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Evaluation of Height Functions for *Acacia seyal* Del. Variety *Seyal* in Natural Stands, South Kordofan Sudan

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Abstract: Height-diameter relationship was constructed for *Acacia seyal* natural stands in Umfakarin Natural Forest Reserve, South Kordofan, Sudan. A total of 3799 single trees, from three different stand densities, were used for this purpose. The three stand densities namely; dense, medium and slight were distinguished based on number of trees per hectare. Diameter at breast height and height of each single tree were measured. Height-diameter relationship was evaluated using six height functions based on data from medium stand density. The criteria for selecting the best model to be used for the *A. seyal* height curve were based on coefficient of determination (R^2), Akaike´s Information Criterion (AIC), asymptote and the plausibility of the curve. The statistical analysis resulted in a very similar values of R^2 (0.237-0.242) and AIC (479.00-486.36) as well. Based on the results of the evaluated models, a Michailow height function ($R^2 = 0.237$) was selected as it also produces a line of the best fit acceptable from a biological point of view. Therefore, Michailow height function can be used for predicting height of *A. seyal* trees in Umfakarin natural forest and other areas of similar climatic conditions.

Key words: Acacia seyal % AIC % Michailow height function % Umfakarin forest

INTRODUCTION

Acacia seyal natural stands in Sudan are managed for firewood and charcoal production in reserved and unreserved natural forests [1]. In Sudan, there are two main varieties of *A. seyal*, variety *seyal* and variety *fistula*, occur naturally in the low rainfall savannah zone [2-4]. The species is distributed throughout its natural range and is usually associated with *Balanites aegyptiaca* in the *Acacia* seyal-*Balanites* woodland area. In such formation, *A. seyal* is the dominant tree species, forming pure dense stands in many areas. According to Mustafa [3], this formation begins to emerge with an increase in the annual rainfall to accumulations of more than 500 mm. As the management of *A. seyal* in Sudan is mainly undertaken for fuelwood, most of the studies were conducted as part of the project “fuelwood development for energy in Sudan”. Volume and height functions have been developed, respectively, for predicting the volume and height of *A. seyal* in natural stands [1].

One of the most important elements of forest structure is the relationship between tree diameters and heights [5]. A stand height curve (Formula 1) is a mathematical or graphical representation that describes the dependency of tree height on diameter [6,7]. The height curve can be used to predict individual tree height when only DBH is measured. Moreover, the predicted height can be used together with DBH to derive single tree volume and/or to produce volume table (2-way volume table) of a forest stands. The stand height curve can also be used to provide information about the development and growth stages of the stand; for example it can help determine whether it is a young or old stand. It also provides information about the competitive situation of the trees in the stand.

$$h = f(d) \tag{1}$$

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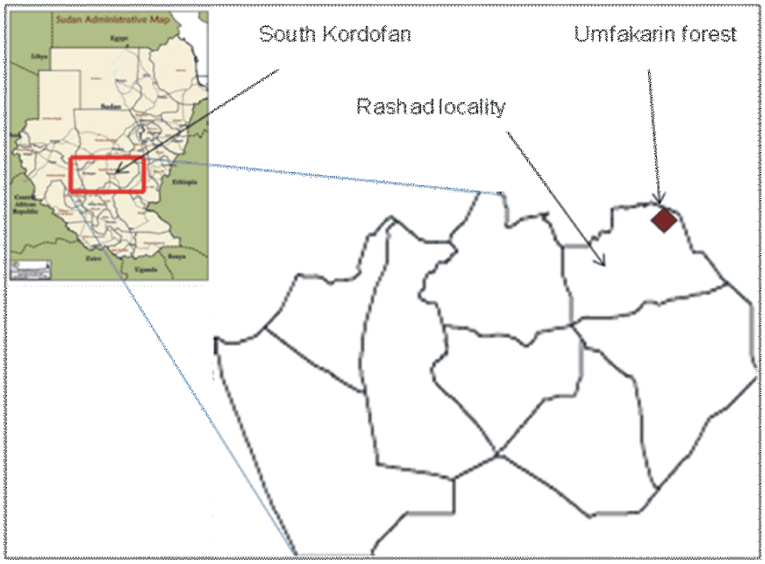

Fig. 1: Location of Umfakarin natural forest reserve, Rashad Locality, South Kordofan, Sudan. Available at http://www.maplibrary.org/index.php and http://www.sd.undp.org/UNDP_protocol_areas.htm

