

Application of Project Scheduling in Agriculture (Case Study: Grape Garden Stabilization)

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Abstract: Some activities of project are critical in the sense that delay in their commencement will delay the overall project completion time. Therefore, management and scheduling of projects is inevitable. In this paper, project scheduling in agriculture, for establishing 300 hectares grape garden in Agricultural Research Center of University of Zabol, is carried out by CPM (Critical Path Method) and PERT (Program Evaluation Review Technique) methods. Results show that the minimum completion time of this project, based on using Normal time and PERT method is 390 days and 364.67 days, respectively. Also the results obtained from employing CPM method indicate that the cost of reducing the project completion time to 365 days is 23643530 Rials.

Key words: Project scheduling • CPM • PERT • Agriculture

INTRODUCTION

Project scheduling and control refers to the planning, scheduling and control of projects, which consist of numerous activities. In the other words, Scheduling issue is a frequent task in the control of various systems such as manufacturing processes [1], project management [2] and service system control (reservation systems, timetabling). Examples can be found in diverse areas such as shipbuilding, road construction, oil refinery maintenance, missile launching and auditing. The management of such projects is problematic, because certain activities must be completed in a prescribed order or sequence. This means that some of these activities are critical so that they delay in their commencement will delay the overall project completion time. In addition, because the basic input parameters for planning (time, cost and resources for each activity) are not deterministic and projects are typically characterized by:

- Uniqueness (no similar experience)
- Variability (trade-off between performance measures like time, cost and quality)
- Ambiguity (lack of clarity, lack of data, lack of structure and bias in estimates) Project planning and scheduling is inevitably involves uncertainty.

Many different techniques and tools have been developed to support better project planning and these

tools are used seriously by a large majority of project managers [3, 4]. In the case of simple projects we can use diagrams to identify critical activities and calculate the minimum time required for their completion and in the case of more complex projects, a simple algorithm is used to identify critical activities and the minimum project completion time. Two techniques associated with the algorithm are the CPM (Critical Path Method) (for developing strategies to complete a project in less than what would normally be regarded as the minimum time) and the PERT (Project Evaluation Review Technique) (for determining probabilities associated with completion times when the activity durations are unknown).

Therefore, the purpose of this paper is studying the application of project scheduling in agriculture, for stabilizing 300 hectares grape garden in Agricultural Research Center of Zabol University, by CPM and PERT methods with WinQsb software.

CRITICAL PATH METHOD (CPM)

During the late 1950's, Engineers from the DuPont Company developed a scheduling technique called CRITICAL PATH PLANNING and SCHEDULING "CPPS" to assist them in minimizing the downtime of industrial facilities during renovations. In 1963, the Associated General Contractor of America "AGC" began a series of conventions devoted to the use of CPM, which

were followed by publications. The intent was to establish CPM methodology for the contracting community. With the acceptance of CPM scheduling by several Federal Government Agencies, including General Services Agency and the Corp of Engineers, the CPM's place in the contracting industry was finally established.

When project managers are required to complete a project in a time that is shorter than the length of the critical path, they can use the CPM. CPM [5] is a deterministic technique, which, by use of a network of dependencies between tasks and given deterministic values for task durations, calculates the earliest network (the 'critical path'), which is the earliest time for project completion. Completing a project in a shorter period of time can be achieved by crashing one or more critical activities by devoting additional resources to these activities so that they are completed in less than normal time. Different costs are typically incurred in crashing different activities, so the problem arises as to which activities to crash and by how much. Also, the activity costs are a function of the activity durations, which are bounded from below (crash duration) and from above (normal duration). Solution procedures for the linear case are proposed by Kelley and Walker [6], Fulkerson [7], Kelley [8], Ford and Fulkerson [9], Siemans [10], Goyal [11], Maghraby and Salem [12].

The two types of cost associated with an activity are the normal-time cost and the crash time cost. The CPM technique for crashing project activities while minimizing cost is as follows: [5]

- Identify the activity on the critical path with the minimum crash-time cost per unit of time. If there is more than one critical path, identify an activity on each path (perhaps a shared activity) such that the total of the crash-time costs per unit of time is a minimum.
- Crash the activity(s) identified in step 1 to the point where another path in the network becomes critical or the activity(s) have been crashed to their the lowest possible values (to avoid over-crashing, it is often circumspect to crash by increments of one time unit).
- Revise the network and determine the critical path, using normal activity times for non-crashed activities and crash times for crashed activities.
- Repeat steps 1 to 3 until all critical activities have been crashed.

