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Anti Fuzzy Structures on Graphs

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Abstract: We introduce the concepts of connected anti fuzzy graphs, self-centroid anti fuzzy graphs, constant anti fuzzy graphs, totally constant anti fuzzy graphs, neighbourly irregular anti fuzzy graphs, neighbourly totally irregular anti fuzzy graphs, highly irregular anti fuzzy graphs and highly totally irregular anti fuzzy graphs and present some of their properties.

Key words: Anti fuzzy graphs, connected anti fuzzy graphs, self-centroid anti fuzzy graphs, irregular anti fuzzy graphs, highly irregular anti fuzzy graphs, anti fuzzy digraph.

INTRODUCTION

In 1736, Euler first introduced the notion of graph theory. In the history of mathematics, the solution given by Euler of the well known Königsberg bridge problem is considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas. It is noted that many real world systems can be modelled using graphs. Any entity involving points and connections between them may be called a graph. The connections may be physical as in electrical networks and computer networks or relationships as in molecules and ecosystems.

In 1975, Rosenfeld [1] discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann [2] in 1973. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Bhattacharya [3] gave some remarks on fuzzy graphs. Mordeson and Peng [4] introduced several concepts on fuzzy graphs. Gani and Latha [5] introduced irregular fuzzy graphs and Karunambigai *et al.*[6] presented some properties of intuitionistic fuzzy graphs. Recently, Akram *et al.* [7-16] introduced many structures of fuzzy graphs including bipolar fuzzy graphs, interval-valued fuzzy line graphs, strong intuitionistic fuzzy graphs, $\mathcal{N}$-graphs, cofuzzy graphs. In this paper, we introduce the concepts of connected anti fuzzy graphs, self-centroid anti fuzzy graphs, constant anti fuzzy graphs, totally constant anti fuzzy graphs, neighbourly irregular anti fuzzy graphs, neighbourly totally irregular anti fuzzy graphs, highly irregular anti fuzzy graphs and highly totally irregular anti fuzzy graphs and present some of their properties.

PRELIMINARIES

By a graph, we mean a pair $G^* = (V, E)$, where V is the set and E is a relation on V. The elements of V are vertices of G^* and the elements of E are edges of G^*. We write $xy \in E$ to mean $\{x, y\} \in E$, and if $e = xy \in E$, we say x and y are *adjacent*. Formally, given a graph $G^* = (V, E)$, two vertices x, $y \in V$ are said to be *neighbors, or adjacent nodes*, if $xy \in E$. The *neighborhood* of a vertex v in a graph G^* is the induced subgraph of G^* consisting of all vertices adjacent to v and all edges connecting two such vertices. The neighborhood is often denoted by $N(v)$. The degree $\deg(v)$ of vertex v is the number of edges incident on v. The set of neighbors, called a *open neighborhood* $N(v)$ for a vertex v in a graph G^*, consists of all vertices adjacent to v but not including v, that is, $N(v) = \{u \in V \mid vu \in E\}$. When v is also included, it is called a *closed neighborhood* $N[v]$, that is, $N[v] = N(v) \cup \{v\}$. A *path* in a graph G is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. The *length* of a path $P : v_1v_2 \cdots v_{n+1}$ $(n > 0)$ in G is n. A path $P : v_1v_2 \cdots v_{n+1}$ in G is called a *cycle* if $v_1 = v_{n+1}$ and $n \geq 3$. An undirected graph G is *connected* if there is a path between each pair of distinct vertices. For a pair of vertices u, v in a connected graph G, the *distance* $d(u,v)$ between u and v is the length of a shortest path connecting u and v. The *eccentricity* $e(v)$ of a vertex v in a graph G is the distance from v to a vertex furthest from v, that is, $e(v) = \max\{d(u,v) \mid u \in V\}$. The *radius* of a connected graph (or weighted graph)G is defined as $\mathrm{rad}(G) = \min\{e(v) \mid v \in V\}$. The *diameter* of a connected graph (or weighted graph)G is defined as $\mathrm{diam}(G) = \max\{e(v) \mid v \in V\}$. The *eccentric set* S of a graph is its set of eccentricities. The *center* $C(G)$ of a graph G is the set of vertices with minimum eccentricity. A graph is *self-centered* if all its vertices lie in the center. Thus, the eccentric set of a self-centered graph contains only one element, that is, all the vertices have the same eccentricity. Equivalently, a self-centered graph is a graph whose diameter equals its radius. A connected *regular graph* is a graph where each vertex has the same number of neighbors, i.e., all the vertices have the same open neighborhood degree. A connected graph is *highly irregular* if each of its vertices is adjacent only to vertices with distinct degrees. Equivalently, a graph G is highly irregular if every two vertices of G connected by a path of length 2 have distinct degrees. A connected graph is said to be *neighbourly irregular* if no two adjacent vertices of G have the same degree. Equivalently, a connected graph G is called neighbourly irregular if every two adjacent vertices of G have distinct degree.

