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Some New Solutions of the Higher-order Sawada-Kotera Equation via the Exp-function Method

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Abstract: In this article, the Exp-function method is applied to construct traveling wave solutions of the fifth-order Sawada-Kotera equation. This method is one of the powerful methods that appear in recent time for establishing exact traveling wave solutions of nonlinear partial differential equations. The solution procedure of this method is implemented by symbolic software, such as, Maple. We obtain some new exact solutions including solitary and periodic wave solutions. It is shown that the Exp-function method is straightforward and effective mathematical tool for solving nonlinear evolution equations in mathematical physics and engineering sciences. In addition, some of solutions are described in the figures with the aid of commercial software Maple.

Key words: The exp-function method . nonlinear partial differential equations . the Sawada-Kotera equation . traveling wave solutions

INTRODUCTION

Many phenomena in physics, applied mathematics and engineering sciences are described by Nonlinear Evolution Equations (NLEEs). The investigation of traveling wave solutions for NLEEs is being a promising subject in different branches of mathematical and physical sciences, such as, biology, chemistry, physics etc. In recent time, many methods have been recommended to obtain exact solutions of nonlinear evolution equations, for instance, the Hirota's bilinear transformation method [1], the Backlund transformation method [2], the inverse scattering method [3], the homotopy analysis method [4-7], the tanh-function method [8], the homogeneous balance method [9], the variational iteration method [10-18], the Jacobi elliptic function expansion method [19] and others [20-29].

Recently, He and Wu [30] presented a straightforward and concise method, called the Exp-function method to obtain generalized solitary wave solutions of NLEEs. The Exp-function method is being effectively used to study various kinds of differential equations for establishing traveling wave solutions, for example, discrete equation [31], high-dimensional equations [32-34], modified Zakharov-Kuznetsov and Zakharov-Kuznetsov-modified equal width equation [35], KdV-Burger's equation [36], equations with variable-coefficients [37, 38] reaction-diffusion equations [39], a system of nonlinear partial differential equations [40], nonlinear evolution equations with higher order nonlinearity [41], generalized fisher equation [42], sixth-order Boussinesq equation and regularized long wave equations [43], higher dimensional nonlinear partial differential equation [44], two-dimensional Ginzburg-Landau equation [45], nonlinear Beam equation [46] and so on.

In this article, we apply the Exp-function method to construct exact solutions including solitary wave solutions and periodic wave solutions for the fifth-order Sawada-Kotera equation.

BASIC IDEA OF THE EXP-FUNCTION METHOD

This paper is dealt with the fifth-order Sawada-Kotera equation Referred by Liu and Dai [47]:

$$u_t + 45u^2u_x + 15u_x u_{xx} + 15uu_{xxx} + u_{xxxxx} = 0 \tag{1}$$

Now, we present the Exp-function method in solving the nonlinear partial differential equation of the form:

$$Q(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt} ...) = 0 \tag{2}$$

where Q is a polynomial in u = u(x,t) and the subscripts indicate the partial derivatives. The main steps of the Exp-function method [30] are as follows:

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Step 1: Consider the traveling wave variable

$$u(x,t) = v(\eta),\ \eta = x + st \qquad (3)$$

Now using the traveling wave variable (3), Eq. (2) becomes an ordinary differential equation (ODE) in the form:

$$P(v, v', v'', v''', \ldots) = 0 \qquad (4)$$

where primes denote the derivatives with respect to η.

Step 2: Suppose the solution of the ODE (4) can be expressed in the following form [30]:

$$v(\eta) = \frac{\sum_{n=-c}^{d} a_n \exp(n\eta)}{\sum_{m=-p}^{q} b_m \exp(m\eta)} = \frac{a_{-c}\exp(-c\eta) + \cdots + a_d \exp(d\eta)}{b_{-p}\exp(-p\eta) + \cdots + b_q \exp(q\eta)} \qquad (5)$$

where c,d,p and q are positive integers which are unknown to be determined, a_n and b_m are unknown constants. Eq. (5) can be rewritten in the following equivalent form:

$$v(\eta) = \frac{a_c \exp(c\eta) + \ldots + a_{-d}\exp(-d\eta)}{b_p \exp(p\eta) + \ldots + b_{-q}\exp(-q\eta)} \qquad (6)$$

This equivalent presentation plays an important and fundamental role for searching analytical solutions of NLEEs.

