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A New Modified of McDougall-Wotherspoon Method for Solving Nonlinear Equationsby Using Geometric Mean Concept

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Abstract: Finding the roots of nonlinear algebraic equations is an important problem in science and engineering. Many mathematical models in physics, engineering and applied science, are applied with nonlinear equations. Later many methods developed for solving nonlinear equations. The efficient methods to find the roots of nonlinear equations has been developed in recent years [1-35], Natasˇa Glisˇovic´*et al.* [1] developed the method of T.J. McDougall and J. Wotherspoonwhich have derived a multistep iterative method with memory as a new modificationof the classical Newton's method [2]. We verified on a number of examples and numerical results obtained show the efficiency of the present method which is convergencesbetter than of the modification of Glisˇovic´ *et al*.

Key words: Nonlinear equations · Newton's Method · Method of Glisˇovic´ · T.J. McDougall and J. Wotherspoon · Geometric Mean · Harmonic mean · Arithmetic mean

INTRODUCTION

Solving nonlinear equations (1), is one of the most important problem in scientific and engineering applications. There are several well-known methods for solving nonlinearalgebraic equations of the form:

$$f(x) = 0 \tag{1}$$

where f denote a continuously differentiable function on [a, b] C R and has at least one root α, in [a, b] Such as Newton's Method, Bisection method, Regula Falsi method, Nonlinear Regression Method and several another methods see forexample [3-35]. Here we describe a new method by using geometric Meanof *xn* and *yn* instead of harmonic meanused by Glisˇovic´*et al.* [1] which wasreplaced byarithmetic mean of *xn* and *yn* used by T.J. McDougall and J. Wotherspoon [2] which presented a simple modification of Newton's Method which converges faster than the Newton's Method with a convergence order of $1+\sqrt{2}\approx 2.4142$. We verified on a number of examples and numerical results obtained show that the present method convergences better than of the modification of Glisˇovic´ *et al*.

The Present Method: Consider a nonlinear equation (1), consider the following iterative method proposed by T.J. McDougall and J. Wotherspoon Which have derived a multistep iterative method with memory [2],

$$y_0 = x_0 \tag{2}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'\left(\frac{1}{2}(y_0 + x_0\right)} = x_0 - \frac{f(x_0)}{f'(x_0)} \tag{3}$$

followed by (for n ≥ 1)

$$y_n = x_n - \frac{f(x_n)}{f'(\frac{1}{2}(x_n - 1 + y_{n-1}))} \tag{4}$$

$$X_{n+1} = X_n - \frac{f(x_n)}{f'(\frac{1}{2}(x_n - y_n))} \tag{5}$$

Glisˇovic´ *et al* replace in this method of T.J. McDougall and J. Wotherspoon, harmonic mean by arithmetic mean of *xn* and *yn*, then new iterative scheme obtained for n ≥ 1, preserving y_0 and x_1.

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$$y_n = x_n - \frac{f(x_n)}{f'\left(\frac{2x_{n-1}.y_{n-1}}{x_{n-1}+y_{n-1}}\right)} \tag{6}$$

$$X_{n+1} = x_n - \frac{f(x_n)}{f'\left(\frac{2x_n.y_n}{x_n+y_n}\right)} \tag{7}$$

Now, in present method, we replace arithmetic mean of x_n and y_n by geometric mean, then we obtain the following New scheme,

$$y_0 = x_0 \tag{8}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(\sqrt{x_0.y_0})} = x_0 - \frac{f(x_0)}{f'(x_0)} \tag{9}$$

followed by (for n $\geq$ 1)

$$y_n = x_n - \frac{f(x_n)}{f'\left(\sqrt{x_{n-1}.y_{n-1}}\right)} \tag{10}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'\left(\sqrt{x_n.y_n}\right)} \tag{11}$$

Algorithm of the Present Method:

- Give x_0 initial value (number real), give the tolerance number ε (for stopping) and take $y_0 = x_0$.
- Calculus of $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
- Calculus (for n $\geq$ 1): $y_n = x_n - \frac{f(x_n)}{f'(\sqrt{x_{n-1}.y_{n-1}})}$ and $x_{n+1} = x_n - \frac{f(x_n)}{f'(\sqrt{x_n.y_n})}$
- Calculus of stopping condition: if $\left|\frac{x_{n+1}-x_n}{x_{n+1}}\right| \leq \varepsilon$ *then stop, else,*
- Take n=n+1 and return to (3).