PROGRAM EVALUATION REVIEW TECHNIQUE (PERT)

Ever since PERT method was first used in the 1950s to estimate the duration of tasks in a ballistic missile program, this technique has been satisfactorily used by experts both for the theoretical treatment of the duration of tasks within a project and for the cash flow within an investment, as well as for other types of problem. However, during the 1960s and 1970s, various authors proposed a revision of the hypotheses put forward by the creators of the PERT method: Heally [13], Clark [14], Grubbs [15], MacCrimmon and Ryavec [16], Pulido et al. [17], Vazsonyi [18], among others. In the 1980s, Sasieni [19] reopened the debate, questioning the reasoning behind the classical values of the PERT parameters. Immediate replies came from Gallagher [20], Littlefield and Randolpli [21]. Since then, numerous authors have participated in the same debate expressing similar doubts Farnum and Stanton [23], Berny [23], Herrerias [24], Troutt [25], Chae and Kim [26], Moitra [27], Keefer and Verdini [28] and many more.

The best-known technique to support project scheduling is the Critical Path Method (CPM). This technique, which is incorporated into the most widely used project management software tools, is purely deterministic. It makes no attempt to handle or quantify uncertainty. However, another well-used technique Program Evaluation and Review Technique (PERT) [5, 29] does incorporate uncertainty in a restricted sense, by using a probability distribution for each task. Instead of having a single deterministic value, three different estimates (pessimistic, optimistic and most likely) are approximated. Then the 'critical path' and the start and finish date are calculated by use of distributions' means and applying probability rules. Results in PERT are more realistic than CPM but PERT does not address explicitly any of the sources of uncertainty listed above.

The foregoing has assumed that the activity times are known with certainty. If they are not known with certainty they should be treated as random variables. In the Program Evaluation Review Technique (PERT) it is assumed that each activity time is a random variable, which follows a Beta distribution¹. Under this assumption it can be shown that the means and variances of the activity times can be respectively approximated by:

¹Beta distribution: (WinQSB 1.0's help)

$f(x)=P(x/b,v-1)P(1-x/b,w-1)/b/B(v,w)$, for $0 < x < b$,

Where $f(x)$ is Probability density function (pdf), b is the scale parameter, v and w are the shape parameters and $b>0, v>0, w>0$. $\mu = bv / (v+w)$, $\sigma^2 = b^2 v w / (v+w)^2 / (v+w+1)$.

$$E\{t_i\} = \frac{a+4m+b}{6}$$

and

$$Var\{t_i\} = \frac{(b-a)^2}{36}$$

Where:

- a_i = is the optimistic time and is an estimate of the shortest possible time in which activity i can be accomplished.
- m_i = is the most probable time and is an estimate of the time which would occur most often if activity i were repeated a large number of times.
- b_i = is the pessimistic time and is an estimate of the longest time which activity i could take if "everything went wrong".

In practice, for determining such things as the critical path, the mean activity time $E\{t_i\}$ is substituted for the unknown activity time t_i in our algorithms and T denote the time required to complete a group G of activities. If the activity times are statistically independent then the mean and variance of T can be approximated by:

$$E\{T\} = \sum_{i \in G} E\{t_i\}; \text{ (i.e. sum of } E\{t_i\} \text{ over all activities}$$

in group G) and $Var\{T\} = Var\{t_i\}$; (i.e. sum of $Var\{t_i\}$ over activities in group G)

If there are enough activities in the group (i.e. ≥ 30), then the Central Limit Theorem can be used to demonstrate that T is approximately normally distributed. This means that we can make probability statements concerning T . These probability statements are of considerable use in the special case where the activities of interest are on the critical path-they are probability statements concerning the completion of the project in the minimum time.

RESULTS AND DISCUSSION

Minimum Project Completion Time (Using Normal Time): The results of Activity Analysis for Project Scheduling (Minimum Project Completion time, Critical Activities, Earliest and Latest Start time, Earliest and Latest Finish time and Slack time) of 300 hectares grape garden stabilization in Agricultural Research Center of Zubol University, by using WinQsb software and based on using Normal time, have been shown in Table 1.