Step 3: For determining the values of c and p, we balance the linear term of the highest order to the highest order nonlinear term and for determining the values of d and q, we balance the lowest order linear term to the lowest order nonlinear term in Eq. (4). This completes the determination of the values of c,d,p and q.

Step 4: Putting the values of c,d,p and q into Eq. (6), then substituting Eq. (6) into Eq. (4) and simplifying, we obtain

$$\sum_j C_j \exp(j\eta) = 0 \qquad (7)$$

Setting each coefficient $C_j = 0$, yields a set of algebraic equations for a_c's and b_p's.

Step 5: Suppose the unknown a_c's and b_p's can be obtained by solving a set algebraic equations obtained in step 4. Substituting these values into Eq. (6), we obtain exact traveling wave solutions of the Eq. (2).

APPLICATIONS OF THE METHOD

In this section, we use the Exp-function method to construct generalized solitary and periodic solutions of the fifth-order Sawada-Kotera equation (1). The obtained solutions and the solutions obtained in previous literature are discussed in this section. Furthermore, the obtained solutions are depicted in graphs with the aid of commercial software Maple.

Solutions of the Sawada-Kotera equation: In this subsection, we apply the Exp-function method [30] to the fifth-order Sawada-Kotera equation (1). Using the transformation (3), Eq. (1) becomes

$$sv' + 45v^2 v' + 15v'v'' + 15vv''' + v^{(5)} = 0 \qquad (8)$$

where primes denote the derivatives with respect to η. According to step 2, solution of Eq. (8) can be written in the form of (6).

To determine the values of c and p, according to step 3, we balance the linear term of the highest order in Eq. (8) with the highest order nonlinear term. With the aid of Maple, we obtain

$$v^{(5)} = \frac{c_1 \exp[(5p + c)\xi] + \ldots}{c_2 \exp[6p\xi] + \ldots} \qquad (9)$$

and

$$vv''' = \frac{c_3 \exp[2(2p + c)\xi] + \ldots}{c_4 \exp[6p\xi] + \ldots} \qquad (10)$$

where c_i's are determined coefficients only for simplicity. Balancing the highest order of the Exp-function, from (9) and (10), we obtain 5p+c = 2(2p+c), which leads to the result: p = c.

To determine the values of q and d, we balance the linear term of the lowest order in Eq. (8) with the lowest order nonlinear term. We have

$$v^{(5)} = \frac{\ldots + d_1 \exp[-(5q + d)\xi]}{\ldots + d_2 \exp[-6q\xi]} \qquad (11)$$

$$vv''' = \frac{\ldots + d_3 \exp[-2(2q + d)\xi]}{\ldots + d_4 \exp[-6q\xi]} \qquad (12)$$

Balancing the lowest order of the Exp-function, from Eqs. (11) and (12), we obtain -(5q+d) = -2(2q+d) which leads to the result: q = d.

Here, c and d are free parameters, therefore, they can take any values. But, the final solution does not depend upon the choice of values of c and d.

Case 1: Choose p = c = 1, q = d = 1.
For this case, the trial solution Eq. (6) reduces to

$$v(\eta)=\frac{a_1e^{\eta}+a_0+a_{-1}e^{-\eta}}{b_1e^{\eta}+b_0+b_{-1}e^{-\eta}} \tag{13}$$

In case $b_1\neq 0$, Eq. (13) can be simplified as:

$$v(\eta)=\frac{a_1e^{\eta}+a_0+a_{-1}e^{-\eta}}{e^{\eta}+b_0+b_{-1}e^{-\eta}} \tag{14}$$