MATERIALS AND METHODS

The Study Area: Data for the present study were collected from Umfakarin natural reserve forest (Lat. 12° 29´- 12° 35´ N and Long. 31° 17´ - 31° 20´ E), South Kordofan, Sudan, between September 2007 and February 2008 (Figure 1). The forest covers an area of about 2,689 hectares. Annual rainfall ranges from 400 to 1000 mm and the species flourishes along seasonal water courses in areas where annual rainfall is less than 400 mm. Seasonal flooding is the most conspicuous feature in the Umfakarin forest. Temperatures range from 30-35°C. Sandy, cracking clay and non-cracking sandy clay (local name: *Gardud*) are the prevailing soils in the study area. In general, the forest reserve can be described as a slightly undulating land surface with the exception to a few seasonal streams that penetrate some parts of the forest. No physical features seem to be clearly bounded by the forest reserve. In the northern parts of South Kordofan some scattered thorny trees (acacias) dominate the vegetation cover. The density of vegetation cover increases from north to south where formations of poor *A. senegal* and *A. mellifera* pave the way for *Acacia seyal-Banalities* woodland and other plant formations.

Generally, in semi-arid savannas the growth rate of *A. seyal* is low; however, early growth rates can be quite fast, trees can reach up to 1 meter in 3 months on favorable sites [8]. As indicated by Mustafa [3], *A. seyal* trees can reach its reproductive stage rapidly, within 5 years in a natural stand, unless the growth is retarded by local events such as intensive browsing or fire. The periodic increment (PI) of the diameter at breast height and volume, respectively, does not exceed 1.3 cm and 5 m^3/ha in 3 years [9]. In Sudan, the growth and yield of *A. seyal* vary according to region. For example, the mean annual increment (MAI), of *A. seyal*, in Garri forest, Blue Nile, ranged between 1.6-2.4 m^3/ha/year during 1963-1966, where recorded annual rainfall was 657 to 718 mm. However, the MAI ranged between 1-1.5 m^3/ha in the Rawashda forest in eastern Sudan [9], where annual rainfall ranges between 450-500 mm. Trees managed on a 10-15 year rotation yield 10-35 m^3/ha of fuel-wood per year [10].

Data Collection: Data for the present study were collected from *Acacia seyal* natural stands in Umfakarin natural forest reserve, South Kordofan, Sudan. A total of 3799 individual trees were measured covering a range of 5-40 cm diameter in three different stand densities determined according to stand density. These three stand densities are later called dense (396 trees/ha), medium (271 trees/ha) and slight (209 trees/ha). Diameter at breast height (DBH, in cm) and height (h, in m) of individual trees were measured using diameter tape and Hypsometer, respectively.

Computational Statistics and Analysis: Data entry and mathematical processes were conducted using an Excel spreadsheet in Microsoft Office Excel 2003, while data analysis based on nonlinear regression [11] was performed using SPSS Statistics 17.0 Release 17.0.0 Aug 23, 2008.

Quadratic Mean Diameter: The quadratic mean diameter, i.e. DBH corresponding to the average basal area of trees in the stand was obtained (Formula 2) according to Kramer and Akça [6], Vink [12] and West [7].

$$D = \sqrt{\sum_{i=1}^{n} \frac{d_i^2}{n}} \qquad (2)$$

Where: D is the quadratic mean diameter (QMD, in cm); n is the number of observations; and d_i is the diameter at breast height (DBH, in cm) of i tree.

Stand Basal Area: Basal area per hectare was calculated by multiplying the square mean diameter with number of trees per hectare (Formula 3).

$$BA = D^2 * p / 40000 * N \qquad (3)$$

Where BA is the basal area per hectare (m²); D is the QMD; and N represents the number of trees per hectare.

Stand Mean Height: The mean height of the stand was calculated following Kramer and Akça [6]and Van Laar and Akça [13] by regressing the Michailow stand height curve on the mean DBH (Formula 4).

$$H = 1.3 + a_0 * e^{(a_1 / D)} \qquad (4)$$

Where H is the mean height (m); D is the QMD; e is the base of natural logarithm; and a_0 and a_1 represent coefficients.

Single tree volume was obtained using Formula 5, as illustrate below:

$$v = ba * h * ff \qquad (5)$$

Where ba represents the tree's basal area (in m^2, $ba = d^2 * B / 40000$); d represents tree diameter (DBH); h represents the observed tree height (m); ff is the form factor (ff = 0.5 according to [14]; and B = 3.14159265.

