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Some Applications of the Newton's Formula

J. López-Bonilla, R. López-Vázquez and S. Vidal-Beltrán

ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso,
Col. Lindavista CP 07738, CDMX, México

Abstract: We exhibit the applicability of the Newton's formula to the gamma function, Bernoulli and Stirling numbers, characteristic polynomial and harmonic sums.

Key words: Bernoulli numbers · Gamma function · Characteristic equation · Stirling numbers · Harmonic sums · Newton's formula

INTRODUCTION

In the Newton's expression [1]:

$$a_0 = 1, \;\; ra_r + s_1\, a_{r-1} + s_2 a_{r-2} + \ldots + s_{r-1} a_1 + s_r = 0, \;\; r = 1, 2, \ldots, n, \tag{1}$$

the quantities s_r are given and the values of the a_j are solutions of the triangular linear system [2, 3]:

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & \cdots & 0 \\ s_1 & 1 & 0 & \cdots & \cdots & 0 \\ s_2 & s_1 & 3 & \cdots & \cdots & \vdots \\ \vdots & \vdots & s_1 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ s_{n-1} & s_{n-2} & s_{n-3} & \cdots & \cdots & n \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ \vdots \\ a_n \end{pmatrix} = -\begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ \vdots \\ s_n \end{pmatrix}, \tag{2}$$

that is [4-6]:

$$m!\; a_m = (-1)^m \begin{vmatrix} s_1 & s_2 & s_3 & \cdots & s_{m-1} & s_m \\ m-1 & s_1 & s_2 & \cdots & s_{m-2} & s_{m-1} \\ 0 & m-2 & s_1 & \cdots & s_{m-3} & s_{m-2} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & s_1 \end{vmatrix}, \quad m = 1, \ldots, n. \tag{3}$$

which, for example, is applicable to the following situations:

- If A_{nxn} is an arbitrary matrix, then s_r = trace A^r and the a_j are the coefficients of its characteristic equation [1]:

$$\lambda^n + a_1 \lambda^{n-1} + \ldots + a_{n-1}\lambda + a_n = 0. \tag{4}$$

- For the harmonic sums [7]:

Corresponding Author: J. López-Bonilla, ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México. E-mail: jlopezb@ipn.mx.

$$S_n(m) = \sum_{k=1}^{n} \binom{n}{k} \frac{(-1)^{k-1}}{k^m}, \quad m,n \geq 1, \tag{5}$$

we have [8]:

$$a_k = S_n(k), \quad s_k = -H_n^{(k)} = -\sum_{j=1}^{n} \frac{1}{j^k}, \quad k = 1,\dots,n. \tag{6}$$

- The Stirling numbers of the first kind [9] verify (1) with [8, 10]:

$$a_r = \frac{(-1)^{n-r-1}}{(n-1)!} S_n^{(r+1)}, \quad s_r = (-1)^r H_{n-1}^{(r)}, \quad 1 \leq r \leq n, \tag{7}$$

and also for [10]:

$$a_j = S_n^{(n-j)}, \quad s_j = H_{n-1}^{(-j)}, \quad j = 1,\dots,n. \tag{8}$$

- The derivatives of gamma function [11] satisfy (1) for [12, 13]:

$$a_k = \frac{1}{k!} \Gamma^{(k)}(1), \quad s_k = (-1)^{k+1} \zeta(k), \tag{9}$$

involving the Riemann zeta function [11].

- The Bernoulli numbers [9] have the property (1) with [13, 14]:

$$a_r = \frac{1}{r!} B_r, \quad s_r = \frac{(-1)^r}{r!} B_r. \tag{10}$$

Hence the expressions (2) and (3) are applicable to (4), … , (10).

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