By substituting Eq. (14) into Eq. (8) and equating the coefficients of $\exp(\pm n\eta)$, n = 0,1,2,3,…, with the aid of algebraic software Maple, we obtain a set of algebraic equations in terms of a_{-1}, a_0, a_1, b_{-1}, b_0 and s

$$\begin{aligned}&\frac{1}{A}(C_5e^{5\eta}+C_4e^{4\eta}+C_3e^{3\eta}+C_2e^{2\eta}+C_1e^{\eta}+C_0+C_{-1}e^{-\eta}\\&+C_{-2}e^{-2\eta}+C_{-3}e^{-3\eta}+C_{-4}e^{-4\eta}+C_{-5}e^{-5\eta})=0\end{aligned} \tag{15a}$$

and setting each coefficient of $\exp(\pm n\eta)$, n = 0,1,2,3,…, to zero, we obtain

$$\begin{aligned}&C_5=0, C_4=0, C_3=0, C_2=0, C_1=0, C_0=0,\\&C_{-1}=0, C_{-2}=0, C_{-3}=0, C_{-4}=0, C_{-5}=0\end{aligned} \tag{15b}$$

For determining unknowns, the obtained above system of algebraic Eqs. (15b) have been solved with the aid of commercial software Maple, we obtain the following solutions.

$$\begin{aligned}&b_0=b_0,\quad a_0=\frac{5}{3}b_0,\quad a_1=-\frac{1}{3},\\&a_{-1}=-\frac{1}{12}b_0^2,\quad b_{-1}=\frac{1}{4}b_0^2,\quad s=-1\end{aligned} \tag{16}$$

where b_0 is arbitrary constant.

$$\begin{aligned}&a_1=a_1,\quad b_0=b_0,\quad a_{-1}=\frac{1}{4}a_1b_0^2,\quad a_0=a_1b_0+b_0,\\&b_{-1}=\frac{1}{4}b_0^2,\quad s=-45a_1^2-15a_1-1\end{aligned} \tag{17}$$

where a_1 and b_0 are arbitrary constants.

Now substituting Eq. (16) into Eq. (14), we obtain following solution:

$$v(\eta)=\frac{-4e^{\eta}+20b_0-b_0^2e^{-\eta}}{12e^{\eta}+12b_0+3b_0^2e^{-\eta}} \tag{18}$$

Eq. (18) can be simplified as follows:

$$v(\eta)=\frac{-1}{3}+\frac{8b_0}{(4+b_0^2)\cosh\eta+(4-b_0^2)\sinh\eta+4b_0} \tag{19}$$

where $\eta = x - t$.
If $b_0 = 2$, Eq. (19) becomes

$$v(x,t)=\frac{-1}{3}+\frac{2}{1+\cosh(x-t)} \tag{20}$$

Again, substituting Eq. (17) into Eq. (14) and simplifying, we obtain

$$v(\eta)=a_1+\frac{4b_0}{(4+b_0^2)\cosh\eta+(4-b_0^2)\sinh\eta+4b_0} \tag{21}$$

where $\eta=x-(45a_1^2+15a_1+1)t$.

If $b_0 = 2$, Eq. (21) becomes

$$v(x,t)=a_1+\frac{1}{1+\cosh\left[x-(45a_1^2+15a_1+1)t\right]} \tag{22}$$

Case 2: p = c = 2, q = d = 1.
For this case, the trial solution Eq. (6) becomes

$$v(\eta)=\frac{a_2e^{2\eta}+a_1e^{\eta}+a_0+a_{-1}e^{-\eta}}{b_2e^{2\eta}+b_1e^{\eta}+b_0+b_{-1}e^{-\eta}} \tag{23}$$

Since, there are some free parameters in Eq. (23), for simplicity, we may consider $b_2 = 1$ and $b_{-1} = 0$. Then the solution Eq. (23) is simplified as follows:

$$v(\eta)=\frac{a_2e^{2\eta}+a_1e^{\eta}+a_0+a_{-1}e^{-\eta}}{e^{2\eta}+b_1e^{\eta}+b_0} \tag{24}$$

