2n}$;…; $X_{n1}, X_{n2},\ldots X_{nn}$ be the n groups of n independent random variables, which have the same p.d.f. $f(.)$. Let $X_{i(1)}$, $X_{i(2)},\ldots X_{i(n)}$ be the order statistics of the random sample $X_{i1}, X_{i2},\ldots X_{in}$ in the i-th group (I = 1,2,…,n). Then $X_{1(1)}, X_{2(2)},\ldots X_{n(n)}$ denotes the ranked set sample, where $X_{i(i)}$ is the i-th order statistics in the i-th group. To simplify the notation, $X_{i(i)}$ will be denoted by Y_i throughout this paper.

To test the hypothesis (2.1) based on the RSS, we propose the following test

$$\varphi_{RSS1}(\mathbf{x}) = \begin{cases} 1 & \min\{Y_i\} < 1 \ \text{ or } \ \min\{Y_i\} > c_{RSS}(n,\alpha) \\ 0 & \text{otherwise} \end{cases} \quad (2.4)$$

To find the value of $C_{RSS}(n,\alpha)$ we must solve the equation

$$\alpha = P_{\mu=1}(\min\{Y_i\} > c_{RSS}(n,\alpha)) = \prod_{i=1}^{n}\left(P_{\mu=1}(Y_i > c_{RSS}(n,\alpha)\right)$$

$$= \prod_{i=1}^{n}\left\{\int_{c_{RSS}}^{+\infty} \frac{n!}{(i-1)!(n-i)!}\left[1-e^{-(y_i-\mu)}\right]^{i-1}.\left[e^{-(y_i-\mu)}\right]^{n-i+1} dy_i\right\} \quad (2.5)$$

Then, the power function of this test is

$$\pi_{\varphi_{RSS1}}(\mu) = 1 - P_\mu(\min\{Y_i\} \geq 1) + P_\mu(\min\{Y_i\} > c_{RSS}(n,\alpha))$$

$$= \begin{cases} 1 - \prod_{i=1}^{n} P_\mu(Y_i > 1) + \prod_{i=1}^{n} P_\mu(Y_i > c_{RSS}) & \mu < 1 \\ \prod_{i=1}^{n} P_\mu(Y_i > c_{RSS}) & 1 < \mu < c_{RSS} \\ 1 & \mu > c_{RSS} \end{cases} \quad (2.6)$$

To construct the test based on ERSS, we choose from the i-th group $X_{i1}, X_{i2},\ldots X_{in}$, the first order statistics, i.e., $X_{i(1)}$, i = 1,…,n. Therefore, $X_{1(1)}, X_{2(1)},\ldots X_{n(1)}$ denotes the extreme ranked set sample. To simplify the notation, $X_{i(1)}$ will be denoted by Z_i throughout this paper.

To test the hypothesis (2.1) based on the ERSS, we propose the following test

$$\varphi_{ERSS1}(\underset{\sim}{x}) = \begin{cases} 1 & \min\{Z_i\} < 1 \text{ or } \min\{Z_i\} > c_{ERSS}(n,\alpha) \\ 0 & \text{otherwise} \end{cases} \quad (2.7)$$

To find the value of $c_{ERSS}(n,\alpha)$ we must solve the equation

$$\alpha = P_{\mu=1}(\min\{Z_i\} > c_{ERSS}(n,\alpha)) = \prod_{i=1}^{n}\left(P_{\mu=1}(Z_i > c_{ERSS}(n,\alpha)\right)$$

$$= \prod_{i=1}^{n}\left\{\int_{c_{ERSS}}^{+\infty} ne^{-n(z-1)}dz\right\} = \left\{\int_{c_{ERSS}}^{+\infty} ne^{-n(z-1)}dz\right\}^n = \left\{e^{-n(c_{ERSS}-1)}\right\}^n \quad (2.8)$$

Therefore, $c_{ERSS} = 1 - \frac{1}{n^2}\ln a$. The power function of the φ_{ERSS1} is given by

$$p_{j_{ERSS1}}(m) = 1 - P_m(\min\{Z_i\} \geq 1) + P_m(\min\{Z_i\} > c_{ERSS}(n,a))$$

$$= \begin{cases} 1 - \left\{e^{-n(1-\mu)}\right\}^n + \left\{e^{-n(c_{ERSS}-\mu)}\right\}^n & \mu < 1 \\ \left\{e^{-n(c_{ERSS}-\mu)}\right\}^n & 1 < \mu < c_{ERSS} \\ 1 & \mu > c_{ERSS} \end{cases} \quad (2.9)$$

