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On the Zlobec's Theorem for the Moore-Penrose Generalized Inverse

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Abstract: We employ the SVD expression for the Moore-Penrose pseudoinverse to motivate a Zlobec's theorem about this inverse.

Key words: Faddeev-Sominsky's method · Zlobec's theorem · Moore-Penrose's generalized inverse · Full-rank factorization · Singular Value Decomposition

INTRODUCTION

For an arbitrary matrix A_{nxm}, its Moore-Penrose pseudoinverse A^+ [1-4] verifies the properties [2, 4-9]:

$$AA^+A = A, \quad A^+AA^+ = A^+ \quad (AA^+)^T = AA^+, \quad (A^+A)^T = A^+A \tag{1}$$

and is given by [2, 4, 10, 11]:

$$A^+{}_{mxn} = V_{mxp}\Lambda^{-1}{}_{pxp}U^T{}_{pxn}, \tag{2}$$

in terms of the matrices U,Λ and V generated by the corresponding Singular Value Decomposition (SVD) of A [10, 12-17], such that $p = rank\ A$ and:

$$A = U\Lambda V^T, \quad U^TU = V^TV = I_{pxp}, \quad \Lambda = Diag\,(\lambda_1,...,\lambda_p), \quad \lambda_j > 0, \quad j = 1,...,p. \tag{3}$$

In [9, 18-21] A^+ is obtained via the Faddeev-Sominsky procedure [22-25] and in [11, 26, 27] the expression (2) is deduced using the full-rank factorization technique suggested by MacDuffee (1959) [6].

If $P_{R(A)}$ is the orthogonal projection matrix onto range space of A, then from the SVD are immediate the relations:

$$P_{R(A)} = UU^T, \quad P_{R(A^T)} = VV^T, \tag{4}$$

and with (2) and (3) is simple to prove the relations:

$$A^+A = P_{R(A^T)}, \quad AA^+ = P_{R(A)}, \quad A^+P_{R(A)} = A^+, \tag{5}$$

which are a strong motivation for the following Zlobeck's theorem [9, 28] when $n = m$:

"If B verifies the conditions $BA = P_{R(A^T)}$, $AB = P_{R(A)}$ and $BP_{R(A)} = B$, then $B = A+$ '', (6)

thus in the general case the Moore-Penrose generalized inverse is characterized by (1), or by (6) for square matrices.

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