Executing the same procedure as described in case 1, we obtain

$$\begin{aligned}&b_1=b_1,\quad a_{-1}=0,\quad a_0=-\frac{1}{12}b_1^2,\\&a_1=\frac{5}{3}b_1,\quad a_2=-\frac{1}{3},\quad b_0=\frac{1}{4}b_1^2,\quad s=-1\end{aligned} \tag{25}$$

where b_1 is free parameter.

$$\begin{aligned}&a_2=a_2,\quad b_1=b_1,\quad a_{-1}=0,\quad a_0=\frac{1}{4}a_2b_1^2,\\&a_1=a_2b_1+b_1,\quad b_0=\frac{1}{4}b_1^2,\quad s=-45a_2^2-15a_2-1\end{aligned} \tag{26}$$

where a_2 and b_1 are free parameters.

Substituting Eq. (25) into Eq. (24) and simplifying, we obtain

$$v(\eta)=\frac{-1}{3}+\frac{8b_1}{\left(4+b_1^2\right)\cosh\eta+\left(4-b_1^2\right)\sinh\eta+4b_1} \tag{27}$$

where $\eta = x\text{-}t$.

If $b_1 = 2$, Eq. (27) becomes

$$v(x,t)=\frac{-1}{3}+\frac{2}{1+\cosh(x-t)} \tag{28}$$

Again, substituting Eq. (26) into Eq. (24) and simplifying, we obtain

$$v(\eta)=a_2+\frac{4b_1}{\left(4+b_1^2\right)\cosh\eta+\left(4-b_1^2\right)\sinh\eta+4b_1} \tag{29}$$

where $\eta = x-\left(45a_2^2+15a_2+1\right)t$.

If $b_1 = 2$, Eq. (29) becomes

$$v(x,t)=a_2+\frac{1}{1+\cosh\left(x-\left(45a_2^2+15a_2+1\right)t\right)} \tag{30}$$

It is noted that solutions (20) and (28) are identical. And if we set $a_2 = a_1$, solution (30) is identical to solution (22).

Case 3: Choose p = c = 2, q = d = 2.

For this case, the trial solution Eq. (6) becomes

$$v(\eta)=\frac{a_2e^{2\eta}+a_1e^{\eta}+a_0+a_{-1}e^{-\eta}+a_{-2}e^{-2\eta}}{b_2e^{2\eta}+b_1e^{\eta}+b_0+b_{-1}e^{-\eta}+b_{-2}e^{-2\eta}} \tag{31}$$

For simplicity, we may consider $b_2 = 1$ and $b_{-1} = b_{-2} = 0$. Then solution Eq. (31) is simplified as follows:

$$v(\eta)=\frac{a_2e^{2\eta}+a_1e^{\eta}+a_0+a_{-1}e^{-\eta}+a_{-2}e^{-2\eta}}{e^{2\eta}+b_1e^{\eta}+b_0} \tag{32}$$

Executing the same procedure as described in case 1, we obtain

$$\begin{aligned}&b_1=b_1,\quad a_{-1}=0,\quad a_{-2}=0,\quad a_0=\frac{-1}{12}b_1^2,\\&a_1=\frac{5}{3}b_1,\quad a_2=-\frac{1}{3},\quad b_0=\frac{1}{4}b_1^2,\quad s=-1\end{aligned} \tag{33}$$

where b_1 is free parameter.

$$\begin{aligned}&a_2=a_2,\quad b_1=b_1,\quad a_{-2}=0,\quad a_{-1}=0,\quad a_0=\frac{1}{4}a_2b_1^2,\\&a_1=a_2b_1+b_1,\quad b_0=\frac{1}{4}b_1^2,\quad s=-45a_2^2-15a_2-1\end{aligned} \tag{34}$$

where a_2 and b_1 are free parameters.

Substituting the Eq. (33) into Eq. (32) and simplifying, we obtain

$$v(\eta)=\frac{-1}{3}+\frac{8b_1}{\left(4+b_1^2\right)\cosh\eta+\left(4-b_1^2\right)\sinh\eta+4b_1} \tag{35}$$

where $\eta = x\text{-}t$.

