Computational and Applied Mathematical Sciences 1 (1): 05-11, 2016
ISSN 2222-1328

DOI: 10.5829/idosi.cams.2016.05.11

Thermal Slip Effect on Heat Transfer Falkner-Skan Boundary Layer Flow over a Moving Wedge

[1]*Muatazz Abdolhadi Bashir* and [1,2]*Mustafa Mamat*

[1]Department of Mathematics, Faculty of Science, University of Benghazi UOB, Benghazi- Libya
[2]Department of Mathematics, Institute of Oceanography and Environment,
Universiti Malaysia Terengganu 21030 K. Terengganu, Malaysia

Abstract: The problem of the heat transfer for Falkner-Skan boundary layer flow past a moving wedge with thermal slip condition is investigated. The partial differential equations that govern the problem are transformed into a set of ordinary differential equations using scaling transformation. The shooting method is used to obtain the ordinary differential equations numerically. It also explores the effect of the influences parameters on velocity and temperature. Comparisons with published results are presented.

Key words: Thermal Slip Condition · Falkner-Skan · Similarity Solution · Scaling transformations

INTRODUCTION

Improving and raising the efficiency of the heat transfer is very important, particularly in areas such as engineering, biochemical process and chemical process due to convection. Local similarity solutions obtained on a study conducted by Ali [1], where he studied the temperature dependence viscosity impact on a laminar mixed convection boundary layer flow, as well as, heat transfer on a continually moving vertical isothermal surface. another study by Jayanthi and Kumari [2] "The effect of variable viscosity on non-Darcy free or mixed convective heat transfer along a vertical surface embedded in a porous medium saturated with a non-Newtonian fluid". As a result of their analysis, both of heat transfer rate and velocity on the surface significantly affected by each of viscosity parameter variable transfer rate, viscosity index, mixed load teacher and a number of Argun, the number of Peclet, Rayleigh number and the temperature parameter variation. A result obtained by Chin *et al*. [3], shows in the case of opposing flow that there exist dual solutions and appearance of boundary separation when examining a combination of forced and free convection of boundary layer flow over a vertical impregnable surface embedded in a porous medium when the fluid viscosity varies inversely as a linear function of the temperature. Hayat *et al*. [4], built a clarified analytical results for temperature profiles and velocity by revealing a solution for the stream of a third-grade fluid in a pipe using homotopy analysis method (HAM). The fluid in this construction considered to be with variable space dependence viscosity and the pipe temperature is taken to be higher than the fluid temperature. Pal and Mondal [5], conducted a study analysing the properties of the heat transfer incompressible Newtonian electrically conducting and heat generating/absorbing fluid with the temperature-dependent viscosity over a non-isothermal wedge as well as the existence of thermal radiation. Furthermore, a direct numerical investigation has been done by Pantokratoras [6] on the MHD boundary layer flow over a heated stretching sheet to obtain a solution of the boundary layer equations with respect to viscosity and Prandtl number variation along the boundary layer. Aziz [7], publish a paper on the possibility of obtaining a similarity solution for the thermal boundary layer over a flat plate in a uniform stream of fluid together with the condition of convective boundary whereas the convective heat transfer and the hot fluid are associated on the lower surface of the plate and is proportional to x-12. Makinde and Aziz [8] offered a numerical approach toward studying the heat and mass transfer from a vertical plate embedded in a porous medium experiencing a first-order chemical reaction and exposed to a transverse magnetic field.

Corresponding Author: Muatazz Abdolhadi Bashir, Department of Mathematics, Faculty of Science, University of Benghazi UOB, Benghazi- Libya.