Definition 1. [17,18]. *A fuzzy subset μ on a set X is a map $\mu : X \to [0,1]$. A fuzzy binary relation on X is a fuzzy subset μ on $X \times X$. By a fuzzy relation we mean a fuzzy binary relation given by $\mu : X \times X \to [0,1]$.*

Definition 2. [1]. *By a fuzzy graph G of G^*, we mean a pair $G = (\mu, \nu)$ where μ is a fuzzy set on V and ν is a fuzzy relation on E such that $\nu(\{x,y\}) \leq \min(\mu(x), \mu(y))$ for all x, $y \in V$.*

Fuzzy graph is a graph consists of pairs of vertex and edge that have degree of membership containing closed interval of real number [0,1] on each edge and vertex.

ANTI FUZZY STRUCTURES ON GRAPHS

Throughout this paper, G^* is a crisp graph and G is an anti fuzzy graph.

Definition 3. *By an anti fuzzy graph G of a graph G^*, we mean a pair $G = (\mu, \nu)$ where μ is a fuzzy set in V and ν is an anti fuzzy relation on E such that*

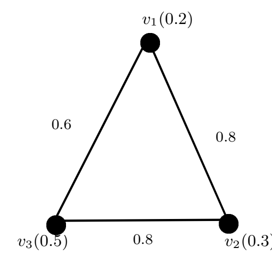
$$\nu(uv) \geq \max(\mu(u), \mu(v))$$

for all $uv \in E$. We note that ν is symmetric relation.

Example 4. *Consider a graph $G^* = (V,E)$ such that $V = \{v_1, v_2, v_3\}$, $E = \{v_1v_2, v_2v_3, v_3v_1\}$. Let μ be a fuzzy set of V and let ν be an anti fuzzy function of $E \subseteq V \times V$ defined by*

	v_1	v_2	v_3
μ	*0.2*	*0.3*	*0.5*

	v_1v_2	v_2v_3	v_3v_1
ν	*0.6*	*0.8*	*0.8*



By routine computations, it is easy to see that $G = (\mu, \nu)$ is an anti fuzzy graph of G^*. Adjacency matrix of the anti fuzzy graph is

$$A = \begin{pmatrix} 0 & 0.8 & 0.6 \\ 0.8 & 0 & 0.8 \\ 0.6 & 0.8 & 0 \end{pmatrix}.$$

Definition 5. *Let G be an anti fuzzy graph. The degree of a vertex u in G is defined by $\deg(u) = \sum_{uv \in E} \nu(uv)$. Note that $\nu(uv) > 0$ for $uv \in E$ and $\nu(uv) = 0$ for $uv \notin E$.*

Definition 6. *The number of vertices, the cardinality of V, is called the order of an anti fuzzy graph G and denoted by $|V|$ (or $O(G)$), and defined by*

$$O(G) = |V| = \sum_{u \in V} \mu(u).$$

The number of edges, the cardinality of E, is called the size of an anti fuzzy graph G and denoted by $|E|$ (or $S(G)$), and defined by

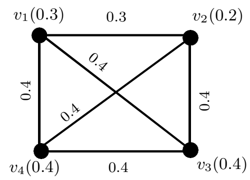
$$S(G) = |E| = \sum_{uv \in E} \nu(uv).$$