Table 1: Critical values of the size-α SRS, RSS and ERSS-based tests for the location parameter of two parameter exponential distribution

	α = 0.01			α = 0.05			α = 0.1		
n	c_{SRS}	c_{RRS}	c_{ERSS}	c_{SRS}	c_{RRS}	c_{ERSS}	c_{SRS}	c_{RRS}	c_{ERSS}
3	2.5351	2.0249	1.5117	1.9986	1.7113	1.3329	1.7675	1.5684	1.2558
4	2.1513	1.7227	1.2878	1.7489	1.5087	1.1872	1.5756	1.4095	1.1439
5	1.9210	1.5574	1.1842	1.5991	1.3957	1.1198	1.4605	1.3199	1.0921
6	1.7675	1.4534	1.1279	1.4993	1.3237	1.0832	1.3838	1.2624	1.0640
7	1.6579	1.3821	1.0940	1.4280	1.2739	1.0611	1.3289	1.2224	1.0470
8	1.5756	1.3301	1.0720	1.3745	1.2373	1.0468	1.2878	1.1930	1.0360

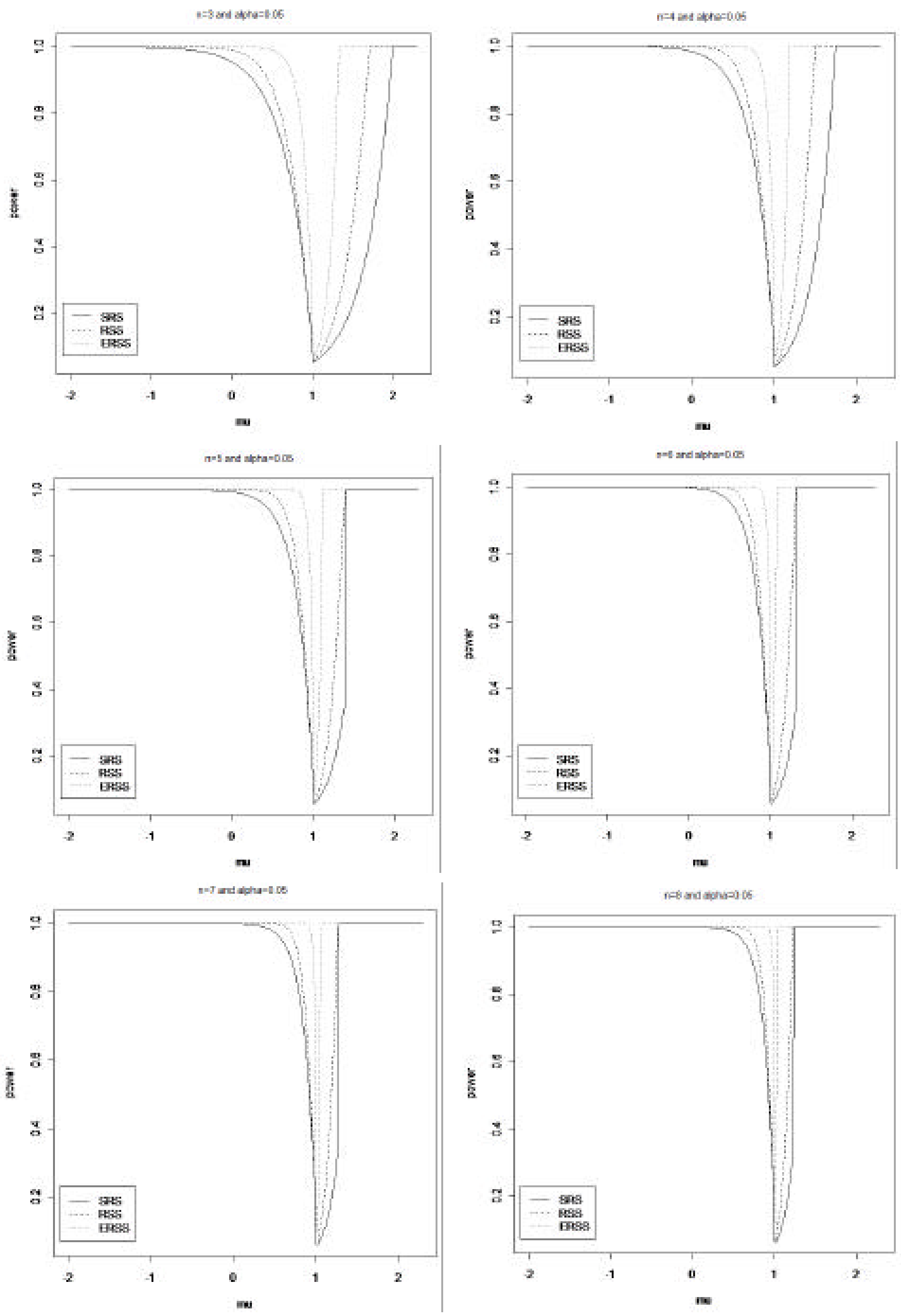

Fig. 1: $\pi_{\varphi_{UMPT1}}(\mu)$, $\pi_{\varphi_{RSS1}}(\mu)$ and $\pi_{\varphi_{ERSS1}}(\mu)$ for $\alpha = 0.05$ and $n = 3,4,\ldots,8$

The values of c_{SRS}, c_{RSS} and c_{ERSS} for different values of α and for $n = 3,4,\ldots,8$ are calculated and are given in Table 1 (Computations done by numerical integration using computer software Maple 11).

In Fig. 1, plots of the $\pi_{\varphi_{ERSS1}}$, $\pi_{\varphi_{RSS1}}$ and $\pi_{\varphi_{UMPT1}}$ for $n = 3,4,\ldots,8$ and $\alpha = 0.05$ are given. These figures show that for all n, φ_{ERSS1} is more powerful than φ_{RSS1} and φ_{UMPT1}. Also, for $n = 3,4,\ldots,8$, φ_{RSS1} is more powerful than φ_{UMPT1}.