Table 1: Activity Analysis for Project Scheduling of 300 hectares grape garden stabilization

Activity name	On critical path	Activity time	Earliest start	Earliest finish	Latest start	Latest finish	Slack(LS-ES)
1	Yes	36	0	36	0	36	0
2	Yes	10	36	46	36	46	0
3	Yes	7	46	53	46	53	0
4	Yes	33	53	86	53	86	0
5	Yes	147	86	233	86	233	0
6	Yes	10	233	243	233	243	0
7	Yes	147	243	390	243	390	0
8	No	21	243	264	271	292	28
9	No	10	53	63	66	76	13
10	No	14	264	278	333	347	69
11	No	5	278	283	347	352	69
12	No	2	63	65	76	78	13
13	No	148	65	213	242	390	177
14	No	2	65	67	78	80	13
15	No	198	67	265	192	390	125
16	No	4	67	71	80	84	13
17	No	148	71	219	242	390	171
18	No	148	71	219	242	390	171
19	No	2	71	73	84	86	13
20	No	298	73	371	92	390	19
21	No	2	73	75	86	88	13
22	No	98	75	173	292	390	217
23	No	4	75	79	88	92	13
24	No	298	79	377	92	390	13
25	No	298	79	377	92	390	13
26	No	2	79	81	290	292	211
27	No	73	81	154	317	390	236
28	No	60	264	324	292	352	28
29	No	38	324	362	352	390	28

Project Completion Time = 390 days

Number of Critical Path(s) = 32

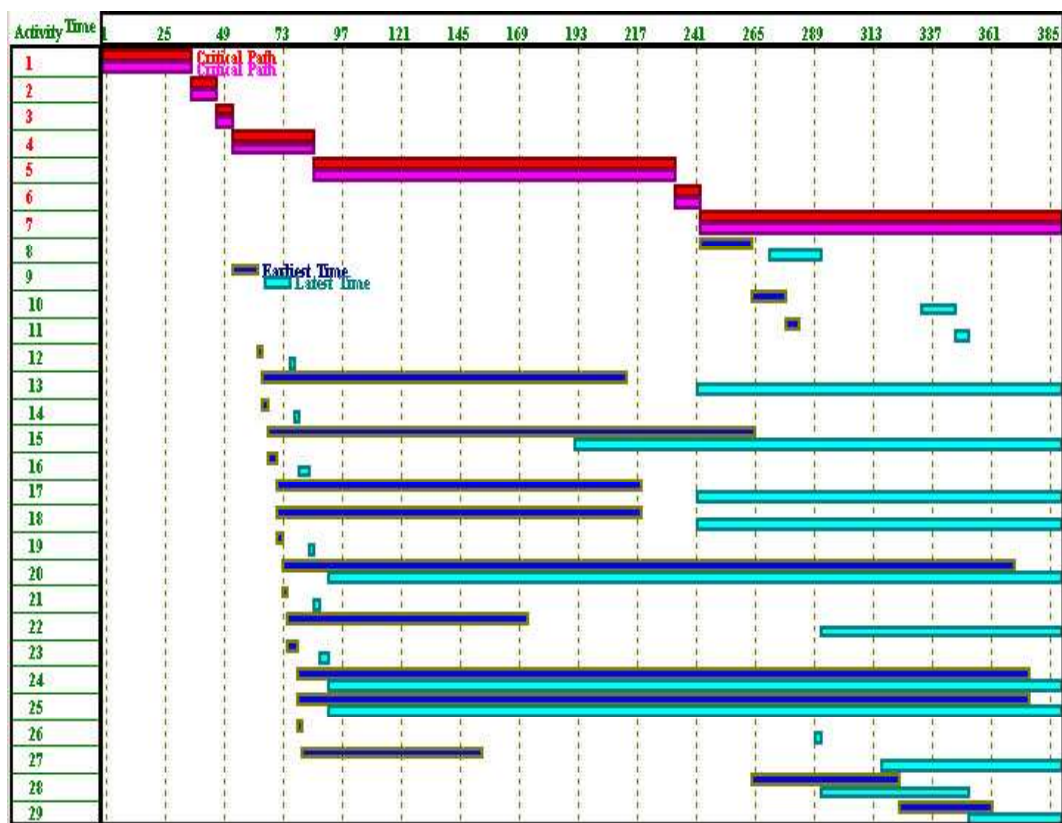


Fig. 1: The Gantt Chart for Project Scheduling of 300 hectares grape garden stabilization

The Table 1 shows that, there are 32 Critical Paths and the activities 1, 2, 3, 4, 5, 6 and 7 are Critical. In this means that, delays in their commencement will delay the overall project completion time. In addition, the Table 1 shows that the minimum project completion time is 390 days.

