Optimal Solution for Multi-Objective Two Stage Fuzzy Transportation Problem

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Abstract: The linear multiobjective transportation problem is a special type of vector minimum problem in which constraints are all equality type and the objectives are conflicting in nature. It gives an optimal compromise solution. The obtained result has been compared with the solution obtained by using a linear membership function. To illustrate the methodology some numerical examples are presented.

Key words: Multi-objective Transportation problem • Fuzzy set • Trapezoidal Fuzzy number

INTRODUCTION

Transportation is an essential part of modern society. It is not possible for each individual of family to produce his own food, clothing, etc. Goods can be produced more efficiently in factories, large farms, etc. but this necessitates the movement of both goods and people. The whole structure of society involves a trade-off between the economies of scale and focusing activities or groups of activities (factories, schools, office buildings and cities) and the cost of transporting people from home to work places and goods from factories to consumers thus, the structure generates a problem known as Transportation Problem. The Transportation Problem is a classic Operations Research Problem where the objective is to determine the schedule for transporting goods from source to destination in a way that minimizes the shipping cost while satisfying supply and demand constraints. Although it can be solved as a Linear Programming problem, other methods exist.

The Transportation Problem (TP) was first developed and proposed by F.L. Hitchcock since 1941, the Hitchcock-Koopman's transportation problem is expressed as a linear transportation model as follows:

Minimize \( z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} \)

Subject to \( \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,.....,m \) (Supply)

\( \sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,.....,n \) (Demand)

\( x_{ij} \geq 0 \) for all \( i \) and \( j \)

where,

- \( x_{ij} = \) The amount of goods moved from origin \( i \) to destination
- \( c_{ij} = \) The cost of moving a unit amount goods from origin \( i \) to destination \( j \)
- \( a_i = \) The supply available at each origin \( i \)
- \( b_j = \) The demand available at each destination \( j \)
- \( m = \) Total number of origins (Sources)
- \( n = \) Total number of destinations (Sinks)

This problem can be solved by classical transportation methods.

The transportation problem makes an important role in real life as for example minimization of total cost, consumption of certain scarce resources such as energy, total deterioration of goods during transportation, vehicle scheduling in public transit etc. From the investigation, the entire existing objectives in single objective transportation models are represented by quantitative information. This may cause the negligence of some crucial points which cannot be described by quantitative data. Real life decision making takes into account multiple, often conflicting, criteria. For example in shortest path problem, where cyclists aims to reach their destination in minimal time but along a safe route.
Factors that may influence route choice are road traffic, road condition and presence of dedicated cycling facilities. Therefore, it is reasonable to formulate cyclist route choice as a bi-objective problem with travel time as one objective, whereas all other route choice factors are combined into a second objective that we call attractiveness. That means in reality, considering only one objective of TP is not sufficient because it may not lead to the practical optimal solution. Thus the Decision Maker (DM) is rather to pay attention on several objectives at same time or in other words we can describe this as the limitation of single objective TP. This limitation can be sought out by generating multi-objective TP.


Nagoor Gani and Abdul Razak [9] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers.

**Terminology:** In this section some basic definitions of fuzzy set theory are reviewed (Dubois and Prade, 1980), (Kauffman and Gupta, 1991).

**Definition:** The characteristic function \( \mu_A(x) \) of a crisp set \( A \subset X \) assigns a value either 0 or 1 to each member in \( X \). This function can be generalized to a function \( \mu_\tilde{A}(x) \) such that the value assigned to the element of the universal set \( X \) fall within a specified range i.e. \( \mu_\tilde{A}:X \rightarrow [0,1] \). The assigned value indicate the membership grade of the element in the set \( \tilde{A} \). The function \( \mu_\tilde{A}(x) \) is called the membership function and the set \( \tilde{A} = \{(x, \mu_\tilde{A}(x)) : x \in A \text{ and } \mu_\tilde{A}(x) \in [0,1] \} \) is called a fuzzy set.

**Definition:** A fuzzy set \( \tilde{A} \), defined on the set of real numbers \( R \) is said to be a fuzzy number if its membership function \( \mu_\tilde{A}:R \rightarrow [0,1] \) has the following characteristics.

- \( \tilde{A} \) is normal. It means that there exists an \( x \in R \) such that \( \mu_\tilde{A}(x) = 1 \)
- \( \tilde{A} \) is convex. It means that for every \( x_1, x_2 \in R \),
  \[ \mu_\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_\tilde{A}(x_1), \mu_\tilde{A}(x_2)\}, \lambda \in [0,1] \]
- \( \mu_\tilde{A} \) is upper semi-continuous.
- \( \text{Supp}(\tilde{A}) \) is bounded in \( R \).