To get more quantitative comparison, let MRI denote the maximum rate of the improvement, which is defined by

Table 2: The comparison between $\pi_{\varphi_{ERSS1}}(\mu^*)$ and $\pi_{\varphi_{RSS1}}(\mu^*)$ and the maximum rate of improvement ($\alpha = 0.05$)

	$\alpha = 0.05$			
n	μ^*	$\pi_{\varphi_{ERSS1}}(\mu^*)$	$\pi_{\varphi_{RSS1}}(\mu^*)$	MRI
3	1.33	1.0000	0.2456	307.1846
4	1.18	1.0000	0.2270	417.8286
5	1.11	1.0000	0.1509	562.6905
6	1.08	1.0000	0.1361	634.9385
7	1.06	1.0000	0.1268	688.5745
8	1.04	1.0000	0.1090	817.5901

$$MRI = \frac{\pi_{\varphi_{ERSS1}}(\mu^*) - \pi_{\varphi_{RSS1}}(\mu^*)}{\pi_{\varphi_{RSS1}}(\mu^*)} \times 100\%$$

where $\mu^* = \mu^*(n)$, for given n, is the value such that $\pi_{\varphi_{ERSS1}}(\mu^*) - \pi_{\varphi_{RSS1}}(\mu^*) = \max(\pi_{\varphi_{ERSS1}}(\mu) - \pi_{\varphi_{RSS1}}(\mu))$. The comparison between φ_{ERSS1} and φ_{RSS1} in terms of MRI, for n = 3,4,…,8, is presented in Table 2. Note that all values of MRI are above 307%; which indicate that the improvement of φ_{ERSS1} over φ_{RSS1} is substantial, hence practically significant.

TESTING THE SCALE PARAMETER OF THE PARETO DISTRIBUTION

Let $X_1, X_2, \ldots, X_n$ be a random sample from the pareto distribution with p.d.f.

$$f(x) = \frac{\beta\theta^{\beta}}{x^{\beta+1}} I_{(\theta,+\infty)}(x), \qquad \theta > 0 \ , \ \beta > 0$$