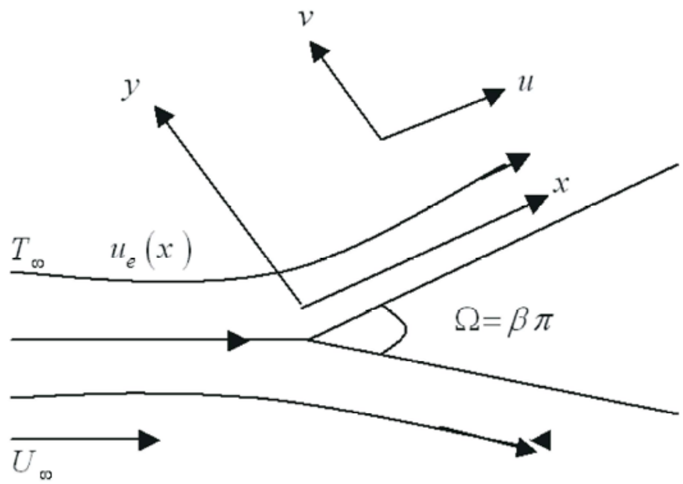

Fig. 1: The Physical model and coordinate system

Lately, some researchers have studied Falkner-Skan equation. Liu and Chang [9], introduced a precise and simple method without utilizing any repetition procedure to estimate unknown initial conditions by using the Lie-group shooting method on the Blasius and Falkner–Skan equations. Alizadeh *et al*. [10], implemented the Adomian Decomposition Method (ADM) to solve the two-dimensional boundary layer Falkner–Skan equation of the laminar flow within a wedge. Afzal [11], conducted a study on the influence of suction and infusion on a tangential movement of a nonlinear power-law stretching surface, which is governed by the laminar boundary layer flow of a viscous and incompressible fluid. Parand *et al*. [12], utilized Hermite functions pseudo-spectral method in order to reach an approximate solution for the third-order nonlinear ordinary differential laminar boundary layer Falkner–Skan equations. Postelnicu and Pop [13], investigated the steady two-dimensional laminar boundary layer flow of a power-law fluid past a permeable stretching wedge beneath a variable free stream.

Going from the foregoing, the aim of this paper is investigating the influence of the thermal slip on the steady two-dimensional heat transfer Falkner-Skan boundary layer flow on a moving wedge, as well as, exploring the effect of thermal slip parameter, power law parameter on the flow field, moving parameter, wall shear stress and wall heat transfer rate.

Mathematical Formulation of the Problem: Figure (1) shows the physical model of a steady two-dimensional heat transfer Falkner-Skan boundary layer flow past a moving wedge, by considering the thermal slip condition effects. It is also assumed that the free stream velocity is $u_e = U_\infty x^m$ [14]. *Xy*-axes are considered to be the coordinates used respectively to measure the surface of the wedge and normal to it. The partial differential equations that govern the problem with the previous condition and the corresponding boundary conditions are given by:

☻ (1)

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y^2}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (3)$$

With respect to the boundary conditions [15],

$$y=0, u_w(x), v=0, T=T_w+D_1(x)\frac{\partial T}{\partial y} \quad (4)$$

$y \to \infty$, $u = u_e(x)$, $T = T_\infty$

where u and v are the velocity components along the xy axes and v is the kinematic viscosity, α is the thermal diffusivity. D_1 is the parameter of thermal slip, T is the temperature and T_∞ is the free stream temperature.

We introduce the following relations for u, ν and θ.

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, u=\frac{\partial\psi}{\partial y}, v \quad (5)$$

where, ψ is the stream function and θ is the dimensionless temperature. Then equations (1) and (3) with the boundary conditions (4) transform as follows,

$$\psi_y \psi_{x,y} - \psi_x \psi_{yy} = u_e \frac{du_e}{dx} + \nu \psi_{yyy} \quad (6)$$

$$\psi_y \theta_x - \psi_x \theta_y = \alpha \frac{\partial^2 \theta}{\partial y^2} \quad (7)$$

With respect to the boundary conditions,

$$y = 0, \psi_y = u_w(x), \psi_x = 0, \theta = 1 + D_1(x)\theta_y \quad (8)$$

$$y \to \infty, \psi_y = u_e(x), \theta = 0 \quad (6)$$

Then, we transform this system into an ordinary system using scaling transformations [16],

$$x^* = \lambda^{sc1} x, y^* = \lambda^{sc}{}_2 y, \psi^* = \lambda^{sc} \psi, \theta^* \quad (9)$$

where c's are constants. This implies that, (6) and (8) will remain invariant under the group of transformations (9) if,

$$C_2 = \frac{1}{2}(1-m)C_1, C_3 = \frac{1}{2}(1+m)C_1, \quad (10)$$

The characteristic equations are,

$$\frac{d_x}{c_1 x} = \frac{dy}{\frac{1}{2}(1-m)c_1 y} = \frac{d\psi}{\frac{1}{2}(1+m)c} \quad (11)$$