Definition 7. *An anti fuzzy graph G is called complete if*

$$\nu(uv) = \max(\mu(u), \mu(v)) \quad \text{for every} \quad u, v \in V.$$

Example 8. *Consider a complete anti fuzzy graph G such that $V = \{v_1, v_2, v_3, v_4\}$,*

$$E = \{v_1v_2, v_2v_3, v_1v_4, v_3v_4, v_1v_3, v_4v_2\}.$$



By routine calculations, we haves
(i) *order of a complete anti fuzzy graph= $O(G)$=1.3.*
(iii) *degree of each vertex in G is*

$$\deg(v_1) = \deg(v_2) = 1.1, \quad \deg(v_3) = \deg(v_4) = 1.2.$$

Definition 9. *The complement of an anti fuzzy graph $G = (\mu, \nu)$ of $G^* = (V, E)$ is an anti fuzzy graph $\overline{G} = (\overline{\mu}, \overline{\nu})$ on $\overline{G^*}$ where $\overline{\mu}$ and $\overline{\nu}$ are defined by*

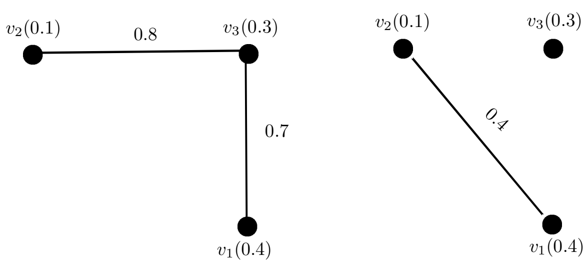
(1)
$$\overline{V} = V,$$

(2)
$$\overline{\mu}(x) = \mu(x) \qquad \forall \;\; x \in V,$$

(3)
$$\overline{\nu}(xy) = \begin{cases} 0 & if \;\; \nu(xy) > 0, \\ \max(\mu(x), \mu(y)) & if \;\; if\, \nu(xy) = 0. \end{cases}$$

Definition 10. *An anti fuzzy graph $G = (\mu, \nu)$ is called self complementary if $G \approx \overline{G}$.*

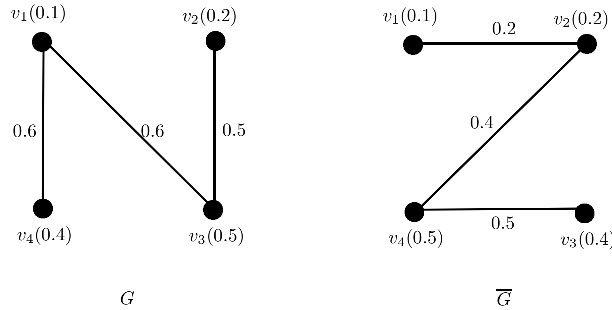
Example 11. *Consider an anti fuzzy graph G*



Clearly, graph G is not isomorphic to its complement $\overline{G}$. Hence G is not self complementary.

Example 12. *Consider an anti fuzzy graph G*

Clearly, graph G is isomorphic to its complement $\overline{G}$. Hence G is self complementary.

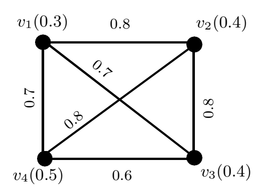


Definition 13. *A fuzzy path P in an anti fuzzy graph G is a sequence of distinct vertices P : $v_0v_1v_2 \ldots v_n$ such that $\nu(v_{i-1}v_i) > 0$ for $1 \le i \le n$.*

Definition 14. *An anti fuzzy graph G is connected if any two vertices are joined by a fuzzy path. That is, an anti fuzzy graph G is connected if $(\nu)^\infty(xy) > 0$, $xy \in E$.*

Definition 15. *Let G be a connected anti fuzzy graph. If deletion of a vertex x reduces the strength of connectedness between some other pair of vertices, a vertex x is an anti fuzzy cut vertex of G.*

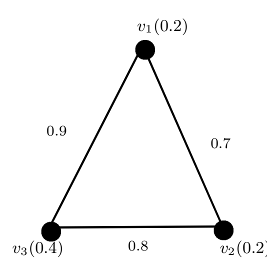
Example 16. *Consider connected anti fuzzy graph, G such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1, v_1v_3, v_2v_4\}$.*



In the above Figure, removal of a vertex v_4, reduces the strength between v_1 and v_3 from 0.6 to 0.7. Hence v_4 is an anti fuzzy cut vertex.