We want to test

$$H_0 : \theta = \theta_0 \quad \text{vs} \quad H_0 : \theta \neq \theta_0 \qquad (3.1)$$

without loss of generality, we assume that $\theta_0 = 1$ and $\beta = 1$. It can be easily show that the UMP test of size α for testing (3.1) is given by

$$\varphi_{UMPT2}(\underset{\sim}{x}) = \begin{cases} 1 & \min\{x_i\} < 1 \ \text{ or } \ \min\{x_i\} > k_{SRS} \\ 0 & \text{otherwise} \end{cases} \qquad (3.2)$$

where $k_{SRS} = \frac{1}{\sqrt[n]{\alpha}}$. The power function of this test is given by

$$\pi_{\varphi_{UMPT2}} = \begin{cases} 1-\theta^n(1-\alpha) & \theta < 1 \\ \alpha\theta^n & 1 < \theta < k_{SRS} \\ 1 & \theta > k_{SRS} \end{cases} \qquad (3.3)$$

Let $Y_i = X_{i(i)}$ be the i-th order statistics in the i-th group (i = 1,2,…,n), which was denoted in Section 2.

To test the hypothesis (3.1) based on the RSS we propose the following test

$$\varphi_{RSS2}(\underset{\sim}{x}) = \begin{cases} 1 & \min\{Y_i\} < 1 \ \text{ or } \ \min\{Y_i\} > k_{RSS}(n,\alpha) \\ 0 & \text{otherwise} \end{cases} \qquad (3.4)$$

To find the value of $k_{RSS}(n,\alpha)$ we must solve the equation

$$\alpha = P_{\theta=1}(\min\{Y_i\} > k_{RSS}(n,\alpha)) = \prod_{i=1}^{n}\left(P_{\theta=1}(Y_i > k_{RSS}(n,\alpha))\right)$$

$$= \prod_{i=1}^{n}\left\{\int_{k_{RSS}}^{+\infty} \frac{n!}{(i-1)!(n-i)!}\left[1-\frac{1}{y_i}\right]^{i-1} \cdot \left[\frac{1}{y_i}\right]^{n-i+2} dy_i\right\}$$

$$= \prod_{i=1}^{n}\left\{\frac{n!}{(i-1)!(n-i)!}\sum_{k=0}^{i-1}\binom{i-1}{k}(-1)^k \frac{1}{n+k-i+1}\cdot\frac{1}{(k_{RSS})^{n+k-i+1}}\right\} \qquad (3.5)$$

Furthermore the power function of this test can be written as

$$\pi_{\varphi_{RSS2}} = \begin{cases} 1-\prod_{i=1}^{n} P_\theta(Y_i > 1) + \prod_{i=1}^{n} P_\theta(Y_i > k_{RSS}) & \theta < 1 \\ \prod_{i=1}^{n} P_\theta(Y_i > k_{RSS}) & 1 < \theta < k_{RSS} \\ 1 & \theta > k_{RSS} \end{cases} \qquad (3.6)$$

To obtain the test based on ERSS method, from the sampling method given in Section 2, we have the sample S = {$Z_1, Z_2, \ldots Z_n$}. Now for testing the hypothesis (3.1) with size α, we propose the following test

$$\varphi_{ERSS2}(\underset{\sim}{x}) = \begin{cases} 1 & \min\{Z_i\} < 1 \ \text{ or } \ \min\{Z_i\} > k_{ERSS}(n,\alpha) \\ 0 & \text{otherwise} \end{cases} \qquad (3.7)$$

where

$$\alpha = P_{\theta=1}(\min\{Z_i\} > k_{ERSS}(n,\alpha)) = \prod_{i=1}^{n}\left(P_{\theta=1}(Z_i > k_{ERSS}(n,\alpha))\right)$$

$$= \prod_{i=1}^{n}\left\{\int_{k_{ERSS}}^{\infty} nz^{-n-1}dz\right\} = \frac{1}{(k_{ERSS})^{n^2}}$$

Therefore $k_{ERSS} = \frac{1}{\sqrt[n^2]{\alpha}}$. Also the power function of φ_{ERSS2} is given by

The values of k_{ERSS}, k_{RSS} and k_{SRS} for different values of α and for n = 3,4,…,8 are calculated and are given in Table 3 (Computations done by numerical integration using computer software Maple 11).