If $b_1 = 2$, Eq. (35) becomes

$$v(x,t)=\frac{-1}{3}+\frac{2}{1+\cosh(x-t)} \tag{36}$$

Again, substituting Eq. (34) into Eq. (32) and simplifying, we obtain

$$v(\eta)=a_2+\frac{4b_1}{\left(4+b_1^2\right)\cosh\eta+\left(4-b_1^2\right)\sinh\eta+4b_1} \tag{37}$$

where $\eta = x-\left(45a_2^2+15a_2+1\right)t$.

If $b_1 = 2$, Eq. (37) becomes

$$v(x,t)=a_2+\frac{1}{1+\cosh\left(x-\left(45a_2^2+15a_2+1\right)t\right)} \tag{38}$$

Now, we have observed that solution (36) is identical to solutions (20) and (28). And if we set $a_2 = a_1$, solution (38) is identical to solutions (22) and (30).

Case 4: Now choose p = c = 3 and q = d = 2

For this case, the trial solution Eq. (6) becomes

$$v(\eta)=\frac{a_3e^{3\eta}+a_2e^{2\eta}+a_1e^{\eta}+a_0+a_{-1}e^{-\eta}+a_{-2}e^{-2\eta}}{b_3e^{3\eta}+b_2e^{2\eta}+b_1e^{\eta}+b_0+b_{-1}e^{-\eta}+b_{-2}e^{-2\eta}} \tag{39}$$

Eq. (39) can be rewritten as:

$$v(\eta)=\frac{a_3e^{2\xi}+a_2e^{\xi}+a_1+a_0e^{-\xi}+a_{-1}e^{-2\xi}+a_{-2}e^{-3\xi}}{b_3e^{2\xi}+b_2e^{\xi}+b_1+b_0e^{-\xi}+b_{-1}e^{-2\xi}+b_{-2}e^{-3\xi}} \tag{40}$$

Since, there are some free parameters in Eq. (40), we may consider $a_{-2} = 0$, $b_3 = 1$, $b_0 = 0$, $b_{-1} = 0$ and $b_{-2} = 0$, so that the Eq. (40) reduces to the Eq. (32). This indicates that the case 4 is equivalent to the case 3. Similarly, if we put $a_{-2} = 0$, $a_{-1} = 0$, $b_3 = 1$, $b_0 = 0$, $b_{-1} = 0$

and $b_{-2} = 0$ into Eq. (40), we obtain the trial solution form as Eq. (24). i.e., case 4 is also equivalent to the case 2.

Again, Eq. (40) can be rewritten in the form:

$$v(\eta) = \frac{a_3 e^{\eta} + a_2 + a_1 e^{-\eta} + a_0 e^{-2\eta} + a_{-1} e^{-3\eta} + a_{-2} e^{-4\eta}}{b_3 e^{\eta} + b_2 + b_1 e^{-\eta} + b_0 e^{-2\eta} + b_{-1} e^{-3\eta} + b_{-2} e^{-4\eta}} \quad (41)$$

If we set $a_{-2} = 0$, $a_{-1} = 0$, $a_0 = 0$, $b_3 = 1$, $b_{-2} = 0$, $b_{-1} = 0$ and $b_0 = 0$ into Eq.(41), we obtain the same form as Eq. (14). This implies that the case 4 is equivalent to the case 1.

If we consider $p = c = 3$ and $q = d = 3$, it can be shown that this Case is also equivalent to the case 1, case 2 and case 3.

Results and discussion: Many authors obtained traveling wave solutions of the fifth order Sawada-Kotera equation by applying different methods, such as, Liu and Dai [47] studied this equation by using Hirota's bilinear method. Feng and Zheng [48] established traveling wave solutions of the same equation via the (G'/G)-expansion method. Wazwaz [49] concerned about the extended tanh method to obtain exact solutions of this equation. Salas [50] implemented the projective Riccati equation method to construct analytical solutions of the same equation. But, to the best of our knowledge, the fifth-order Sawada-Kotera equation is not investigated to construct exact traveling wave solutions by appying the Exp-function method. The solutions are obtained in this article are new and have not been found in the previous literature.

