

## Two Applications of the Melvin-Swamy's Identity

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**Abstract:** We show that the Melvin-Swamy's identity for the hypergeometric function  ${}_3F_2$  gives the Sheppard and Morales relations.

**Key words:** Hypergeometric function • Morse potential

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### INTRODUCTION

Melvin-Swamy [1] obtained the following identity for the hypergeometric function  ${}_3F_2$ :

$${}_3F_2(\alpha_1, \alpha_2, \alpha_3; \lambda_1, \lambda_2; 1) = \frac{\Gamma(\lambda_1)\Gamma(\lambda_1 - \alpha_1 - \alpha_2)}{\Gamma(\lambda_1 - \alpha_1)\Gamma(\lambda_1 - \alpha_2)} {}_3F_2(\alpha_1, \alpha_2, \lambda_2 - \alpha_3; \alpha_1 + \alpha_2 - \lambda_1 + 1, \lambda_2; 1), \quad (1)$$

where we can employ the values:

$$\alpha_1 = -m, \quad \alpha_2 = 1 - \beta, \quad \alpha_3 = 1 - k + m, \quad \lambda_1 = 1 - n - \beta, \quad \lambda_2 = 2 - k + n - \beta, \quad (2)$$

to deduce the Morales expression [2]:

$${}_3F_2(-m, 1 - \beta, 1 - k + m; 1 - n - \beta, 2 - k + n - \beta; 1) = \frac{n! \Gamma(n - m + \beta)}{(n - m)! \Gamma(n + \beta)} x \quad (3)$$

$$x \cdot {}_3F_2(-m, 1 - \beta, 1 - \beta + n - m; 1 + n - m, 2 - k + n - \beta; 1),$$

which is useful to study the matrix elements for the Morse potential [3].

Now if in (1) we use the values:

$$\alpha_1 = -n, \quad \alpha_2 = \alpha, \quad \alpha_3 = b; \quad \lambda_1 = e, \quad \lambda_2 = d, \quad (4)$$

we obtain the Sheppard's identity [4, 5]:

$${}_3F_2(-n, a, b; e, d; 1) = \frac{\Gamma(e)\Gamma(e + n - a)}{\Gamma(e + n)\Gamma(e - a)} {}_3F_2(-n, a, d - b; a - n - e + 1, d; 1), \quad (5)$$

### REFERENCES

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