Akaike´s Information Criterion (AIC): AIC is an index proposed by Akaike [15] to measure the fit of the model [16]. The AIC is recently used for judging the performance of several candidate models. The minimum value of the index, the adequate model fit to the data. The mathematical formula for calculating AIC is as follows:

$$AIC = 2k + n[\text{In(RSS)}] \qquad (6)$$

Where: AIC is Akaike's Information Criterion; k is the number of parameters in the model; n is number of observations; ln is the natural logarithm; RSS is the residual sum of squares.

Evaluation of Stand Height Functions: Height-diameter relationship was evaluated using six height functions (Table 1) based on data from medium stand density. The criteria for selecting the best model to be used for the *A. seyal* height curve were based on R², AIC and the plausibility of the curve. Stand height curves were produced for dense, medium and slight stand densities using the selected allometric height function.

Table 1: Potential models used for evaluation of height-diameter relationship of *A. seyal*

Function nane	Mathematical Formula	Number
Parabolic	$h = a_0 + a_1 * d + a_2 * d^2$	(7)
Petterson	$h = 1.3 + \left(\frac{d}{a_0 + a_1 * d} \right)^2$	(8)
Prodan	$h = 1.3 + d^2 / (a_0 + a_1 * d + a_2 * d^2)$	(9)
Michailow	$h = 1.3 + a_0 * e^{(a_1 / d)}$	(10)
Hendrickson	$h = a_0 + a_1 * \log d$	(11)
Van laar	$h = e^{\left(a_0 + a_1 / d + a_2 / d^2\right)}$	(12)

Source: [6]

Where: h = total height (m); d = diameter at breast height (DBH, in cm); log = natural logarithm; e = base of natural logarithm (˜ 2.7183); a_0, a_1 and a_2 are coefficients. Observations (n = 1235) based on data from medium stand density.

RESULTS AND DISCUSSION

Forest Composition and Structure: Stand structure is the distribution of species and tree sizes in a forest area [17]. As previously mentioned three stands densities, based on the number of trees per unit area, of *A. seyal* in the Umfakarin Natural Forest Reserve were distinguished, namely dense, medium and slight. Important features of these stands can be described as mono-species, single-layered and naturally regenerated. Additionally, there have been no silvicultural treatments or felling operations in the forest, with the exception of illegal felling, in some parts of the forest, by local people who live adjacent to the reserve. The stands are typical for the Garri forest in Blue Nile, Sudan which was described by Vink [9] as "*A. seyal* formation of 10-20 years old, of natural origin, pure, no understory, no natural regeneration and the site is being grazed by villages´ livestock".

Diameter and Height Distribution: Diameter at breast height (DBH) and height frequency distribution for the three stand densities was identified. Figure 2 illustrates