If we set $a_1 = a_2 = 0$ in (22), (30) and (38). Then Eqs. (22), (30) and (38) become Liu and Dai's [47] solution which reads:

$$u_1(x,t) = \frac{1}{2}\operatorname{sech}^2\left(\frac{x-t}{2}\right), \text{when } P = \frac{1}{2} \quad (42)$$

Graphical representations of the solutions: With the aid of Maple, the graphical illustrations of solutions are demonstrated in the Fig. 1-12:

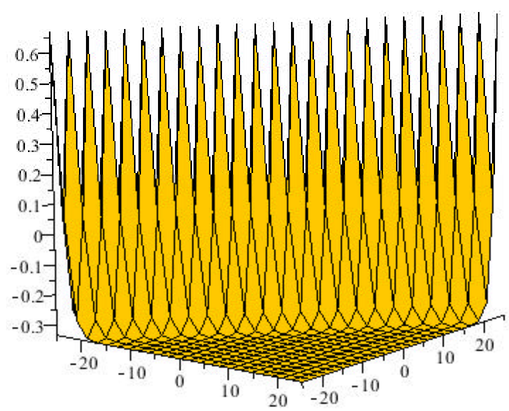

Fig. 1: Solitons solution

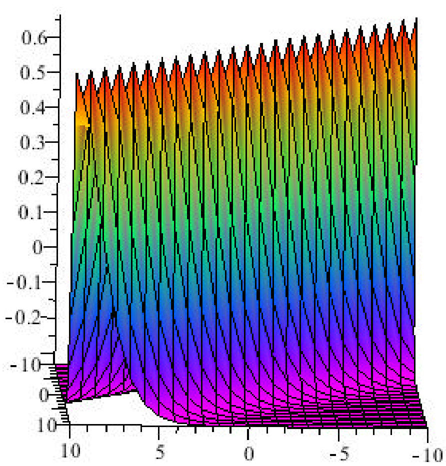

Fig. 2: Solitons solution

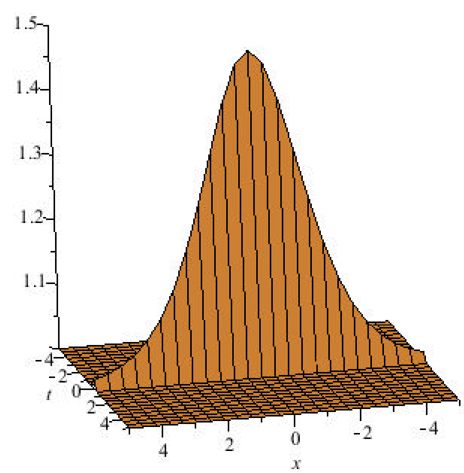

Fig. 3: Solitons solution for $a_1 = 1$

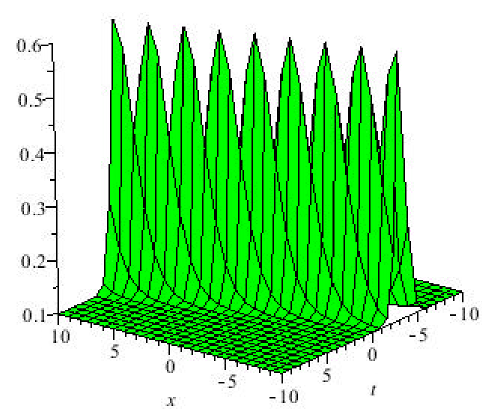

