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Comments on Some Combinatorial Identities

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Abstract: We realize comments on some combinatorial identities obtained by Cheon-Seol-Elmikkawy.

Key words: Stirling numbers · Binomial coefficients · Gauss hypergeometric function

INTRODUCTION

In [1] we find the following combinatorial identities:

$$\sum_{k=0}^{n} k\binom{m}{k}\binom{n}{k} = n\binom{m+n-1}{n}, \tag{1}$$

$$\sum_{k=0}^{n}\binom{n}{k}k^r = \sum_{k=0}^{r} k!\binom{n}{k} S_r^{[k]}\ 2^{n-k}, \tag{2}$$

$$\sum_{k=0}^{n}\binom{n}{k}k^r x^{n-k} = \sum_{k=0}^{r} k!\binom{n}{k} s_r^{[k]}(1+x)^{n-k}, \tag{3}$$

$$A \equiv \sum_{k=0}^{n} k^2\binom{m}{k}\binom{n}{k} = mn\binom{m+n-2}{n-1}. \tag{4}$$

The relation (1) is the expression (3.30) of Gould [2]; the identity (2) is in Spivey [3] and corresponds to (9.58) of Quaintance-Gould [4]. From (1.126) in [2] or (9.57) in [4]:

$$\sum_{k=0}^{n}\binom{n}{k}k^r y^k = \sum_{k=0}^{r} k!\binom{n}{k} y^k (1+y)^{n-k} S_r^{[k]}, \tag{5}$$

where we can employ $y = \frac{1}{x}$ and after to multiply by x^n to obtain (3).

Besides, it is easy to see that:

$$A = m\,n\sum_{k=0}^{\infty} t_k,\quad t_k = \frac{(k+1)^2}{m\,n}\binom{m}{k+1}\binom{n}{k+1} \qquad \therefore \quad \frac{t_{k+1}}{t_k} = \frac{(k+1-m)(k+1-n)}{(k+1)^2}, \tag{6}$$

Hence [5-12]:

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$$A = m\,n\;{}_2F_1(1-m,\,1-n;1;1) = m\,n\,\frac{(m+n-2)!}{(m-1)!(n-1)!}, \qquad (7)$$

Which is equivalent to (4). In (7) we have the participation of the Gauss hypergeometric function with the following property in terms of the Gamma function [13, 14]:

$${}_2F_1\;(a,b;c;1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}. \qquad (8)$$

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