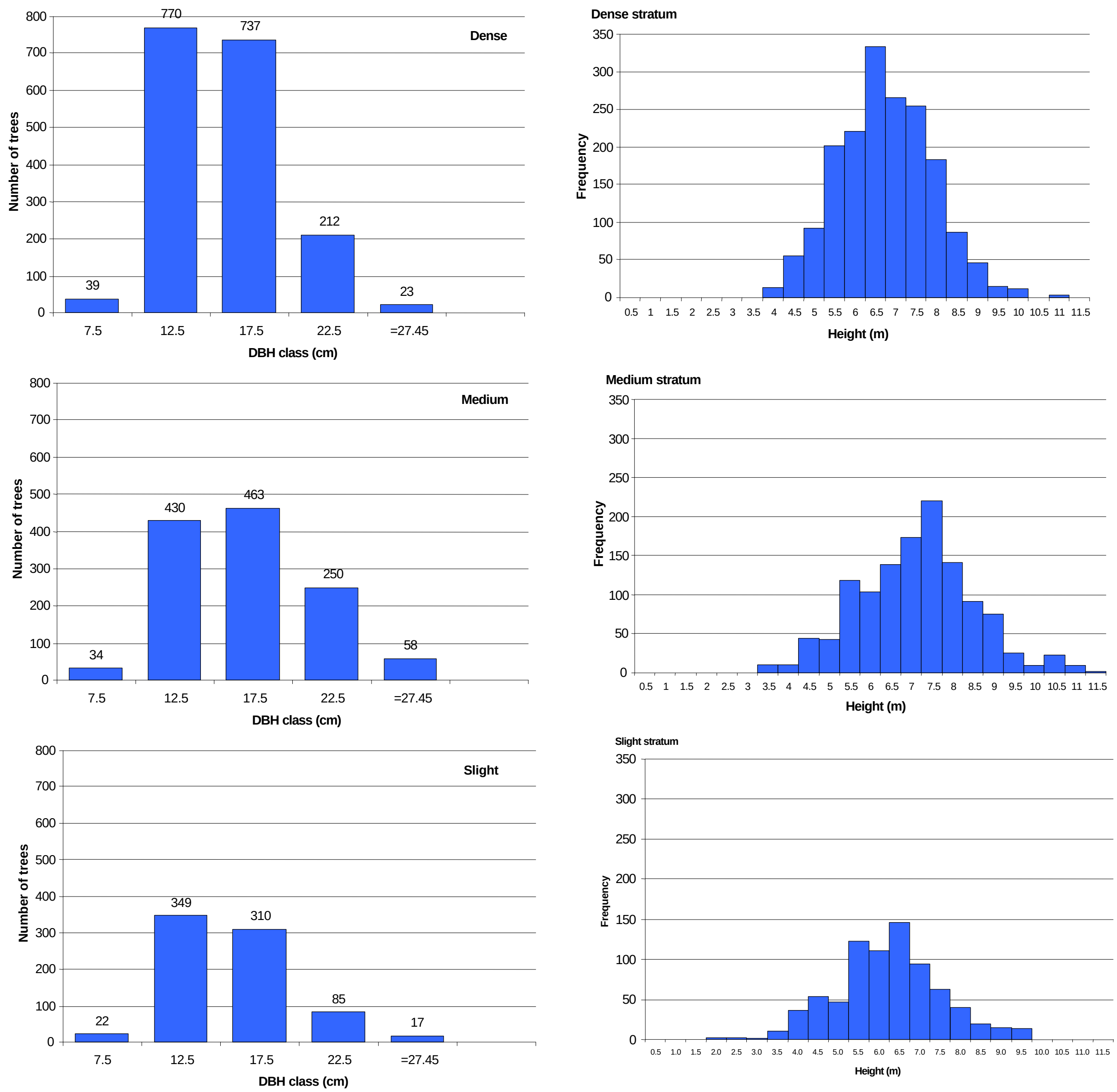

Fig. 2: DBH and height frequency distribution for *A. seyal* in the Umfakarin Forest Number of observations: dense = 1781, medium = 1235 and slight = 783.

Table 2: Number per hectare of natural regeneration greater and less than 1.3 m height

	Acacia seyal		Associated species	
	$ 1.3m	#1.3m	#1.3 m	#1.3 m
Stand density	Number of seedlings per hectare			
Dense	50	-	-	8
Medium	9	-	-	-

Table 3: Stand values of natural A. seyal in different stand densities

Stand density	DBH (cm)	h (m)	BA/ha (m²)	V/ha (m³)	N/ha
Dense	12. 2	7.2	5.93	22.10	396
Medium	13.4	7.6	4.87	19.45	271
Slight	12.2	6.7	3.09	10.86	209

Where: DBH is the quadratic mean diameter at breast height (over bark, cm); h is the mean tree height (m); BA is the basal area (m^2); V is the stand volume (m^3); N is the number of trees per hectare.

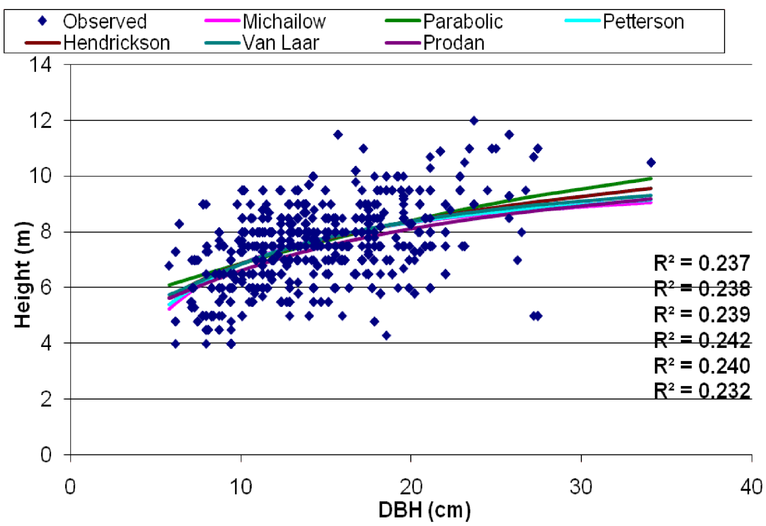

Fig. 3: Observed data and the lines of best fit (evaluated height functions) for *A. seyal*