Table 3: Critical values of the size-α SRS, RSS and ERSS-based tests for the scale parameter of pareto distribution

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.1$		
n	k_{SRS}	k_{RSS}	k_{ERSS}	k_{SRS}	k_{RSS}	k_{ERSS}	k_{SRS}	k_{RSS}	k_{ERSS}
3	4.6416	2.7868	1.6681	2.7144	2.0366	1.3950	2.1544	1.7654	1.2916
4	3.1623	2.0599	1.3335	2.1147	1.6632	1.2059	1.7783	1.5060	1.1548
5	2.5119	1.7461	1.2023	1.8206	1.4855	1.1273	1.5849	1.3770	1.0965
6	2.1544	1.5737	1.1365	1.6475	1.3823	1.0868	1.4678	1.3001	1.0661
7	1.9307	1.4653	1.0985	1.5341	1.3150	1.0630	1.3895	1.2493	1.0481
8	1.7783	1.3911	1.0746	1.4542	1.2678	1.0479	1.3335	1.2129	1.0366

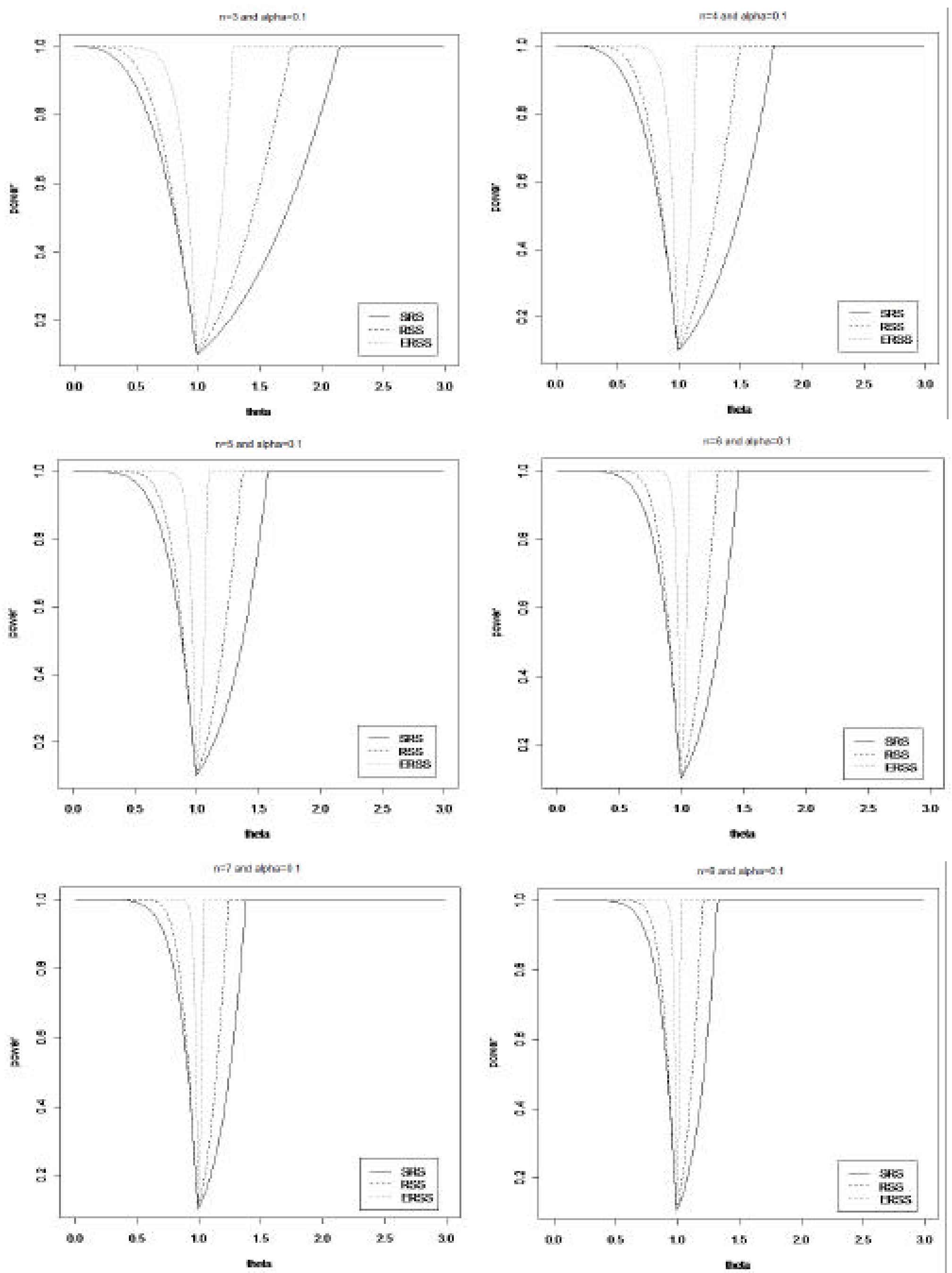

Fig. 2: $\pi_{\varphi_{UMPT2}}(\theta)$, $\pi_{\varphi_{RSS2}}(\theta)$ and $\pi_{\varphi_{ERSS2}}(\theta)$ for $\alpha = 0.1$ and n = 3,4,…,8