Definition 17. *Let G be an anti fuzzy graph such that G^* is a cycle. Then G is an anti fuzzy cycle if it has more than one weakest vertex.*

Example 18. *Consider an anti fuzzy graph, G such that $V = \{v_1, v_2, v_3\}$, $E = \{v_1v_2, v_2v_3, v_3v_1\}$.*

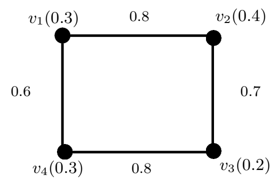


Clearly, G has an anti fuzzy cycle.

Definition 19. *Let G be a connected anti fuzzy graph. If deletion of xy reduces the strength of connectedness between pair of vertices, an edge xy is an anti fuzzy bridge of G.*

Definition 20. *A connected anti fuzzy graph G with no anti fuzzy cut vertices is called an anti fuzzy block.*

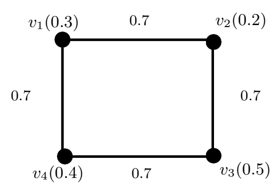
Example 21. *Consider an anti fuzzy graph, G such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$.*



In the above Figure, removal of an edge (v_4v_1) reduces the strength between v_1 and v_4 from 0.6 to 0.7. Hence (v_4v_1) is an anti fuzzy bridge. It is also anti fuzzy block.

Definition 22. *An anti fuzzy graph G is called constant if degree of each vertex is k. That is, $\deg(x) = k$ for all $x \in V$.*

Example 23. *Consider an anti fuzzy graph G such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$.*



Clearly, G is constant anti fuzzy graph since the degree of v_1, v_2, v_3 and v_4 is 1.4.

Theorem 24. *Let G be an anti fuzzy graph with the set of vertices V. Then:*

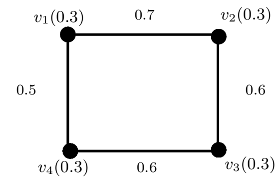
(i) *If ν is constant function for all $uv \in E$, then G has no anti fuzzy bridge.*

(ii) *If ν is not constant function for all $uv \in E$, then G has at least one anti fuzzy bridge.*

Definition 25. *A regular anti fuzzy graph is an anti fuzzy graph where each vertex has the same number of open neighbors degree. It is denoted by* $\deg(v)$.

The following example shows that there is no relationship between regular anti fuzzy graph and constant anti fuzzy graph.

Example 26. *Consider an anti fuzzy graph G such that $V = \{v_1, v_2, v_3, v_4\}$.*



By routine calculations show that G is regular anti fuzzy graph since each open neighbors degree is same, that is, 0.6. But it is not constant anti fuzzy graph since degree of each vertex is not same.

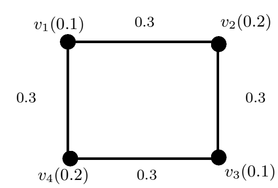
Definition 27. *Let G be an anti fuzzy graph. The totally degree of a vertex u is defined by*

$$Td(u) = \sum_{uv \in E} \nu(uv) + \mu(u) = \deg(u) + \mu(u).$$

If each vertex of G has totally same degree m, then G is called a m-totally constant anti fuzzy graph.

Definition 28. *A totally regular anti fuzzy graph is an anti fuzzy graph where each vertex has the same number of closed neighbors degree. it is denoted by* $\deg[v]$.

Example 29. *Consider an anti fuzzy graph G such that $V = \{v_1, v_2, v_3, v_4\}$.*



By routine calculations show that it is constant anti fuzzy graph G since the degree of v_1,v_2,v_3 and v_4 is 0.6. *It is neither totally regular anti fuzzy graph nor totally constant anti fuzzy graph.*

We state the following Theorem without its proof.