the DBH and the height frequency distribution of natural *A. seyal* stands in the Umfakarin Forest. As illustrated by this figure, the frequency distribution of DBH classes (at 5 cm intervals) and the height classes at 1.5 m intervals, of the three stand densities, shows an irregularly distributed form, especially height in medium and slight densities. Normality test was conducted and revealed that DBH and height are not normally distributed ($P < 0.000$).

Distribution of diameters and the heights of trees, in a specific stand, is a good criterion not only for describing the horizontal and vertical structure of the stand but to also provide basic information for forest resource management. Moreover, trees of different diameters may be used for different purposes and have different values per cubic meter of wood [18]. In the present study, the pattern of DBH distribution appears to be similar in the three stand densities. The number of individuals is higher in the second and third diameter classes. In contrast, there are few stems in the small (DBH = 7.5 cm) and large diameter classes (DBH = 22.5⁺ cm). The lower number of stems in the 22.5 diameter class and upwards could be attributed to illegal felling in some parts of the forest. Egadu *et al*. [19], in their study of the population of acacia tree species producing gum Arabic in Karamoja, Uganda, revealed a higher number of stems of *A. seyal* in smaller diameter classes indicating good regeneration. Conversely, in this study few stems were detected in the smaller diameter class indicating poor natural regeneration (Table 2). This table illustrates the number of natural regeneration (greater or less than 1.3 m in height) per hectare by stratum of *A. seyal* and other associated species. In general, all stands had a very limited number of seedlings. *A. seyal* occurs at all strata; however, only 9-50 seedlings per hectare, = 1.3 m in height, were recorded. Seedlings from other species of = 1.3 m height occur in dense and slight strata with a density of 8 and 15 seedlings per hectare, respectively.

Table 4: Models evaluated for the height curve of A. seyal natural stands in the Umfakarin Forest, South Kordofan, Sudan.

Function name	Mathematical formula	a_0	a_1	a_2	R²	AIC
Parabolic	$h = a_0 + a_1 * d + a_2 * d^2$	4.908	0.216	-0.002	0.238	486.36
Petterson	$h = 1.3 + \left(\frac{d}{a_0 + a_1 * d}\right)^2$	0.961	0.328		0.239	483.68
Prodan	$h = 1.3 + d^2 / (a_0 + a_1 * d + a_2 * d^2)$	-1.267	0.977	0.095	0.240	483.26
Michailow	$h = 1.3 + a_0 * e^{(a_1 / d)}$	8.917	-4.714		0.237	485.94
Hendrickson	$h = a_0 + a_1 * \log d$	1.762	-4.714		0.242	479.00
Van Laar	$h = e^{(a0+a1/d+a2/d2)}$	2.399	-6.125	13.654	0.240	484.32

Where: h = total height (m); d = diameter at breast height (DBH, in cm); log = natural logarithm; e = base of natural logarithm (˜ 2.7183); a_0, a_1 and a_2 are parameters to be estimated by nonlinear regression; and R² = coefficient of determination. AIC = Akaike´s information Criterion; Observations (n = 1235) based on data from medium stratum.

Stand values, such as basal area (BA, in m² per hectare) and the number of trees per hectare, besides mean diameter at breast height (DBH, in cm), mean height (h, in m) and volume (V, in m^3 per hectare) were obtained, a summary of which can be found in Table 4. Stand BA ranged from 3.09 m²/ha in slight stratum to 5.93 m²/ha in dense stratum. The three stand categories vary in terms of the number of trees per unit area, ranging from 209 to 396 stems per hectare, with standing volume ranging from 10.86 to 22.10 m^3/ha. Stands of dense, medium and slight density have mean DBH 12.2, 13.4 and 12.2 cm with mean height 7.2, 7.6 and 6.7 m, respectively.

