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New Complex Travelling Wave Solutions to the Nonlinear Broer-Kaup-Kupershmidt System

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Abstract: In this present work we applied the direct algebraic method to Broer-Kaup-Kupershmidt system. Then new types of complex solutions are obtained to the Broer-Kaup-Kupershmidt system.

Key words: Plasma · Nonlinear · Complex solution · Broer-Kaup-Kupershmidt system

INTRODUCTION

The new application of direct algebraic method was developed by F. Noo *et al*. [1]. The method is now used by many researchers in a variety of scientific fields. The method has been proved by many authors [2-4]. Recently, many powerful methods have been established and improved. Among these methods, we cite the, the hyperbolic tangent expansion method [7, 8], the trial function method [9], the homogeneous balance method [5, 6], the tanh-method [10-14], the inverse scattering transform [15], the Backlund transform [16, 17], the Hirota's bilinear method [18, 19], the motivation of the present paper is to explore the possibilities of solving such equations, the balance numbers of which are not positive integers, using the direct algebraic method. The direct algebraic method is based on the assumptions that the travelling wave solutions can be expressed by a polynomial in $u(\xi)=\sum_{i=0}^{n} a_i F^i(\xi)$. The paper is arranged as follows. In Section 2, we describe briefly the direct algebraic method method. In Sections 3, we apply the method to the combined Broer-Kaup-Kupershmidt system. In Section 4 some conclusions are given.

Description of Direct Algebraic Method: For a given partial differential equation

$$G(u,u_x,u_t,u_{xx},u_{tt},....), \tag{1}$$

Our method mainly consists of four steps:

Step 1: We seek complex solutions of Eq. (1) as the following form:

c
where k and c are real constants. Under the transformation (2), Eq. (1) becomes an ordinary differential equation

$$N(u,iku',-ikcu',-k^2u'',.....), \tag{2}$$

where $u'=\frac{du}{d\xi}$.

Step 2: We assume that the solution of Eq. (3) is of the form

$$u(\xi)=\sum_{i=0}^{n} a_i F^i(\xi), \tag{4}$$

where $a_i(i = 1,2,...,n)$ are real constants to be determined later. $F(\xi)$ expresses the solution of the auxiliary ordinary differential equation

$$F'(\xi)=b+F^2(\xi), \tag{5}$$

Eq. (5) admits the following solutions:

$$F(\xi)=\begin{cases}-\sqrt{-b}\tanh(\sqrt{-b}\xi), & b\prec 0\\ -\sqrt{-b}\coth(\sqrt{-b}\xi), & b\prec 0\end{cases}$$
$$F(\xi)=\begin{cases}\sqrt{b}\tan(\sqrt{b}\xi), & b\succ 0\\ -\sqrt{b}\cot(\sqrt{b}\xi), & b\succ 0\end{cases} \tag{6}$$
$$F(\xi)=-\frac{1}{\xi}, \qquad b=0$$

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Integer n in (4) can be determined by considering homogeneous balance [3] between the nonlinear terms and the highest derivatives of $u(\xi)$ in Eq. (3).

Step 3: Substituting (4) into (3) with (5), then the left hand side of Eq. (3) is converted into a polynomial in $F(\xi)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i,k,c.

Step 4: Solving the algebraic equations obtained in step 3 and substituting the results into (4), then we obtain the exact traveling wave solutions for Eq. (1).

Applications of Direct Algebraic Method Broer-kaup-kupershmidt: Consider the Broer-Kaup-Kupershmidt system as

$$\begin{cases} u_{ty} - u_{xxy} + 2(uu_x)_y + 2v_{xx} = 0 \\ v_t + v_{xx} + 2(uv)_x = 0 \end{cases} \tag{7}$$

We may choose the following complex travelling wave transformation:

$$\begin{cases} u = u(\xi), & \xi = ik(x+y-ct) \\ v = v(\xi) & \xi = ik(x+y-ct) \end{cases} \tag{8}$$

where c,k are constants to be determined later. Using the complex traveling wave solutions (8) we have the nonlinear ordinary differential equations

$$\begin{cases} -(ik)^2 u'' - (ik)^3 u''' + (ik)^2 (u)')' + 2(ik)^2 v'' = 0 & (9) \\ -ikcv' + (ik)^2 v'' + 2ik(uv)' = 0 & (10) \end{cases}$$

Integrating (9) with respect to ξ twice and considering the integrating constants to zero and integrating (10) once we obtain

$$\begin{cases} -cu - (ik)u' + u^2 + 2v = 0 & (11) \\ -iv + ikv' + 2uv + C = 0 & (12) \end{cases}$$