Theorem 30. *Let $G = (\mu, \nu)$ be an anti fuzzy graph. Then μ is a constant function if and only if the following are equivalent.*

(i) *G is a constant anti fuzzy graph.*

(ii) *G is a totally constant anti fuzzy graph.*

Theorem 31. *Let G be an anti fuzzy graph where crisp graph G^* is an odd cycle. Then G is constant anti fuzzy graph if and only if ν is a constant function.*

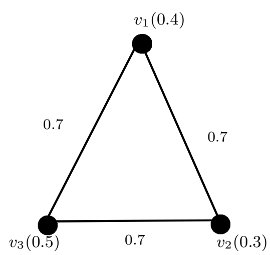
Proof. Suppose that G is a k- constant anti fuzzy graph. Let $e_1, e_2, \cdots, e_{2n+1}$ be the edges of G in that order. Let $\nu(e_1) = c$, $\nu(e_2) = k - c$, $\nu(e_3) = k - (k - c) = c$, $\nu(e_4) = k - c$ and so on. Therefore,

$$\nu(e_i) = \begin{cases} c, \text{if } i \text{ is odd} \\ k - c, \text{if } i \text{ is even} \end{cases}.$$

Hence $\nu(e_1) = \nu(e_{2n+1}) = c$. So, if e_1 and e_{2n+1} incident at a vertex v_1, then $\deg(v_1) = k$, $\deg(e_1) + \deg(e_{n+1}) = k$, $c + c = k$, $2c = k$, $c = \frac{k}{2}$. This shows that ν is constant function. The proof of converse part is obvious. This competes the proof. □

The above theorem does not hold for totally constant anti fuzzy graph.

Example 32. *Consider an anti fuzzy graph G such that $V = \{v_1, v_2, v_3\}$, and $E = \{v_1v_2, v_2v_3, v_1v_3\}$.*



Clearly, G is ν-constant function, but not totally constant anti fuzzy graph.

Theorem 33. *Let G be an anti fuzzy graph where crisp graph G^* is an even cycle. Then G is constant anti fuzzy graph if and only if either ν is a constant function or alternate edges have same membership values.*

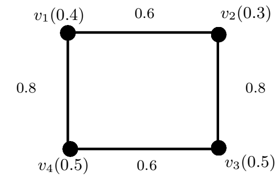
Proof. Suppose G is a k-constant anti fuzzy graph. Let $e_1, e_2, \ldots, e_{2n}$ be the edges of even cycle G^* in that order. Proceeding as in the Theorem 31,

$$\nu(e_i) = \begin{cases} c, \text{if } i \text{ is odd} \\ k - c, \text{if } i \text{ is even} \end{cases}.$$

If $c = k - c$, the ν is a constant function. If $c \neq k - c$, then alternate edges have same membership values. The proof of converse part is obvious. This competes the proof. □

The above theorem does not hold for totally constant anti fuzzy graph.

Example 34. *Consider an anti fuzzy graph G such that such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_4v_3, v_4v_1\}$.*



Clearly, G is ν-constant function but it is not totally constant anti fuzzy graph.

Theorem 35. *Let G be a constant anti fuzzy graph on an even cycle of a crisp graph G^*. Then either G does not have an anti fuzzy bridge or it has $q/2$ anti fuzzy bridges where $q = |E|$. Also G does not have an anti fuzzy cut vertex.*

Proof. Assume that G is a constant anti fuzzy graph on an even cycle of crisp graph G^*. Then by Theorem 33, either ν is a constant function or alternate edges have same membership values.
When ν is a constant function. Then the removal of any edge does not reduce the strength of connectedness between any pair of vertices. Thus G does not have an anti fuzzy bridge and hence does not have an anti fuzzy cut vertex.
When alternate edges have same membership values. Then by Theorem 33, edges with greater membership values are the anti fuzzy bridges of G. There are $q/2$ such edges where $q = |E|$. So G has $q/2$ anti fuzzy bridges. But then no vertex is a common vertex of two anti fuzzy bridges. Hence G does not have a cut vertex. □

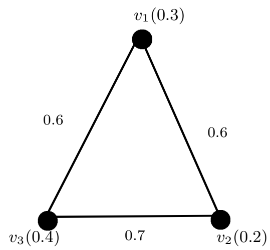
We now introduce the notions of irregular anti fuzzy graphs, neighbourly irregular anti fuzzy graphs, neighbourly totally irregular anti fuzzy graphs, highly irregular anti fuzzy graphs and highly totally irregular anti fuzzy graphs.