Solving the above equations we obtain,

$$\eta = x^{\frac{1-m}{2}} y, \psi = x^{\frac{1+m}{2}} f(\eta), \theta = \theta(\eta) \quad (12)$$

Substituting from (12) into (6) and (7), we get,

$$\nu f''' + \frac{m+1}{2} ff''' - m(f')^2 + mU_\infty^2 = 0 \quad (13)$$

$$\alpha\theta'' + \frac{m+1}{2} f\theta' = 0 \quad (14)$$

Such that, prime is the derivative with respect to η. Then, the boundary conditions become,

$$f)0) = 0, f'(0) = \lambda_1, \theta(0) = 1 + S_1\theta'(0),$$
$$f'(\infty) \to U_\infty, \theta(\infty) = \quad (15)$$

where, $D_1(x) = S_1 x^{(1-m)/2}, u_w(x)$

The following dimensionless variables are introduced:

$$\tilde{\eta} = \sqrt{\frac{U_\infty(m+1)}{2\nu}}\eta, f(\eta) = \sqrt{\frac{2\nu U_\infty}{m+1}} F(\tilde{\eta}) \quad (16)$$

Using (16), equations (13) and (14) become,

$$F''' + FF'' + \frac{2m}{m+1}[1 - (F')^2] = 0 \quad (17)$$

$$\theta'' + \Pr F\theta' = 0 \quad (18)$$

and the corresponding boundary conditions (15) become,

$$F(0) = 0, F'(0) = \lambda, \quad \theta(0) = 1 + S_T\theta'(0) \quad (15)$$
$$f'(\infty) = 1, \theta(\infty) = 0$$

where prime is the derivative with respect to $\tilde{\eta}$, $\lambda = \lambda_1/U_\infty$ and $S_T = S_1\sqrt{\frac{U_\infty(m+1)}{2\nu}}$.

RESULTS AND DISCUSSIONS

The ordinary differential equations (17) and (18) with the boundary conditions (19) have been solved using shooting numerical method. Numerical results are obtained to study the effect of the various values of the parameters Pr, m, S_T and λ on velocity, temperature, shear stress coefficient and heat transfer rate. The numerical findings of the velocity $f(\eta)$ and the temperature $\theta(\eta)$ are shown in Figures 2 to 7, while the numerical results of skin friction coefficient $f`(0)$ and the heat transfer rate at the wall $\theta`(0)$ are given in Tables 2 and 3. We notice from table 2 that the increase of the λ decreases the shear stress, while it increases the heat transfer rate at the wall. Also, one can see that the increase of Falkner–Skan power law parameter m increases the shear stress and the heat transfer rate at the wall. Further, from table 3 we see that the wall heat transfer rate increases when Pr number increases, while it decreases with the increased of the parameter Sr.

Our results have also been compared with those of the corresponding published data by Watanabe [17] and Yih [18] when $\lambda = 0$, as it is shown in Table 1.

The effect of Falkner–Skan power law parameter m on the velocity and the temperature profiles are shown in Figures 2 and 3 respectively. We observe that the velocity increases as m increases while the temperature profiles decreases with the increasing values of m.

Figures 4 and 5 show that the effect of the parameter λ on the velocity and temperature. We see that the velocity increases while the temperature decreases when λ increases.