$$\pi_{\varphi_{ERSS2}}(\theta)=\begin{cases}1-\theta^{n^2}+\left[\dfrac{\theta}{k_{ERSS}}\right]^{n^2} & \theta<1\\ \left[\dfrac{\theta}{k_{ERSS}}\right]^{n^2} & 1<\theta<k_{ERSS}\\ 1 & \theta>k_{ERSS}\end{cases}\qquad(3.8)$$

In Fig. 2, plots of the $\pi_{\varphi_{ERSS2}}$, $\pi_{\varphi_{RSS2}}$ and $\pi_{\varphi_{UMPT2}}$ for n = 3,4,…,8 and α = 0.1 are given. These figures show that for all n, φ_{ERSS2} is more powerful than φ_{RSS2} and φ_{UMPT2}. Also, for n = 3,4,…,8, φ_{RSS2} is more powerful than φ_{UMPT2}.

Similar to the case of exponential distribution, we compute the MRI of φ_{ERSS2} with respect to φ_{RSS2} and observe that all values of MRI are above 208%, which indicate that the improvement of φ_{ERSS2} over φ_{RSS2} is substantial, hence practically significant.

TESTING THE SCALE PARAMETER OF THE UNIFORM DISTRIBUTION

Let X_1, X_2,….,X_n be a random sample from the uniform distribution with probability density function

$$f(x)=\frac{1}{\theta}I_{(0\theta)}(x),\qquad \theta>0\qquad(4.1)$$

We want to test

$$H_0:\theta=\theta_0\quad vs\quad H_0:\theta\neq\theta_0\qquad(4.2)$$

It is well known that the UMP test of size α for testing (4.2) is given by

$$\varphi_{UMPT3}(\underset{\sim}{x})=\begin{cases}1 & \max\{x_i\}>\theta_0\ \text{ or }\ \max\{x_i\}<l_{SRS}(n,\alpha)\\ 0 & \text{otherwise}\end{cases}\qquad(4.3)$$

without lost of generality we take θ_0 = 1. Then $l_{SRS}(n,\alpha)=\sqrt[n]{\alpha}$. The power function of this test is given by

$$\pi_{\varphi_{UMPT3}}(\theta)=P_\theta(\max\{x_i\}>1)+P_\theta(\max\{x_i\}<\sqrt[n]{\alpha})$$
$$=\begin{cases}1 & \theta<\sqrt[n]{\alpha}\\ \left(\dfrac{1}{\theta}\right)^n\alpha & \sqrt[n]{\alpha}<\theta<1\\ 1-(1-\alpha)\left(\dfrac{1}{\theta}\right)^n & \theta>1\end{cases}\qquad(4.4)$$

To test the hypothesis (4.2) based on the RSS, Abu-Dayyeh and Muttlak [3] proposed the following test

$$\varphi_{RSS3}(\underset{\sim}{x})=\begin{cases}1 & \max\{Y_i\}>1\ \text{ or }\ \max\{Y_i\}<l_{RSS}(n,\alpha)\\ 0 & \text{otherwise}\end{cases}\qquad(4.5)$$

where Y_i = $X_{i(i)}$ is the i-th order statistics in the i-th group (i = 1,2,…,n) and l_{RSS} is the solution of the following equation

$$\alpha=\prod_{i=1}^{n}\left\{\int_0^{l_{RSS}}\frac{n!}{(i-1)!(n-i)!}[y_i]^{i-1}.[1-y_i]^{n-i}dy_i\right\}$$
$$=\prod_{i=1}^{n}\left\{\frac{n!}{(i-1)!(n-i)!}\sum_{k=0}^{n-i}\binom{n-i}{k}(-1)^k.\frac{1}{i+k}l_{ERSS}^{i+k}\right\}\qquad(4.6)$$