Gantt Chart: Gantt chart was originated by Gantt [30]. It is a graphical tool to show the start and finish times of each activity. Therefore, Managers find Gantt charts useful because they can simply glance at the chart on a given date to see if the project is being completed on schedule.

In a Gantt chart, time is measured on the horizontal axis, each activity is listed on the vertical axis and a horizontal bar is drawn to depict activity start, finish and duration times. A Gantt chart for Project Scheduling of stabilizing 300 hectares grape garden in agricultural research center of Zabol University by using of WinQsb software is presented in Fig. 1.

In this Gantt chart, for non-critical activities, earliest start and finish times are depicted using the blue bars, while latest start and finish times are depicted using green

bars and for Critical Activities, earliest start and finish times are depicted using the red bars, while latest start and finish times are depicted using pink bars.

Similarly, the Gantt chart shows that the activities 1, 2, 3, 4, 5, 6 and 7 are Critical Activities. Because their Earliest and Latest Start time and Earliest and Latest Finish time are coincident. On the other word, their Slack time is zero.

CPM Analysis: For the project with deterministic activity times, crashing analysis is a process of reducing the activity time to meet the desired completion time. In PERT-CPM, we solve the LP model to obtain the crashing solution. The results of Crashing Analysis for Project Scheduling (new Critical activities, Normal time, Crash time, Suggested time, Additional cost, Suggested cost and total cost of reducing the project completion time to 365 days) by using of WinQsb software, have been shown in Table 2.

The Table 2 shows that, the cost of reducing the project completion time to 360 days is 23643530 Rials. And the activities 1, 2, 3, 4, 5, 6, 7, 9, 12, 14, 16, 19, 21, 23, 24 and 25 become the Critical Activity.

Table 2: Crashing Analysis for Project Scheduling of 300 hectares grape garden stabilization

Activity Name	Critical path	Normal time	Crash time	Suggested time	Additional cost	Normal cost	Suggested cost
1	Yes	36	19	32	1,082,353	0	1,082,353
2	Yes	10	6	6	400,000	0	400,000
3	Yes	7	4	7	0	0	0
4	Yes	33	17	33	0	0	0
5	Yes	147	74	147	0	0	0
6	Yes	10	6	10	0	0	0
7	Yes	147	73	130	204,000	0	204,000
8	No	21	10	21	0	0	0
9	Yes	10	5	10	0	0	0
10	No	14	8	14	0	0	0
11	No	5	3	5	0	0	0
12	Yes	2	1	1	39,000	0	39,000
13	No	148	74	148	0	0	0
14	Yes	2	1	2	0	0	0
15	No	198	99	198	0	0	0
16	Yes	4	2	2	439,000	0	439,000
17	No	148	74	148	0	0	0
18	No	148	74	148	0	0	0
19	Yes	2	1	2	0	0	0
20	No	298	149	298	0	0	0
21	Yes	2	1	1	200,000	0	200,000
22	No	98	49	98	0	0	0
23	Yes	4	2	4	0	0	0
24	Yes	298	149	298	0	0	0
25	Yes	298	149	298	0	0	0
26	No	2	1	2	0	0	0
27	No	73	36	73	0	0	0
28	No	60	60	60	0	0	0
29	No	38	19	38	0	0	0
Overall Project:				365	2,364,353	0	2,364,353

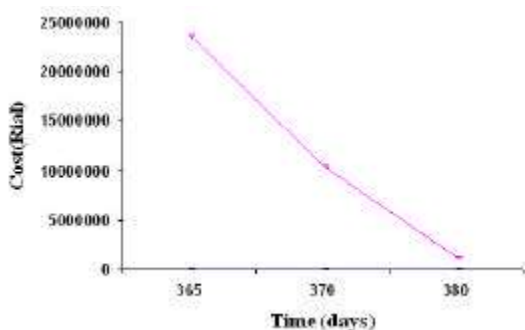


Fig. 2: Cost-time trade-offs in crashing 300 hectares grape garden stabilization

The cost-time trade-offs are depicted in Fig. 2, which can be used to determine the cost of reducing the minimum project completion time to any value between

365 and 380 days. For example, the cost of reducing the project completion time to 370 days is 10505000 Rials.