Table 1: The values of $f''(0)$ for various values of m when $A = 0$ and $N = 0$

m	Wataanbe [17]	Yih [18]	Present results
0	0.46960	0.649600	0.46960007
1/11	0.65498	0.654979	0.65499372
0.2	0.80213	0.802125	0.80212560
1/3	0.92765	0.927653	0.92768004

Table 2: Values of the shear stress coefficient $f''(0)$ and the rate of wall heat transfer $-\theta'(0)$ for different values of m and λ when $Pr = 7$ and $S_T = 0.5$

	$f''(0)$			$-\theta'(0)$		
m	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.1$	$\lambda = 0.5$
0.5	0.97003610	0.80594717	0.61005754	0.76753447	0.84625049	0.90983378
1	1.14656099	0.94681612	0.71329495	0.78369891	0.85578522	0.91517383
2	1.29997295	1.06991893	0.80390718	0.79597263	0.86326133	0.91946542

Table 3: Values of the wall heat transfer $-\theta'(0)$ for different values of Pr and Sr when $\lambda = 0.5$ and $m = 2$

	$-\theta'(0)$		
Pr	$S_T = 0.5$	$S_T = 1$	$S_T = 2$
0.7	0.46165912	0.37507964	0.27276939
7	0.91946542	0.62988615	0.38646021
10	1.00154946	0.66735496	0.40024768
100	1.49557384	0.85569575	0.46111855