Also the power function of this test is given by

$$\pi_{\varphi_{RSS3}}(\theta)=\begin{cases}1 & \theta\le l_{RSS}\\ \prod_{i=1}^{n}P_\theta(Y_i\le l_{RSS}) & l_{RSS}<\theta\le 1\\ 1-\prod_{i=1}^{n}P_\theta(Y_i\le 1)+\prod_{i=1}^{n}P_\theta(Y_i\le l_{RSS}) & \theta>1\end{cases}\qquad(4.7)$$

To obtain the test based on ERSS, we choose from the i-th group $X_{i1},X_{i2},\ldots,X_{in}$, the largest order statistics $X_{i(1)}, i=1,\ldots,n$. Therefore $X_{1(n)},X_{2(n)},\ldots,X_{n(n)}$ denotes the extreme ranked set sample. To simplify the notation, $X_{i(n)}$ will be denoted by W_i throughout this paper.

To test the hypothesis (4.2) based on the ERSS we propose the following test

$$\varphi_{ERSS3}(\underset{\sim}{x})=\begin{cases}1 & \max\{W_i\}>1\ \text{ or }\ \max\{W_i\}<l_{ERSS}(n,\alpha)\\ 0 & \text{otherwise}\end{cases}\qquad(4.8)$$

To find the value of l_{ERSS} (n,α) we must solve the equation

$$\alpha=P_{\theta=1}(\max\{W_i\}<l_{ERSS}(n,\alpha))=\prod_{i=1}^{n}\left(P_{\theta=1}(W_i<l_{ERSS}(n,\alpha)\right)$$
$$=\prod_{i=1}^{n}\left\{\int_0^{l_{ERSS}}nw^{n-1}dw\right\}=\left(l_{ERSS}\right)^{n^2}\qquad(4.9)$$

which reduce to $l_{ERSS}=\sqrt[n^2]{\alpha}$. The power function of the φ_{ERSS3} is given by

Table 4: Critical values of the size-α SRS, RSS and ERSS-based tests for a scale parameter of the uniform distribution

	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.1$		
n	l_{SRS}	l_{RSS}	l_{ERSS}	l_{SRS}	l_{RSS}	l_{ERSS}	l_{SRS}	l_{RSS}	l_{ERSS}
3	0.2154	0.3588	0.5995	0.3684	0.4910	0.7169	0.4642	0.5664	0.7743
4	0.3162	0.4855	0.7499	0.4729	0.6013	0.8293	0.5623	0.6640	0.8660
5	0.3981	0.5727	0.8318	0.5493	0.6732	0.8871	0.6310	0.7262	0.9120
6	0.4642	0.6355	0.8799	0.6070	0.7235	0.9202	0.6813	0.7692	0.9380
7	0.5179	0.6824	0.9103	0.6518	0.7604	0.9407	0.7197	0.8006	0.9541
8	0.5623	0.7188	0.9306	0.6877	0.7888	0.9543	0.7499	0.8245	0.9647