Program Evaluation and Review Technique (PERT): The foregoing has assumed that the activity times are known with certainty. If they are not known with certainty they should be treated as random variables. This is a similar process of CPM method. However, it is used for managing projects with probabilistic activity times. The whole procedure includes determining critical path and computing the expected project completion time. It also involves the probability analysis. The results of Activity Analysis based on PERT (Critical Activities, Activity mean time, Earliest and Latest Start time, Earliest and Latest Finish time and Slack time) by using of WinQsb software, have been shown in Table 3.

Table 3: Activity Analysis for Project Scheduling of 300 hectares grape garden stabilization

Activity Name	On Critical Path	Activity Mean Time	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Slack (LS-ES)	Standard Deviation
1	Yes	36.5	0	36.5	0	36.5	0	1.8333
2	Yes	10.1667	36.5	46.6667	36.5	46.6667	0	1.1667
3	Yes	6.3333	46.6667	53	46.6667	53	0	0.6667
4	No	31.3333	53	84.3333	68.3333	99.6667	15.3333	1.6667
5	No	137.5	84.3333	221.8333	99.6667	237.1667	15.3333	5.8333
6	No	9.5	221.8333	231.3333	237.1667	246.6667	15.3333	0.8333
7	No	111.6667	231.3333	343	253	364.6667	21.6667	5
8	No	20.8333	231.3333	252.1667	246.6667	267.5	15.3333	1.5
9	Yes	9.8333	53	62.8333	53	62.8333	0	0.8333
10	No	14.6667	252.1667	266.8333	307.6667	322.3333	55.5000	1
11	No	5.1667	266.8333	272	322.3333	327.5	55.5	0.5
12	Yes	2.1667	62.8333	65	62.8333	65	0	0.1667
13	No	138.3333	65	203.3333	226.3333	364.6667	161.3333	5
14	Yes	2.1667	65	67.1667	65	67.1667	0	0.1667
15	No	181.6667	67.1667	248.8333	183.0000	364.6667	115.8333	8.3333
16	Yes	4.3333	67.1667	71.5	67.1667	71.5	0	0.3333
17	No	142.5	71.5	214	222.1667	364.6667	150.6667	5.8333
18	No	134.1667	71.5	205.6667	230.5	364.6667	159.0000	5.8333
19	Yes	2.1667	71.5	73.6667	71.5	73.6667	0	0.1667
20	No	26.6667	73.6667	100.3333	338	364.6667	264.3333	1.3333
21	Yes	2.1667	73.6667	75.8333	73.6667	75.8333	0	0.1667
22	No	95.1667	75.8333	171	269.5	364.6667	193.6667	2.5
23	Yes	2.1667	75.8333	78	75.8333	78	0	0.1667
24	No	283.3333	78	361.3333	81.3333	364.6667	3.3333	6.6667
25	Yes	286.6667	78	364.6667	78	364.6667	0	5
26	No	2.1667	78	80.1667	265.3333	267.5	187.3333	0.1667
27	No	68.1667	80.1667	148.3333	296.5	364.6667	216.3333	1.8333
28	No	60	252.1667	312.1667	267.5	327.5	15.3333	0
29	No	37.1667	312.1667	349.3333	327.5	364.6667	15.3333	0.8333

Project Completion time = 364.67 days

Number of Critical Path(s) = 384

Table 4: Probability Analysis for Project Scheduling of 300 hectares grape garden stabilization