**Definition:** A fuzzy number \( \tilde{A} \) is said to be non–negative fuzzy number if and only \( \mu_\tilde{A}(x) = 0, \forall x < 0 \)

**Definition:** A fuzzy number \( \tilde{A} = (a,b,c,d) \) is said to be a trapezoidal fuzzy number if its membership function is given by, where \( a \leq b \leq c \leq d \).

\[
\mu_\tilde{A}(x) = \begin{cases} 
0, & x < a \\
\frac{x-a}{b-a}, & a \leq x \leq b, \\
1, & b < x < c \\
\frac{d-x}{d-c}, & c \leq x \leq d, \\
0, & x > d 
\end{cases}
\]

**Definition:** A trapezoidal fuzzy number \( \tilde{A} = (a,b,c,d) \) is said to be non–negative (non positive) trapezoidal fuzzy number. i.e. \( \tilde{A} \geq 0(\tilde{A} \geq 0) \) if and only if \( a \geq 0(\tilde{A} \leq 0) \). A trapezoidal fuzzy number is said to be positive (negative) trapezoidal fuzzy number i.e. \( \tilde{A} > 0(\tilde{A} < 0) \) if and only if \( a > 0(c < 0) \) [10].

**Definition:** Two trapezoidal fuzzy numbers \( \tilde{A}_1 = (a,b,c,d) \) and \( \tilde{A}_2 = (e,f,g,h) \) are said to be equal. i.e. \( \tilde{A}_1 = (a,b,c,d) \) if and only if \( a=e, b=f, c=g, d=h \).

**Definition:** Let \( \tilde{A}_1 = (a,b,c,d) \) and \( \tilde{A}_2 = (e,f,g,h) \) be two non-negative trapezoidal fuzzy number then.

- \( \tilde{A}_1 \oplus \tilde{A}_2 = (a+b,c+g,d+h) \)
- \( \tilde{A}_1 - \tilde{A}_2 = (a-b,c-g,d-h) \)
- \( \tilde{A}_1 = (a,b,c,d) \)
- \( \tilde{A}_2 = (e,f,g,h) \)
- \( \tilde{A}_1 \otimes \tilde{A}_2 = (ae,bf,gc,hd) \)
- \( \frac{1}{\tilde{A}} = \left( \frac{1}{d}, \frac{1}{c}, \frac{1}{b}, \frac{1}{a} \right) \)
Multi-objective Transportation Problem (MOTP): The multi-objective transportation model is set to solve the transportation problem simultaneously associated with several objectives. Normally, existing multi-objective transportation models use a minimization of the total cost objective as one of their objectives. The other objectives may concern about delivery time, quantity of goods delivered, underused capacity, reliability of delivery, energy consumption, safety of delivery, etc [11]. The multi-objective transportation problem with k objectives can be represented as;

\[
\text{Min } f_j(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^j x_{ij} \\
\ldots
\]

\[
\text{Min } f_k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij}
\]

Subject to \( \sum_{j=1}^{n} x_{ij} \leq a_i, i = 1, 2, \ldots, m \) for all \( i \).

\[
\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \ldots, n \text{ for all } j.
\]

\[
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ And } x_{ij} \geq 0 \text{ for all } i \text{ and } j
\]

where \( c_{ij}^j \) represents the coefficients related to \( x_{ij} \) variable for objective \( k \).

Most of existing research works of a transportation problem has considered depot to customer relationship. However, the relationship between customer and customer is also critical because in fact vehicle route for each depot does not move from depot to customer and returns back from customer to depot as in the transportation model, it moves from depot to customer and moves forward to the other customers. So, it needs also to consider customer to customer relationship to obtain the neighbourhood customers. Then, two objectives are concerned. The first objective is to minimize the total transportation cost which is the baseline objective for all transportation models. It is the depot to customer relationship consideration using quantitative data. The second objective is to minimize the overall independence value between customer and customer, which means the consideration of customer to customer relationship. And, finally this problem introduces an important MOTP to real world [12-15].

New Approach for Solving Fuzzy Transportation Problem: Following are the steps for solving Fuzzy Transportation Problem.

**Step 1:** Select the first row (source) and verify which column (destination) minimum unit has cost. Write that source under column 1 and corresponding destination under column 2. Continue this process for each source. However if any source has more than one same minimum value in different destination then write these entire destination under column 2.

**Step 2:** Select those rows under column-1 which have unique destination. For example, under column-1, sources are S1, S2, S3 have minimum unit cost which represents the destination D1, D1, D3 written under column 2. Here D3 is unique and hence allocate cell (S3, D3) a minimum of demand and supply. For an example if corresponding to that cell supply is 8 and demand is 6, then allocate a value 6 for that cell. However, if destinations are not unique then follow step 3. Next delete that row/column where supply/demand exhausted.

**Step 3:** If destination under column-2 is not unique then select those sources where destinations are identical. Next find the difference between minimum and next minimum unit cost for all those sources where destinations are identical.

**Step 4:** Check the source which has maximum difference. Select that source and allocate a minimum of supply and demand to the corresponding destination. Delete that row/column where supply/demand exhausted.