Fig. 4: Solitons solution for $a_1 = 1\times10^{-1}$

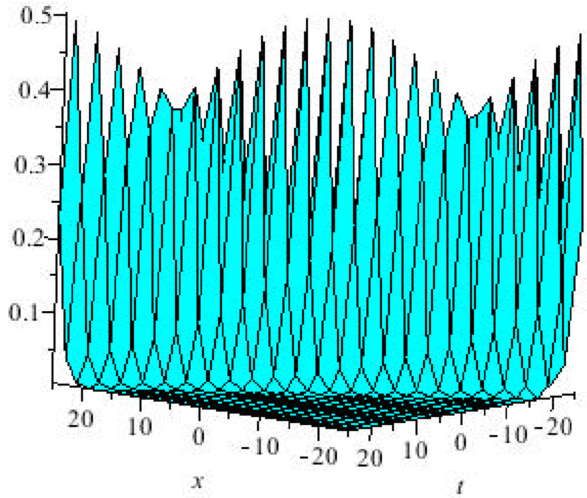

Fig. 5: Solitons solution for $a_1 = 5\times10^{-3}$

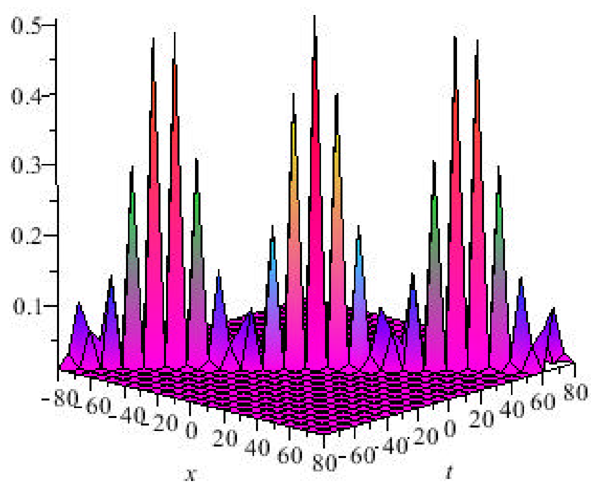

Fig. 6: solitons solution for $a_1 = 1\times10^{-2}$

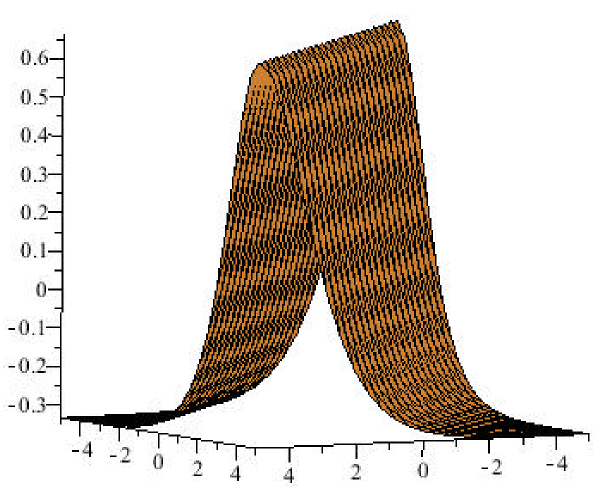

Fig. 7: Periodic solution

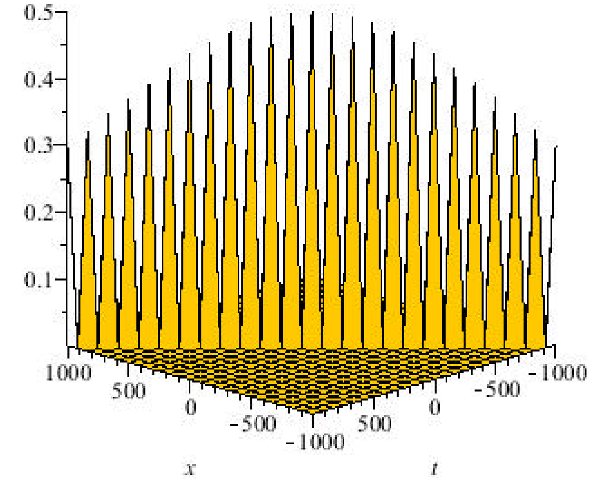

Fig. 8: Solitons solution for $a_1 = 1\times10^{-4}$

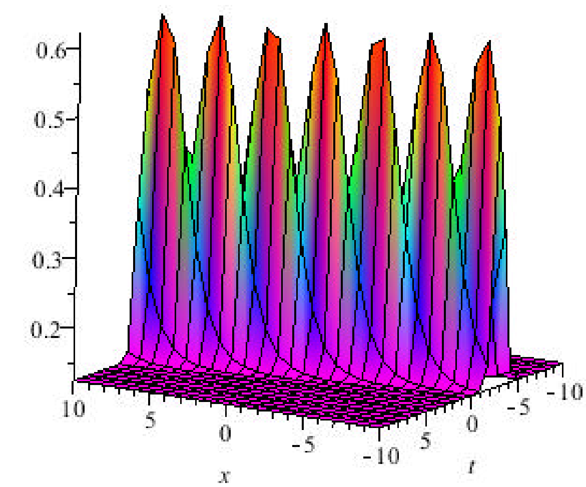

Fig. 9: Solitons solution for $a_1 = 0.125$

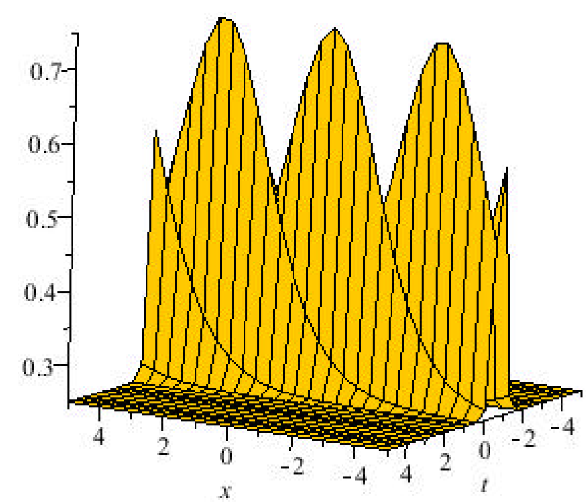

Fig. 10: Solitons solution for $a_1 = 0.25$

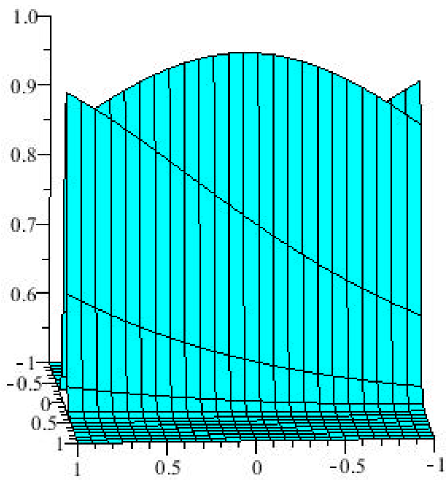

Fig. 11: Periodic solution for $a_1 = 0.5$