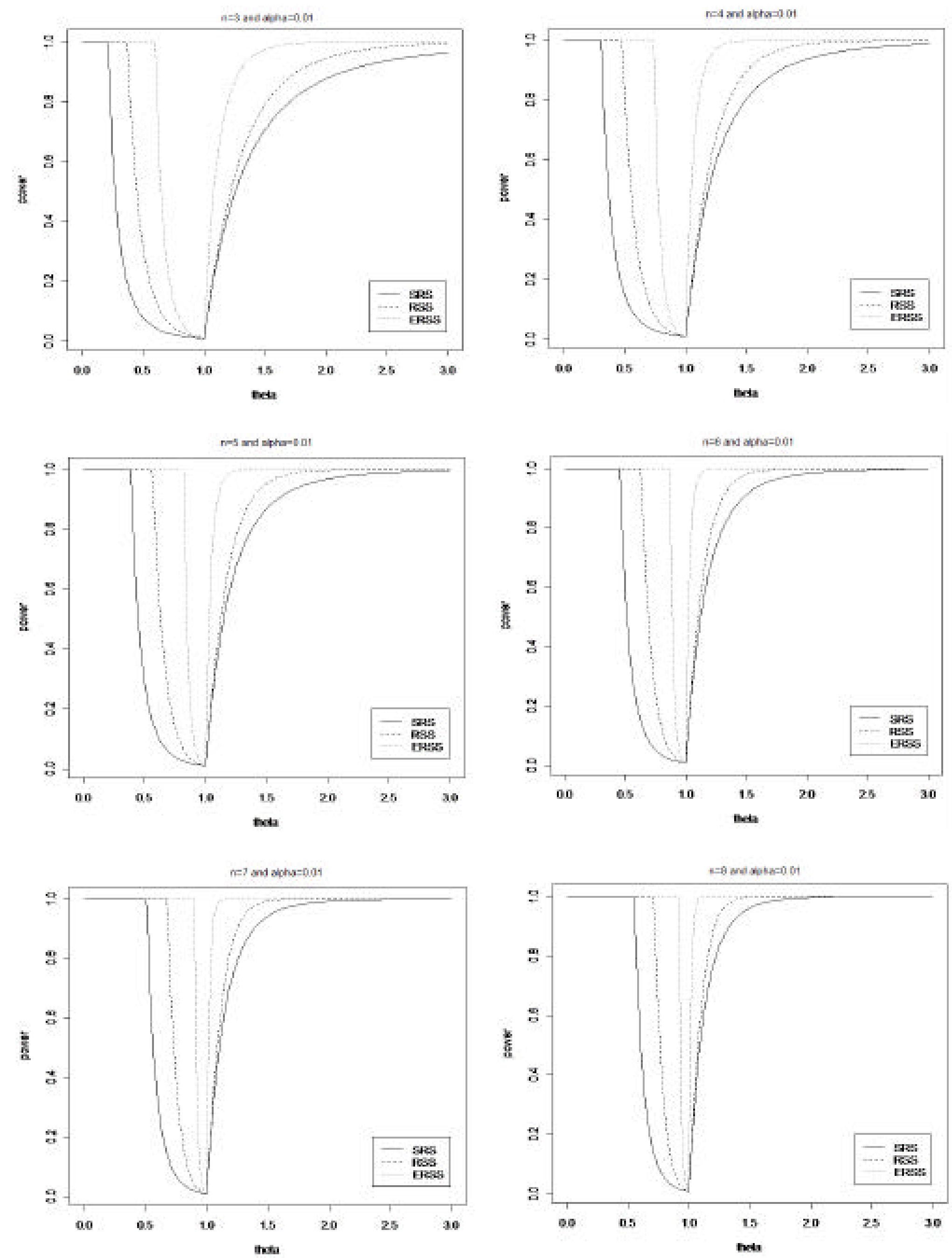

Fig. 3: $\pi_{\varphi_{UMPT3}}(\theta)$, $\pi_{\varphi_{RSS3}}(\theta)$ and $\pi_{\varphi_{ERSS3}}(\theta)$ for $\alpha = 0.01$ and n = 3,4,…,8

$$\pi_{\varphi_{ERSS3}}(\theta) = 1 - P_\theta(\max\{W_i\} \le 1) + P_\theta(\max\{W_i\} \le l_{ERSS}(n,\alpha))$$

$$= \begin{cases} 1 & \theta < \sqrt[n^2]{\alpha} \\ \left(\dfrac{1}{\theta^n}\right)^n \alpha & \sqrt[n^2]{\alpha} < \theta < 1 \\ 1-(1-\alpha)\left(\dfrac{1}{\theta^n}\right)^n & \theta > 1 \end{cases} \quad (4.10)$$

The values of l_{SRS}, l_{RSS} and l_{ERSS}, for different values of α and for n = 3,4,…,8 are calculated and are given in Table 4 (Computations done by numerical integration using computer software Maple 11).

In Fig. 3, plots of the $\pi_{\varphi_{ERSS3}}$, $\pi_{\varphi_{RSS3}}$ and $\pi_{\varphi_{UMPT3}}$ for n = 3,4,…,8 and α = 0.01 are given. From these figures we observe that for all n, φ_{ERSS3} is more powerful than φ_{RSS3} and φ_{UMPT3}. Also, for n = 3,4,…,8, φ_{RSS3} is more powerful than φ_{UMPT3}.

Similarly, we compute the MRI of φ_{ERSS3} with respect to φ_{RSS3} and observe a substantial improvement.

CONCLUSION

According to the results obtained in previous sections, we observe that the test introduced based on ERSS method on location parameter of the two-parameter exponential distribution and scale parameters of the pareto and uniform distributions are more powerful than RSS-based and UMP tests. Also, RSS-based tests are more powerful than UMP test. So, in situations where the measurement of survey variable is costly and/or time-consuming but ranking of sample items relating to the survey variable can easily done, the test based on ERSS is recommended.

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