Definition 36. *Let G be an anti fuzzy graph. The neighbourhood degree of a vertex x in G is defined by $\deg(x) = \sum_{y \in N(x)} \mu(y)$.*

Definition 37. *Let G be an anti fuzzy graph on G^*. If there is a vertex which is adjacent to vertices with distinct neighbourhood degrees, then G is called an irregular anti fuzzy graph. That is, $\deg(x) \neq m$ for all $x \in V$.*

Example 38. *Consider an anti fuzzy graph G such that*

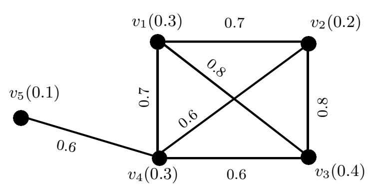
$$V = \{v_1, v_2, v_3\}, E = \{v_1v_2, v_2v_3, v_1v_3\}.$$



By routine computations, it is clear that G is an irregular anti fuzzy graph.

Definition 39. *Let G be an anti fuzzy graph. The closed neighbourhood degree of a vertex x is defined by $\deg[x] = \deg_\nu(x) + \mu(x)$. If there is a vertex which is adjacent to vertices with distinct closed neighbourhood degrees, then G is called a totally irregular anti fuzzy graph.*

Example 40. *Consider an anti fuzzy graph G such that $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{v_1v_2, v_2v_3, v_2v_4, v_3v_1, v_3v_4, v_4v_1, v_4v_5\}$.*

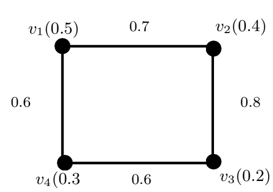


By routine computations, it is clear that G is a totally irregular anti fuzzy graph.

Definition 41. *A connected anti fuzzy graph G is said to be a neighbourly irregular anti fuzzy graph if every two adjacent vertices of G have distinct open neighbourhood degree.*

Example 42. *Consider an anti fuzzy graph G such that*

$$V = \{v_1, v_2, v_3, v_4\}, E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}.$$

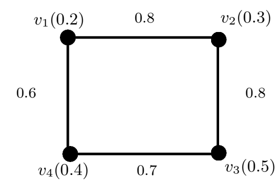


By routine computations, it is clear that G is neighbourly irregular anti fuzzy graph.

Definition 43. *A connected anti fuzzy graph G is said to be a neighbourly totally irregular anti fuzzy graph if every two adjacent vertices of G have distinct closed neighbourhood degree.*

Example 44. *Consider an anti fuzzy graph G such that*

$$V = \{v_1, v_2, v_3\}, E = \{v_1v_2, v_2v_3, v_1v_3\}.$$



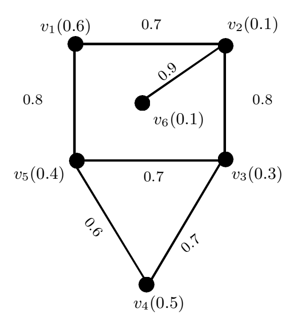
By routine computations, it is clear that G is neighbourly totally irregular anti fuzzy graph.

Definition 45. *Let G be a connected anti fuzzy graph. G is called a highly irregular anti fuzzy graph if every vertex of G is adjacent to vertices with distinct neighbourhood degrees.*

Remark 46. *A highly irregular anti fuzzy graph may not be a neighbourly irregular anti fuzzy graph.*

There is no relation between highly irregular anti fuzzy graphs and neighbourly irregular anti fuzzy graphs. We explain this concept with the following examples.