Modelling Height Curves: The results of the evaluated models were based on data from medium stand density. Lines of the best fit of the evaluated height functions in combination with the observed data were illustrated in Figure 4. Additionally, Table 5 provides details about the evaluated models, their estimated parameters, the coefficient of determination values (R²) and Akaike´s information Criterion (AIC). A model with the greater R² value and smaller AIC is the best. The statistical analysis resulted in a very similar values of R² (0.237-0.242) and AIC (479.00-486.36) as well. Based on the results of the evaluated models, a Michailow height function (R² = 0.237) was selected as it also produces a line of the best fit acceptable from a biological point of view. A scatter plot was created with DBH representing the independent variable on the horizontal axis and height, the dependent variable, plotted on the vertical axis. The distribution of data points on the scatter plot expresses the trend of the height-diameter relationship.

Figure 4 illustrates the Michailow height curves for different stand densities in combination with other observed data. For each density, the data points appear to be randomly scattered, or not around the line of best fit, which indicates a low correlation between height and diameter. Parameters estimated by the Michailow height function for the three stand densities are presented in Table 6. R² values ranged from 0.193 in dense to 0.240 in slight density. Thus, R² value decreases with an increase in the number of trees per unit area. Results of statistical test revealed that the slopes of the height curves in different stand densities are not significantly different (using the conventional significant level, i.e. á = 0.05). Because the slopes for the curves in the three stand densities do not significantly differ, a combined height curve (R² = 0.227) was produced and included (Figure 5, bottom right). The maximum height (asymptote = 10.2 m) is reached by trees in medium stand density. Trees in dense and slight densities reach a maximum height of 9.3 and 9.4 m, respectively.

Height and diameter are the most frequent measurements made by foresters in order to estimate growth and/or the yield of trees in forest stands. El-Juhany and Aref [20] indicated that, under uniform site conditions, trees of the same age grow in height at roughly the same rate but not necessarily the same in diameter. They further noted that, in uniform site conditions, trees of the same diameter do not necessarily having the same height. However, in tropical natural forest stands, it is difficult to estimate tree age, hence trees growing under the same site conditions do not necessarily grow in height at the same rate. Variation in heights and diameters may be due to competition and difference in tree age.

Table 6: Summary of estimated parameters of the Michailow function used for A seyal height estimation in different strata, Umfakarin Forest, South Kordofan, Sudan

Stand density	n	*a*0	*a*1	R^2	Asymptote
Dense	1781	8.039 (0.125)	-3.857 (0.193)	0.193	9.3
Medium	1235	8.917 (0.166)	-4.714 (0.250)	0.237	10.2
Slight	783	8.102 (0.214)	-5.072 (0.328)	0.240	9.4

*a*0 and *a*1 are parameters to be estimated by non-linear regression; in parenthesis is the standard error of parameter; n = number of individual trees; R2 is coefficient of determination