**Remark 1:** For two or more than two sources, if the maximum difference happens to be same then in that case, find the difference between minimum and next to next minimum unit cost for those sources and selects the source having maximum difference. Allocate a minimum of supply and demand to that cell. Next delete that row/column where supply/demand exhausted.

**Step 5:** Repeat steps 3 and 4 for remaining sources and destinations till (m+n-1) cells are allocated.

**Step 6:** Total cost is calculated as sum of the product of cost and corresponding allocated value of supply/demand. That is, Total Cost = \( \Sigma \Sigma C_i X_j \)

**Solution Algorithm:**

**Step 1:** Construct the Transportation problem

**Step 2:** Supply and Demand are fuzzy number \((a_i, a_i, a_i, a_i)\) and \((b_j, b_j, b_j, b_j)\) in the formulation Problem (two stages FCMTP).
Step 3: Convert the problem ($\alpha$-two stage FCMTP) in the form of the ($\alpha'$ - two stage FCMTP)

Step 4: Formulate the ($\alpha'$ - two stage FCMTP) in the parametric form.

Step 5: Apply the new method to get optimal value in stage-I and stage-II.

Step 6: The optimal value of the objective function of the problem is $\text{Min} (C1+C2)$.

Numerical Example: Consider the following two stage cost minimizing transportation problem. Here supplies and demands are trapezoidal fuzzy numbers.

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
<th>D6</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>11</td>
<td>4</td>
<td>2</td>
<td>(4, 5, 7, 8)</td>
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<tr>
<td>S2</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>5</td>
<td>10</td>
<td>4</td>
<td>(6, 7, 8, 9)</td>
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<tr>
<td>S3</td>
<td>12</td>
<td>25</td>
<td>9</td>
<td>6</td>
<td>26</td>
<td>12</td>
<td>(5, 6, 7, 8)</td>
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<tr>
<td>S4</td>
<td>8</td>
<td>7</td>
<td>9</td>
<td>24</td>
<td>10</td>
<td>8</td>
<td>(4, 6, 8, 9)</td>
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<tr>
<td>Demand</td>
<td>(1, 2, 4, 5)</td>
<td>(4, 5, 6, 7)</td>
<td>(3, 4, 5, 7)</td>
<td>(4, 5, 6, 7)</td>
<td>(2, 3, 4, 5)</td>
<td>(3, 4, 5, 6)</td>
<td></td>
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</tbody>
</table>

Consider the $\alpha$ - level set to be $\alpha=0.75$,

Then we get $4.5 \leq a_1 \leq 7.5, 6.5 \leq a_2 \leq 8.5, 5.5 \leq a_3 \leq 7.5, 5.0 \leq a_4 \leq 8.5, 1.5 \leq b_1 \leq 4.5, 4.5 \leq b_2 \leq 6.5, 3.5 \leq b_3 \leq 6.0, 4.5 \leq b_4 \leq 6.5, 2.5 \leq b_5 \leq 4.5, 3.5 \leq b_6 \leq 5.5$.

The $\alpha$-optimal parameters are $a_1 = 6, a_2 = 8, a_3 = 7, a_4 = 4$

$\text{b}_1 = 3, b_2 = 5, b_3 = 5, b_4 = 6, b_5 = 4, b_6 = 5$

Stage I:

We assign $a_1 = 3, a_2 = 4, a_3 = 3, a_4 = 3$

$b_1 = 1, b_2 = 2, b_3 = 3, b_4 = 2, b_5 = 2$

<table>
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<tr>
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<td>3</td>
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</table>

Apply New method which gives following allocation $X_{13} = 1, X_{15} = 2, X_{21} = 1, X_{22} = 1, X_{26} = 2, X_{34} = 3, X_{43} = 3$ and Minimum $Z= 75$.

Stage II:

We assign $a_1 = 3, a_2 = 4, a_3 = 4, a_4 = 4$

$b_1 = 2, b_2 = 3, b_3 = 2, b_4 = 3, b_5 = 2, b_6 = 3$

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<td>Demand</td>
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<td>3</td>
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<td></td>
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</tbody>
</table>

Apply New method which gives following allocation $X_{13} = 1, X_{15} = 2, X_{21} = 2, X_{22} = 2, X_{34} = 1, X_{36} = 3, X_{43} = 1, X_{46} = 3$ and Minimum $Z= 93$.

The optimal value of the objective function is obtained function is obtained by combining stage – I and stage – II, therefore Minimum $Z = 168$.  

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CONCLUSION

Reduction of cost, time and other factors in transportation problem, In this paper Knew method is used to determine the optimal compromise solution for a multi-objective two stage fuzzy transportation problem, in which supplies, demands are trapezoidal fuzzy numbers and fuzzy membership of the objective function is defined. Since the objective value is expressed by membership function rather than by a crisp value, more information is provided for making decisions.

REFERENCES