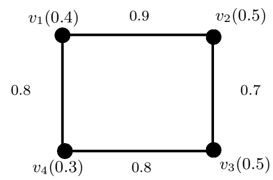
Example 47. *Consider an anti fuzzy graph G such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E = \{v_1v_2, v_2v_3, v_2v_6, v_3v_4, v_3v_5, v_4v_5, v_5v_1\}$.*



By routine computations, we have $\deg(v_1) = 0.5$, $\deg(v_2) = 1.0$, $\deg(v_3) = 1.0$, $\deg(v_4) = 0.7$, $\deg(v_5) = 1.4$ and $\deg(v_6) = 0.1$. Consider a vertex $v_2 \in V$ which is adjacent to the vertices v_1, v_3 and v_6 with distinct neighbourhood degrees. But $\deg(v_2) = \deg(v_3)$. Hence G is highly irregular anti fuzzy graph but it is not a neighborly irregular anti fuzzy graph.

Remark 48. *A neighbourly irregular anti fuzzy graph may not be a highly irregular anti fuzzy graph.*

Example 49. *Consider an anti fuzzy graph G such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$.*



By routine computations, we have $\deg(v_1) = 0.8$, $\deg(v_2) = 0.9$, $\deg(v_3) = 0.8$, $\deg(v_4) = 0.9$. *We see that every two adjacent vertices have distinct neighbourhood degree. But consider a vertex v_2 which is adjacent to the vertices v_1 and v_3 has same degree, that is,* $\deg(v_1) = \deg(v_3)$. *Hence G is neighbourly irregular anti fuzzy graph but not a highly irregular anti fuzzy graph.*

We state the following propositions without their proofs.

Proposition 50. *Let G be an anti fuzzy graph. Then G is highly irregular anti fuzzy graph and neighbourly anti fuzzy graph if and only if the neighbourhood degrees of all the vertices of G are distinct.*

Proposition 51. *an anti fuzzy graph G of G^*, where G^* is a cycle with vertices 3 is neighbourly irregular and highly irregular anti fuzzy graph if and only if the negative value of the vertices between every pair of vertices is distinct.*

Proposition 52. *Let G be an anti fuzzy graph. If G is neighbourly irregular anti fuzzy graph and μ is a constant function, then G is a neighbourly totally irregular anti fuzzy graph.*

Proof. Assume that G is a neighbourly irregular anti fuzzy graph. That is the neighbourhood degrees of every two adjacent vertices are distinct. Let $v_i, v_j \in V$, where v_i and v_j are adjacent vertices with distinct neighbourhood degrees l_1 and l_2, respectively. That is, $\deg(v_i) = l_1$ and $\deg(v_j) = l_2$, where $l_1 \neq 1_2$. Assume that $\nu_1(v_i) = \nu_1(v_j) = c_2$, where c_2 is constant and $c_2 \in [0, 1]$. Therefore, $\deg_\nu[v_i] = \deg_\nu(v_i) + \nu_1(v_i) = l_1 + c_2$, $\deg_\nu[v_j] = \deg_\nu(v_j) + \nu_1(v_j) = l_2 + c_2$
Claim: $\deg_\nu[v_i] \neq \deg_\nu[v_j]$. Suppose that $\deg_\nu[v_i] = \deg_\nu[v_j]$. Consider

$$\begin{aligned}
\deg_\nu[v_i] &= \deg_\nu[v_j] \\
l_1 + c_2 &= l_2 + c_2 \\
l_1 - l_2 &= c_2 - c_2 = 0 \\
l_1 &= l_2, \text{ which is a contradiction to } l_1 \neq l_2.
\end{aligned}$$

Therefore, $\deg_\nu[v_i] \neq \deg_\nu[v_j]$. Hence G is a neighbourly totally irregular anti fuzzy graph. □

We state the following Theorem without its proof.

Theorem 53. *Let G be an anti fuzzy graph. If G is a neighbourly totally irregular and μ is a constant function, then G is a neighbourly irregular anti fuzzy graph.*

We now introduce the notion of self-centroid anti fuzzy graphs.

