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Factorization of the Lorentz Matrix

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Abstract: We exhibit a factorization for an arbitrary Lorentz matrix.

Key words: Lorentz transformation - Special relativity - Factorization of a matrix

INTRODUCTION

The Olinde Rodrigues [1]-Cartan [2] expression:

$$\begin{pmatrix} \tilde{x}^0+\tilde{x}^3 & \tilde{x}^1+i\,\tilde{x}^2 \\ \tilde{x}^1-i\,\tilde{x}^2 & \tilde{x}^0-\tilde{x}^3 \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} x^0+x^3 & x^1+i\,x^2 \\ x^1-i\,x^2 & x^0-x^3 \end{pmatrix} \begin{pmatrix} \bar{\alpha} & \bar{\gamma} \\ \bar{\beta} & \bar{\delta} \end{pmatrix}, \tag{1}$$

where α, β, γ, δ are arbitrary complex numbers verifying the condition $\alpha\delta - \beta\gamma = 1$, implies six degrees of freedom for the Lorentz matrix $L = (L^{\nu}{}_{\mu})$ between the frames of reference $(x^{\mu}) = (ct, x, y, z)$ and $(\tilde{x}^{\nu})$:

$$\tilde{x}^{\nu} = L^{\nu}{}_{\mu}\, x^{\mu}\,. \tag{2}$$

From (1) and (2) we obtain the relations [3-9]:

$$\begin{gathered}
L^0{}_0 = \tfrac{1}{2}\left(\alpha\bar{\alpha} + \beta\bar{\beta} + \gamma\bar{\gamma} + \delta\bar{\delta}\right), L^1{}_0 = \tfrac{1}{2}\left(\bar{\alpha}\,\gamma + \bar{\beta}\delta\right) + cc, L^2{}_0 = -\tfrac{i}{2}\left(\alpha\bar{\gamma} - \bar{\beta}\delta\right) + cc\,, \\
L^0{}_1 = \tfrac{1}{2}(\bar{\alpha}\beta + \bar{\gamma}\delta) + cc, L^1{}_1 = \tfrac{1}{2}(\bar{\alpha}\delta + \beta\bar{\gamma}) + cc, L^2{}_1 = -\tfrac{i}{2}\left(\alpha\bar{\delta} + \beta\bar{\gamma}\right) + cc, \\
L^0{}_2 = -\tfrac{i}{2}(\bar{\alpha}\beta + \bar{\gamma}\delta) + cc, L^1{}_2 = -\tfrac{i}{2}(\bar{\alpha}\delta + \beta\bar{\gamma}) + cc, L^2{}_2 = \tfrac{1}{2}\left(\bar{\alpha}\delta - \bar{\beta}\gamma\right) + cc\,, \\
L^0{}_3 = \tfrac{1}{2}\left(\alpha\bar{\alpha} - \beta\bar{\beta} + \gamma\bar{\gamma} - \delta\bar{\delta}\right), L^1{}_3 = \tfrac{1}{2}\left(\bar{\alpha}\gamma - \bar{\beta}\delta\right) + cc, L^2{}_3 = -\tfrac{i}{2}\left(\alpha\bar{\gamma} + \bar{\beta}\delta\right) + cc\,, \\
L^3{}_0 = \tfrac{1}{2}\left(\alpha\bar{\alpha} + \beta\bar{\beta} - \gamma\bar{\gamma} - \delta\bar{\delta}\right), L^3{}_1 = \tfrac{1}{2}(\bar{\alpha}\beta - \bar{\gamma}\delta) + cc, L^3{}_2 = -\tfrac{i}{2}(\bar{\alpha}\beta - \bar{\gamma}\delta) + cc\,, \\
L^3{}_3 = \tfrac{1}{2}\left(\alpha\bar{\alpha} - \beta\bar{\beta} - \gamma\bar{\gamma} + \delta\bar{\delta}\right), \alpha\delta - \beta\gamma = 1,
\end{gathered} \tag{3}$$

where *cc* means the complex conjugate of all the previous terms.

An analysis of the expressions (3) shows that the corresponding Lorentz matrix admits the following factorization:

$$\left(L^{\nu}{}_{\mu}\right) = \frac{1}{2}\begin{pmatrix} \alpha & \beta & \gamma & \delta \\ \gamma & \delta & \alpha & \beta \\ i\gamma & i\delta & -i\alpha & -i\beta \\ \alpha & \beta & -\gamma & -\delta \end{pmatrix} \begin{pmatrix} \bar{\alpha} & \bar{\beta} & i\bar{\beta} & \bar{\alpha} \\ \bar{\beta} & \bar{\alpha} & -i\bar{\alpha} & -\bar{\beta} \\ \bar{\gamma} & \bar{\delta} & i\bar{\delta} & \bar{\gamma} \\ \bar{\delta} & \bar{\gamma} & -i\bar{\gamma} & -\bar{\delta} \end{pmatrix}, \tag{4}$$

and we consider that it is interesting to analyze the possible physical meaning of this splitting (4), and in this direction we are studying 3-rotations $\delta = \bar{\alpha}, \gamma = -\bar{\beta}, \alpha\bar{\alpha} + \beta\bar{\beta} = 1$ and boosts $\alpha = \bar{\alpha}, \delta = \bar{\delta}, \gamma = \bar{\beta}, \alpha\delta - \beta\bar{\beta} = 1$, [10]. Besides, each matrix factor into (4) can be studied via the Dirac matrices [11, 12].

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REFERENCES

1. Olinde Rodrigues, B., 1840. Des lois géométriques qui régissent les déplacements d'un systéme solide, Journal de Math. (Liouville), 5: 380-440.
2. Cartan, E., 1913. Les groupes projectifs qui ne laissent invariante aucune multiplicité plane, Bull. Soc. Math. de France, 41: 53-96.
3. Ju. Rumer, 1936. Spinorial analysis, Moscow.
4. Aharoni, J., 1959. The special theory of relativity, Clarendon Press, Oxford.
5. Synge, J.L., 1965. Relativity: the special theory, North-Holland, Amsterdam.
6. Penrose, R. and W. Rindler, 1984. Spinors and space-time. I, Cambridge University Press.
7. López-Bonilla, J., J. Morales and G. Ovando, 2002. On the homogeneous Lorentz transformation, Bull. Allahabad Math. Soc., 17: 53-58.
8. Acevedo, M., J. López-Bonilla and M. Sánchez, 2005. Quaternions, Maxwell equations and Lorentz transformations, Apeiron, 12(4): 371-384.
9. Ahsan, Z., J. López-Bonilla and B.M. Tuladhar, 2014. Lorentz transformations via Pauli matrices, J. of Advances in Natural Sci., 2(1): 49-51.
10. Steane, A.M., 2013. An introduction to spinors, arXiv:1312.3824v1 [math-ph] 13 Dec 2013.
11. López-Bonilla, J., L. Rosales-Roldán and A. Zúñiga-Segundo, 2009. Dirac matrices via quaternions, J. Sci. Res. (India), 53: 253-255.
12. Lam-Estrada, P., J. López-Bonilla, R. López-Vázquez and M.R. Maldonado-Ramírez, 2016. Transformation of Dirac spinor under boosts and 3-rotations, Prespacetime J., 7(1): 148-153.