Examples: In this section, we shall check the effectiveness of present method (8)-(11). First numerical comparison for the following test examples taken in [1, 2] and from [26-31] have been employed, we compare present method (PM) with the Newton's method (NM), the Weerakoon-Fernando method (WFM), Ozban's variant of method (OM), the Frontini-Sormani method (FSM), the Kou-Li-Wang method (KLWM), Wang's method (WM), McDougall-Wotherspoon method (McDWM) and Glisˇovic´*et al* method (GOM).

Example 1:

$f_1(x) = x^2 - e^x -3x+2$, $x_0=3$, $\alpha = 0.257302854\ldots$

Table 1: Numerical results for $f_1(x)$

Method	Number of iteration(i)	$\lvert f(x_i)\rvert$
NM	8	$2.28.10^{-25}$
WFM	6	$2.80.10^{-16}$
OM	6	$1.33.10^{-22}$
FSM	6	$4.85.10^{-25}$
KLWM	6	$5.65.10^{-13}$
WM	5	$1.71.10^{-33}$
McDWM	7	$5.88.10^{-50}$
GOM	7	$8.97.10^{-25}$
PM	4	$5.89.10^{-197}$

Example 2:

$f_2(x) = xe^{x^2} - \sin^2 x + 3\cos x + 5$, , $x_0 = -2$, $\alpha = -1.207647827\ldots$

Table 2: Numerical results for $f_2(x)$

Method	Number of iteration(i)	$\lvert f(x_i)\rvert$
NM	11	$1.08.10^{-4}$
WFM	7	$1.76.10^{-4}$
OM	7	$5.99.10^{-10}$
FSM	7	$4.66.10^{-7}$
KLWM	7	$2.44.10^{-10}$
WM	7	$6.22.10^{-6}$
McDWM	9	$1.19.10^{-10}$
GOM	9	$8.83.10^{-10}$
PM	5	$8.51.10^{-13}$

Example 3:

$f_3(x) = e^{x^2+7x-30} - 1$, $x_0 = 3.25$, $\alpha = 3$

Table 3: Numerical results for $f_3(x)$

Method	Number of iteration (i)	$\lvert f(x_i)\rvert$
NM	11	$1.58.10^{-4}$
WFM	7	$1.86.10^{-4}$
OM	7	$1.83.10^{-9}$
FSM	7	$2.47.10^{-6}$
KLWM	7	$2.74.10^{-7}$
WM	7	$1.53.10^{-5}$
McDWM	9	$2.95.10^{-9}$
GOM	9	$2.85.10^{-9}$
PM	4	$6.75.10^{-33}$

Example 4:

$f_4(x) = \ln(x^2 + x + 2) - x + 1$, $x_0 = 3$, $\alpha = 4.152590736\ldots$

Table 4: Numerical results for $f_4(x)$

Method	Number of iteration (i)	$\lvert f(x_i)\rvert$
NM	7	$7.03.10^{-68}$
WFM	4	$1.22.10^{-116}$
OM	5	$3.66.10^{-88}$
FSM	5	$4.74.10^{-80}$
KLWM	5	$3.39.10^{-53}$
WM	5	$3.36.10^{-86}$
McDWM	6	$2.00.10^{-169}$
GOM	6	$2.73.10^{-168}$
PM	4	$4.99.10^{-258}$

CONCLUSIONS

In this work, we have proposed a new iterative method by using the geometric mean. The efficiency of this method is shown for some test problems, comparison of the obtained resultis given with t he existing methods such as with the Newton's method (NM), the Weerakoon-Fernando method (WFM), Ozban's variant of method (OM), the Frontini-Sormani method (FSM), the Kou-Li-Wang method (KLWM), Wang's method (WM), McDougall-Wotherspoon method (McDWM) and Glisˇovic´*et al.* method (GOM), it is shown that this new methodis more efficient than these existing methods and this method has lowest number of iteration and converges faster than the other methods.

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