From equation (11) w e obtain

$$v = \frac{1}{2}(cu + iku' - u^2) \tag{13}$$

And substituting (13)into (12) we have

$$\frac{3}{2}cu^2 - \frac{1}{2}c^2 u - \frac{1}{2}k^2 u'' - u^3 = 0 \tag{14}$$

Considering the homogeneous balance between u^5 and n'' in (1), we required that $3m = m + 2 \Rightarrow m = 1$. So

$$u = A_1 F + A_0, \tag{15}$$

By substituting (11) - (14) into Eq. (8) and collecting all terms With the same power of F together, the left-hand side of Eq. (8) is converted into another polynomial in F. Equating each coefficient of this polynomial to zero, yields a set of simultaneous algebraic equations for A_1,A_0,k,b as follows:

$$\frac{3}{2}cA_0{}^2 - \frac{1}{2}c^2 A_0 - A_0^3 + C = 0$$
$$3cA_1 A_0 - \frac{1}{2}c^2 A_1 - k^2 A_1 b - 3A_1 A_0^2 = 0$$
$$\frac{3}{2}cA_1{}^2 - 3A_1{}^2 A_0 = 0$$
$$k^2 A_1 - A_1^3 = 0$$

By solving relations above we have

$$\begin{aligned} &c = \pm 2k\sqrt{b} \\ &A_0 = \pm k\sqrt{b} \\ &A_1 = \pm ki \\ &C = 6k^2 b \end{aligned} \tag{16}$$

From (6),(14-15) and (16), we obtain the complex travelling wave solutions of (7) as follows

$$u_1 = \pm ki\left[-\sqrt{-b}\tanh(\sqrt{-b}ik(x+y-6k^2bt))\right] \pm k\sqrt{b},$$

$$v_1 = \frac{1}{2}\Big\{\pm kci\left[-\sqrt{-b}\tanh(\sqrt{-b}ik(x+y-6k^2bt))\right] \pm kc\sqrt{b} \pm bk^2\left[1-\tanh(\sqrt{-b}ik(x+y-6k^2bt))\right]$$
$$-k^2 b\tanh^2(\sqrt{-b}ik(x+y-6k^2bt) \mp 2k^2 i\sqrt{b}\left[-\sqrt{-b}\tanh(\sqrt{-b}ik(x+y-6k^2bt))\right] - k^2 b\Big\}$$

where $b < 0$ and k is an arbitrary real constant.

And

$$u_2 = \pm ki\left[-\sqrt{-b}\coth(\sqrt{-b}ik(x+y-6k^2bt))\right] \pm k\sqrt{b},$$

$$v_2 = \frac{1}{2}\left\{\pm kci\left[-\sqrt{-b}\coth(\sqrt{-b}ik(x+y-6k^2bt))\right] \pm kc\sqrt{b} \mp bk^2\left[1-\coth(\sqrt{-b}ik(x+y-6k^2bt))\right]\right.$$
$$\left.-k^2b\coth^2(\sqrt{-b}ik(x+y-6k^2bt) \mp 2k^2i\sqrt{b}\left[-\sqrt{-b}\coth(\sqrt{-b}ik(x+y-6k^2bt))\right] - k^2b\right\}$$

where $b \prec 0$ and k is an arbitrary real constant.

$$u_3 = \pm ki\left[\sqrt{b}\tan(\sqrt{b}ik(x+y-6k^2bt))\right] \pm k\sqrt{b},$$

$$v_2 = \frac{1}{2}\left\{\pm kci\left[\sqrt{b}\tan(\sqrt{-b}ik(x+y-6k^2bt))\right] \pm kc\sqrt{b} \mp bk^2\left[1+\tan(\sqrt{b}ik(x+y-6k^2bt))^2\right]\right.$$
$$\left.+k^2b\tan^2(\sqrt{b}ik(x+y-6k^2bt) \mp 2k^2i\sqrt{b}\left[\sqrt{b}\tan(\sqrt{b}ik(x+y-6k^2bt))\right] - k^2b\right\}$$

where $b \succ 0$ and k is an arbitrary real constant.

$$u_4 = \pm ki\left[-\sqrt{b}\cot(\sqrt{b}ik(x+y-6k^2bt))\right] \pm k\sqrt{b},$$

$$v_2 = \frac{1}{2}\left\{\pm kci\left[-\sqrt{b}\cot(\sqrt{-b}ik(x+y-6k^2bt))\right] \pm kc\sqrt{b} \mp bk^2\left[-1-\cot(\sqrt{b}ik(x+y-6k^2bt))^2\right]\right.$$
$$\left.+k^2b\cot^2(\sqrt{b}ik(x+y-6k^2bt) \mp 2k^2i\sqrt{b}\left[-\sqrt{b}\cot(\sqrt{b}ik(x+y-6k^2bt))\right] - k^2b\right\}$$

where $b \succ 0$ and k is an arbitrary real constant.

$$u_5 = \pm ki\frac{1}{ik(x+y)} \pm k\sqrt{b},$$

$$v = \frac{1}{2}\left\{\pm\frac{c}{(x+y)} \pm kc\sqrt{b} \mp \frac{ik}{(x+y)^2} - \frac{1}{(x+y)^2} \mp 2k\sqrt{b}\frac{1}{(x+y)} - k^2b\right\}$$

where $b = 0$ and k is an arbitrary real constant.

CONCLUSION

We have noted that the direct algebraic method changes the given difficult problems into simple problems which can be solved easily. This paper presents a wider applicability for handling nonlinear evolution equations using the direct algebraic method. The new type of exact travelling wave solution obtained in this paper might have significant impact on future researches.

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