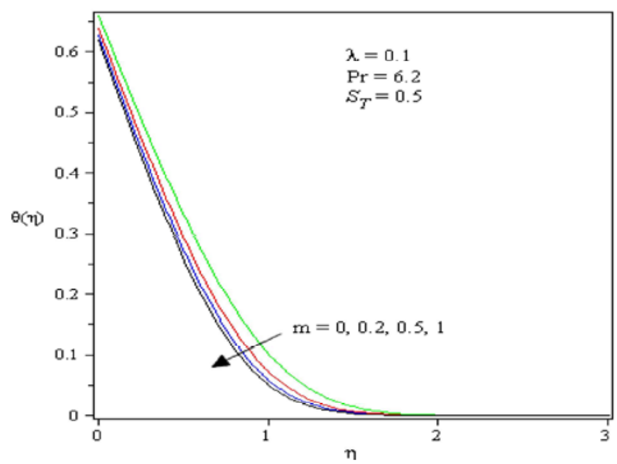

Fig. 2: Effect of m on Velocity

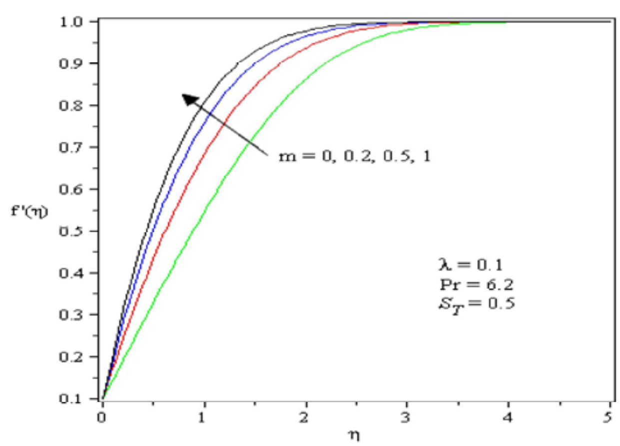

Fig. 3: Effect of *m* on Temperature

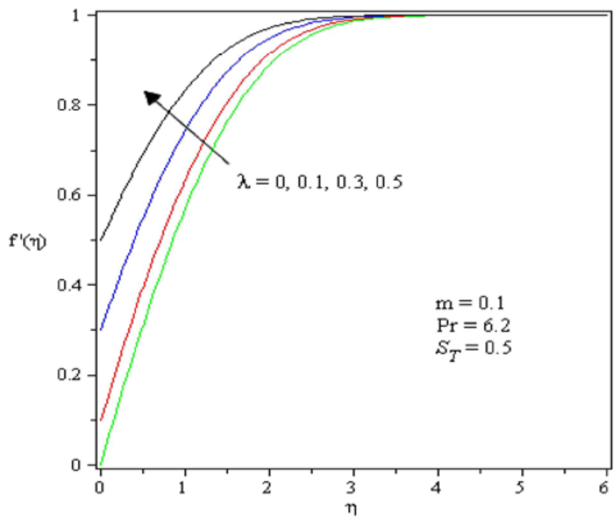

Fig. 3: Effect of λ on Velocity

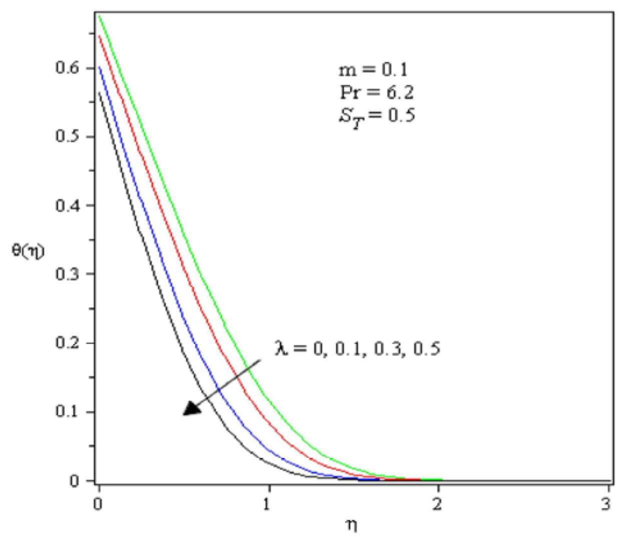

Fig. 4: Effect of λ on Temperature

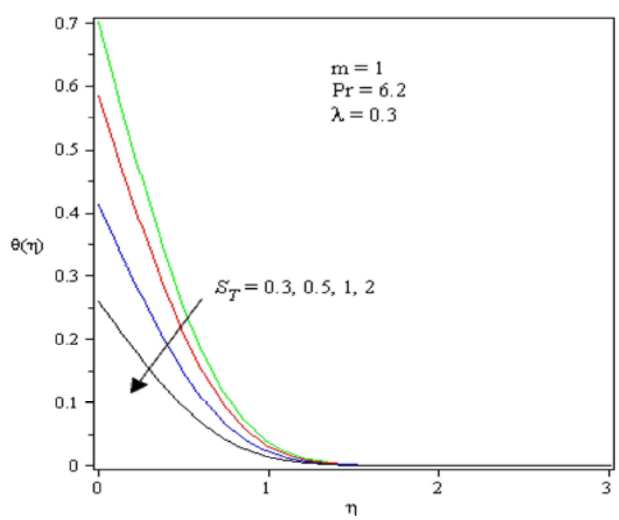

Fig. 5: Effect of S_T on Temperature

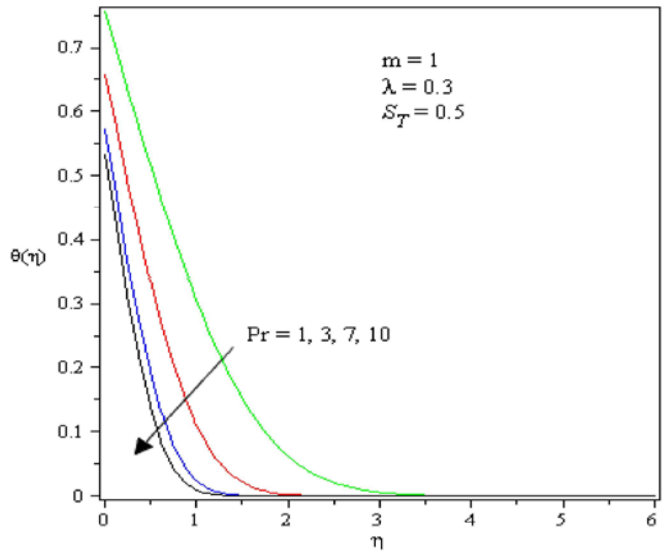

Fig. 6: Effect of *Pr* on Temperature

Figures 6 and 7 depict the effect of the parameters S_T and *Pr* on the temperature profiles, respectively. It is clear from these figures that the temperature decreases with the increase of both of them.