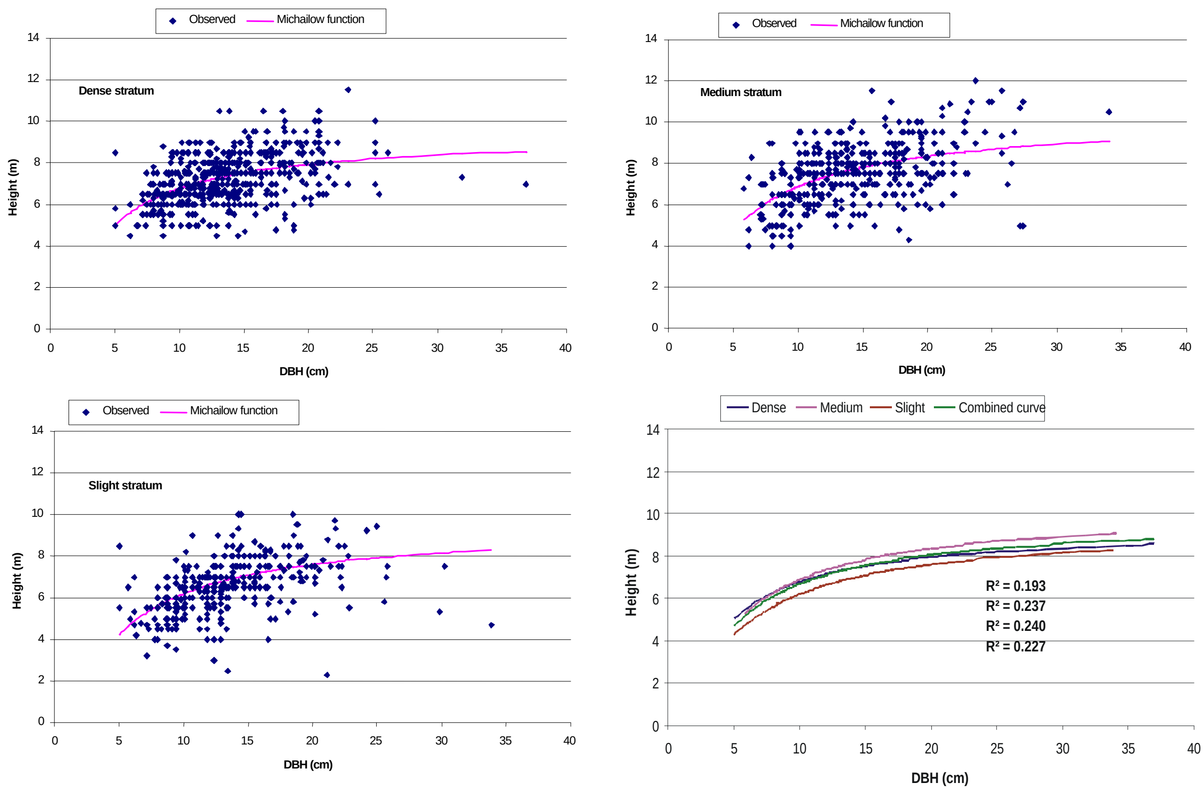

Fig. 5: Observed data and the line of best fit (Michailow height curve) of *A. seyal*

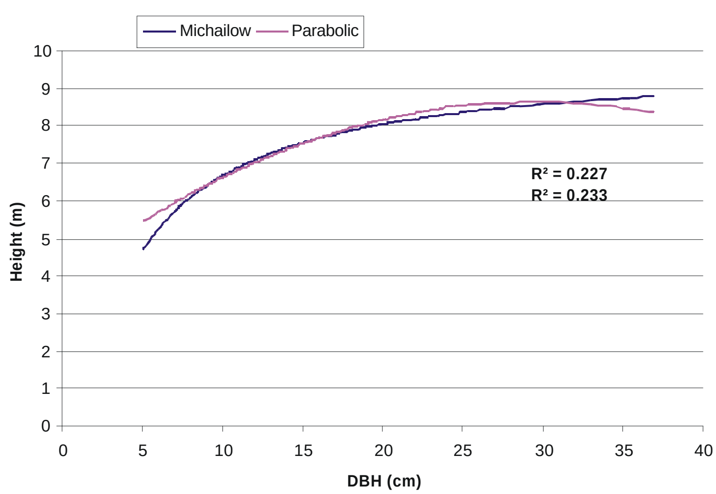

Fig. 6: Michailow and parabolic height curves used for predicting height of A. seyal trees in Umfakarin natural forest, Sudan

The criteria adopted for examining the performance of the height curves were the R^2 value, AIC and the curve shape. However, Van Laar and Akça [13] and Yuancai and Parresol [21] suggested that the fit curve should satisfy specific criteria. These criteria are monotonic increment (increasing height with increasing diameter), inflection point (the point(s) where the curve change its direction) and asymptote (when the diameter goes to infinity). In reality, height increases with increasing diameter but not absolute, it should increase to a certain limit. Based on these criteria Michailow height function was selected because in addition to plausible curve shape, about 23% of the total height variability is explained. Nevertheless, the Michailow height curve was compared to parabolic function which was used by Elsiddig [1] for estimating height of *A. seyal* in eastern Sudan (Figure 6). The later function produces R^2 value greater than that of Michailow but does not satisfy the criteria which proposed by the authors [13, 21]. Both models revealed biologically logical trend up to 30 cm DBH, then height by parabolic decreases with increasing DBH which is not acceptable from biological view point. The Michailow height function also satisfies the asymptote which corresponds to the maximum height (10.2 m) when DBH goes to infinity. The parabolic function does not satisfy this criterion. Based on these findings, Michailow height function (Equation 13) was used for predicting height of *A. seyal* trees in Umfakarin natural forest, Sudan and in other areas of similar climatic conditions.

$$H = 1.3 + 8.486 * e^{(4.604/d)} \quad (13)$$

Parameter as previously explained.

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