Definition 54. *Let G be a connected anti fuzzy graph. The $\mu-$length of a path $P : v_1 v_2 \cdots v_n$ in G, $l_\mu(P)$, is defined as $l_\mu(P) = \sum_{i=1}^{n-1} \frac{1}{\nu(v_i, v_{i+1})}$.*

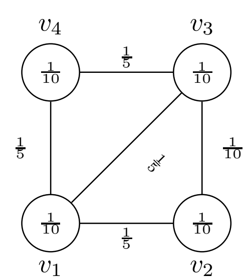
Definition 55. *Let G be a connected anti fuzzy graph. The μ-distance, $\delta_\mu(v_i, v_j)$, is the smallest μ-length of any $v_i - v_j$ path P in G, where $v_i, v_j \in V$. That is, $\delta_\mu(v_i, v_j) = \min(l_\mu(P))$.*

Definition 56. *Let G be a connected anti fuzzy graph. For each $v_i \in V$, the μ-eccentricity of v_i, denoted by $e_\mu(v_i)$ and is defined as $e_\mu(v_i) = \max\{\delta_\mu(v_i, v_j) : v_i \in V, v_i \neq v_j\}$.*

Definition 57. *Let G be a connected anti fuzzy graph. The μ-radius of G is denoted by $r_\mu(G)$ and is defined as $r_\mu(G) = \min\{e_\mu(v_i) : v_i \in V\}$.*

Definition 58. *Let G be a connected anti fuzzy graph. The μ-diameter of G is denoted by $d_\mu(G)$ and is defined as $d_\mu(G) = \max\{e_\mu(v_i) : v_i \in V\}$.*

Example 59. *Consider connected anti fuzzy graph G such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_3v_4\}$.*



G

By routine computations, it is easy to see that:

(1) *v_1v_4 is a path of length 1 and $l_\mu = 5$, $v_1v_3v_4$ is a path of length 2 and $l_\mu = 10$, $v_1v_2v_3v_4$ is a path of length 3 and $l_\mu = 20$.*

(2)

$$\delta_\mu(v_1, v_4) = 5,\ \delta_\mu(v_1, v_2) = 5,\ \delta_\mu(v_1, v_3) = 5,$$

$$\delta_\mu(v_2, v_3) = 10,\ \delta_\mu(v_2, v_4) = 10,\ \delta_\mu(v_3, v_4) = 5.$$

(3) *μ-eccentricity of each vertex is*

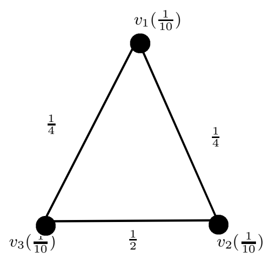
$$e_\mu(a) = 5,\ e_\mu(b) = 10,\ e_\mu(c) = 10,\ e_\mu(d) = 10.$$

(4) *Radius of G is 5, diameter of G is 10.*

Definition 60. *A vertex $v_i \in V$ is called a central vertex of a connected anti fuzzy graph G, if $r_\mu(G) = e_\mu(v_i)$ and the set of all central vertices of a anti fuzzy graph is denoted by $C(G)$.*

Definition 61. *A connected anti fuzzy graph G is a self centered graph, if every vertex of G is a central vertex, that is $r_\mu(G) = e_\mu(v_i)$, $\forall v_i \in V$.*

Example 62. *Consider connected anti fuzzy graph G such that $V = \{v_1, v_2, v_3\}$, $E = \{v_1v_2, v_2v_3, v_3v_1\}$.*



By routine computations, it is easy to see that:

(i) *Distance is*

$$\delta_\mu(v_1, v_2) = 4,\ \delta_\mu(v_1, v_3) = 4,\ \delta_\mu(v_2, v_3) = 2.$$

(ii) *Eccentricity of each vertex is 4.*

(iii) *Radius of G is 4. Hence G is self centered anti fuzzy graph.*

Lemma 63. *An anti fuzzy graph G is a self centered anti fuzzy graph if and only if $r_\mu(G) = d_\mu(G)$.*

Theorem 64. *If G is an anti fuzzy bipartite graph then it has no strong cycle of odd length.*

Theorem 65. *Let G be an anti fuzzy graph. If the graph G is a complete bipartite anti fuzzy graph then the complement of G is a self-centered anti fuzzy graph.*

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