CONCLUSIONS

The problem of the effect of thermal slip on Heat transfer Falkner-Skan boundary layer flow over a moving wedge was investigated. The partial differential equations governing the problem were converted to a non-linear system of ordinary differential equations via scaling transformation method. From the numerical results we conclude that: the velocity, shear stress and the rate of the heat transfer at the wall increase with the Falkner–Skan power law parameter *m*, while the fluid temperature decrease. The increase of the moving parameter λ increases the velocity and the wall heat transfer rate, while it decreases the temperature and shear stress. The fluid temperature decreases with the thermal slip parameter and Prandtl parameter.

REFERENCES

1. Ali, M.E., 2006. The effect of variable viscosity on mixed convection heat transfer along a vertical moving surface, Int. J. Thermal Sci., 45: 60-69.
2. Jayanthi, S. and M. Kumari, 2007. Effect of variable viscosity on non-Darcy free or mixed convection flow on a vertical surface in a non-Newtonian fluid saturated porous medium. Applied Mathematics and Computation, 186: 1643-1659.
3. Chin, K.E., R Nazar, N.M. Arifin and I. Pop, 2007. Effect of variable viscosity on mixed convection boundary layer flow over a vertical surface embedded in a porous medium. Int. Comm. in Heat and Mass Transfer, 34: 464-473.
4. Hayat, T., R. Ellahi and S. Asghar, 2007. The influence of variable viscosity and viscous dissipation on the non-Newtonian flow: An analytical solution. Comm. Nonlinear Sci. Numer. Simul., 12: 300-313.
5. Pal, D. and H. Mondal, 2008. Influence of temperature-dependent viscosity and thermal radiation on MHD forced convection over a non-isothermal wedge, Appl. Math. Comput., 212: 194-208.
6. Pantokratoras, A., 2008. Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity: A numerical reinvestigation. Int. J. of Heat and Mass Transfer, 51: 104-110.
7. Aziz, A., 2009. A similarity solution for laminar thermal boundary layer over flat plate with convective surface boundary condition, Comm. Nonlinear Sci. Numer. Simul., 14: 1064-1068.
8. Makinde, O.D. and A. Aziz, 2010. MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition. Int. J. Thermal Sciences, 49: 1813-1820.
9. Liu, C.S. and J.R. Chang, 2008. The Lie-group shooting method for multiple-solutions of Falkner–Skan equation under suction–injection conditions, International Journal of Non-Linear Mechanics, 43: 844-851.

10. Alizadeh, E., M. Farhadi, K. Sedighi, H.R. Ebrahimi-Kebria and A. Ghafourian, 2009. Solution of the Falkner-Skan equation for wedge by Adomian Decomposition Method, Communications in Nonlinear Science and Numerical Simulation, 14: 724-733.
11. Afzal, N., 2010. Falkner–Skan equation for flow past a stretching surface with suction or blowing, Analytical solutions, Applied Mathematics and Computation, 217: 2724-2736.
12. Parand, K., A.R. Rezaei and S.M. Ghaderi, 2011. An approximate solution of the MHD Falkner–Skan flow by Hermite functions pseudospectral method, Commun Nonlinear Sci Numer Simulat, 16: 274-283.
13. Postelnicu, A. and I. Pop, 2011. Falkner–Skan boundary layer flow of a power-law fluid past a stretching wedge, Applied Mathematics and Computation, 217: 4359-4368.
14. Yacob, N.A., A. Ishak and I. Pop, 2011. Falkner- Skan problem for a static or moving wedge in nanofluids, International Journal of Thermal Sciences, 50: 133-139.
15. Hayat, T., M. Khan and M. Ayub, 2007. The effect of the slip condition on flows of an Oldroyd 6-constant fluid, Journal of Computational and Applied Mathematics, 202: 402-413.
16. Mukhopadhyay, S., G.C. Layek and S.A. Samad, 2005. Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity. International Journal of Heat Mass Transfer, 48(2): 4460-4466.
17. Watanabe, T., 1990. Thermal boundary layers over a wedge with uniform suction or injection in forced flow, Acta Mech., 83: 119-126.
18. Yih, K.A., 1998. Uniform suction/blowing effect on forced convection about a wedge: uniform heat flux, Acta Mech., 128: 173-181.