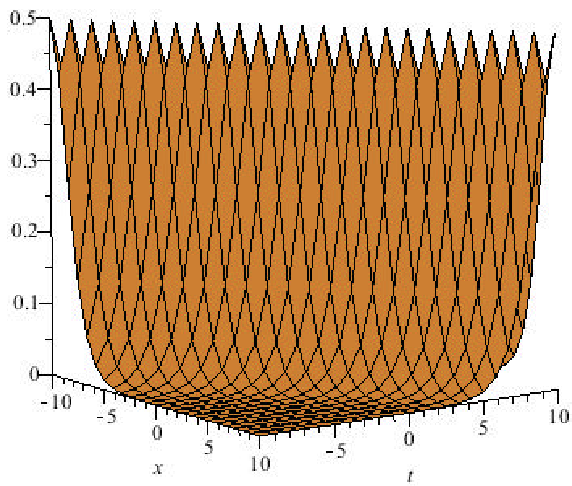

Fig. 12: Solitons solution for $a_1 = 1\times10^{-9}$

CONCLUSIONS

The Exp-function method is successfully applied for constructing some new traveling wave solutions of the fifth-order Sawada-Kotera equation which is highly nonlinear. Our obtained traveling wave solutions demonstrate that the Exp-function method is straightforward and concise mathematical tool to establish analytical solutions of NLEEs. Therefore, we hope that this method can be more effectively used to investigate others NLEEs which are frequently take place in engineering, applied mathematics and physical sciences.

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