Critical Path	Completion Time Std. Dev	Completion Probability in 380 days
1 --> 19 --> 21 --> 23 --> 25	5.3333	0.9980
1 --> 16 --> 21 --> 23 --> 25	5.3411	0.9979
1 --> 9 --> 12 --> 23 --> 25	5.3955	0.9978
1 --> 9 --> 12 --> 21 --> 23 --> 25	5.3980	0.9977
1 --> 3 --> 9 --> 19 --> 21 --> 23 --> 25	5.4391	0.9976
Critical Path	Completion Time Std. Dev	Completion Probability in 370 days
1 --> 12 --> 19 --> 21 --> 25	5.3333	0.8414
1 --> 12 --> 19 --> 21 --> 23 --> 25	5.3359	0.8412
1 --> 12 --> 16 --> 23 --> 25	5.3411	0.8410
1 --> 12 --> 14 --> 16 --> 19 --> 25	5.3437	0.8409
1 --> 14 --> 16 --> 19 --> 21 --> 23 --> 25	5.3463	0.8408
Critical Path	Completion Time Std. Dev	Completion Probability in 365 days
1 --> 14 --> 16 --> 19 --> 21 --> 23 --> 25	5.3463	0.5249
1 --> 3 --> 19 --> 25	5.3697	0.5248
1 --> 3 --> 19 --> 21 --> 23 --> 25	5.3748	0.5247
1 --> 9 --> 12 --> 14 --> 16 --> 19 --> 21 --> 23 --> 25	5.4135	0.5246
1 --> 3 --> 9 --> 19 --> 21 --> 25	5.4365	0.5245

The Table 3 shows that, there is 384 Critical Paths and the activities 1, 2, 3, 9, 12, 14, 16, 19, 21, 23 and 25 are Critical Activities. In addition the Table 3 shows that, the minimum probabilistic project completion time is 364.67 days. Also, the probability of project completion time in 365, 370 and 380 days according to different Critical Paths, has been shown in Table 4.

The above Table 4 shows that, the best Critical Paths for completion this project in 380, 370 and 365 days are (1-->19-->21-->23-->25), (1-->12-->19-->21-->25) and (1-->14-->16-->19-->21-->23-->25), respectively. Also, the Table 4 shows that, the maximum probability of completion this project in 380, 370 and 365 days, is 99.80%, 84.14% and 52.49%, respectively.

CONCLUSIONS

Project scheduling and control refers to the planning, scheduling and control of projects, which consist of numerous activities. In addition, some activities of project are critical in the sense that delays in their commencement will delay the overall project completion time. Therefore management and scheduling of projects is inevitably. In this paper, the application of project scheduling in agriculture, for stabilizing 300 hectares grape garden in Agricultural Research Center of Zabol University, was studied by CPM and PERT methods with WinQsb software.

The main results are as follow:

- Minimum completion time of this project, base on using Normal time and PERT method was 390 days and 364.67 days, respectively.
- Results of CPM method showed that the cost of reducing the project completion time of this project, to 365 days is 23643530 Rials.
- Results of PERT method showed that the maximum probability completion of this project during 380, 370 and 365 days, is 99.80%, 84.14% and 52.49%, respectively.

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Appendix 1: The Activities list of Project Scheduling of 300 hectares Grape Garden Stabilization in Agricultural Research Center of Zabol University

Activity	Predecessor	Normal time	Crash time	Crash cost	Optimistic time	Most likely time	Pessimistic time
1 Land Preparation, Soil Sampling and Analysis	...	36	19	4600000	34	35	45
2 Location Identification and Land Map Preparation	1	10	6	400000	9	9	16
3 Determination of Irrigation System Type	1, 2	7	4	3000000	5	6	9
4 Land Digging and Initial Fixation of Pipes and Links	1, 2, 3	33	17	450000	29	30	39
5 Continue to Fix Pipes and Links	1, 2, 3, 4	147	74	10950000	125	135	160
6 Main Pipe Burying and Primary Welding of Secondary Pipes	1, 2, 3, 4, 5	10	6	720000	8	9	13
7 Continue to Secondary Pipes Welding	1, 2, 3, 4, 5, 6	147	73	888000	100	110	130
8 Pipes Checkout and Burying	1, 2, 3, 4, 5, 6	21	10	420000	18	20	27
9 Land Surveying and Ripping for Windbreak Fixation	1, 2, 3	10	5	1648000	9	9	14
10 Ditch Fix and Windbreaks Drenching	1, 2, 3, 4, 6, 8, 9	14	8	125000	13	14	19
11 Plantation of Windbreak Seedlings	1, 2, 3, 4, 6, 8, 9, 10	5	3	120000	4	5	7
12 Plantation Lines Primary Survey	1, 2, 3, 9	2	1	39000	2	2	3
13 Continue to Plantation lines Survey	1, 2, 3, 9, 12	148	74	2886000	130	135	160
14 Primary Heavy and Semi Heavy Ripping	1, 2, 3, 9, 12	2	1	320000	2	2	3
15 Continue to Heavy and Semi Heavy Ripping	1, 2, 3, 9, 12, 14	198	99	31680000	160	180	210
16 Primary Location Survey and Digging of Pits	1, 2, 3, 9, 12, 14	4	2	439000	4	4	6
17 Continue to Pits Location Survey	1, 2, 3, 9, 12, 16	148	74	2886000	130	140	165
18 Continue to Pits Digging	1, 2, 3, 9, 12, 14, 16	148	74	29600000	125	130	160
19 Arable soil, Gravel and Fertilizer Primary Transfer to Farmland	1, 2, 3, 9, 12, 16	2	1	600000	2	2	3
20 Continue to Arable soil, Gravel and Fertilizer Transfer to Farmland	1, 2, 3, 9, 12, 16, 19	298	149	89400000	24	26	32
21 Primary Preparation of Mixed Soil	1, 2, 3, 9, 12, 16, 19	2	1	200000	2	2	3
22 Continue to Mixed Soil Preparation	1, 2, 3, 9, 12, 16, 19, 21	98	49	9800000	90	94	105
23 Mixed Soil Primary Transfer to Farmland and Primary Filling of Pits with them	1, 2, 3, 9, 12, 16, 19, 21	4	2	1200000	2	2	3
24 Continue to Mixed Soil Transfer to Farmland	1, 2, 3, 9, 12, 16, 19, 21, 23	298	149	119200000	270	280	310
25 Continue to Pits Filling with Mixed Soil	1, 2, 3, 9, 12, 16, 19, 21, 23	298	149	59600000	275	285	305
26 Primary Setting of Pipes (16inches) and Droppers	1, 2, 3, 9, 12, 16, 19, 21, 23	2	1	48000	2	2	3
27 Continue to Setting of Pipes (16inches) and Droppers	1, 2, 3, 9, 12, 16, 19, 21, 23, 26	73	36	1776000	65	67	76
28 Heavy Irrigation	1, 2, 3, 4, 6, 8, 9, 12, 16, 19, 21, 23, 26	60	60	0	60	60	60
29 Plantation of Grape Seedlings	1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 16, 19, 21, 23, 26, 28	38	19	9120000	35	37	40

APPENDIX 2 (Glossary)

- Activity Time: The time to complete a particular activity. The activity time can be deterministic or uncertain (probabilistic). When the activity time is deterministic, the activity is completed in a constant time. When the activity time is probabilistic, the activity is completed in a random time value that may have a certain probability distribution.
- Critical Path: A path that has the longest total activity time.
- Critical Activity: Any activity on the critical path.
- Deterministic Project: A project that all activities finish in constant times.
- End Activity: The activity that is not a predecessor of any other activity is called an end activity of the project.
- Earliest finish (EF): this is the earliest possible time that an activity can be finished (= earliest start time + activity completion time).
- Earliest start (ES): this is the earliest possible time that an activity can begin. All immediate predecessors must be finished before an activity can start.
- Immediate Predecessor: The immediate predecessors of an activity are the activities that must immediately precede the activity.
- Latest finish (LF): this is the latest time that an activity can be finished and not delay the completion time of the overall project (= latest start time + activity completion time). As with start times, the activity is critical if the earliest finish and latest finish times are the same.
- Latest start (LS): this is the latest time that an activity can begin and not delay the completion time of the overall project. If the earliest start and latest start times are the same then the activity is critical.
- Path: A sequence of activities in a project leading from the start activity to the completion activity of the project.
- Precedence Relation: It tells which activity must be completed before another can be started.
- Probabilistic Project: A project that has activities finish in uncertain times.
- Slack time: this is the difference between the earliest start time and the latest start time (which in turn is equal to the difference between the latest start time and the latest finish time), i.e. $Slack = LS - ES = LF - EF$
- Start Activity: The activity that has no immediate predecessor is called a start activity of the project.

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