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Estimation and Validation of Solar Radiation Incident on Horizontal and Tilted Surface at Bhopal, Madhya Pradesh, India

[1]K.N. Shukla, [2]K. Sudhakar and [3]S. Rangnekar

[1]Electrical &Electronics, Lakshmi Narain College of Technology, Bhopal-462021, India
[2,3]Department of Energy, Maulana Azad National Institute of Technology Bhopal-462051, India

Abstract: In this study a standard procedure is adopted for estimation of solar radiation on horizontal and tilted surface at Bhopal climatic conditions, located in central region of India. This estimation is based on the days on which extra-terrestrial radiation is equal to monthly mean value, meteorological parameters like cloud cover, clearness index and precipitation. Estimated solar radiation data compare with ground measured data from Indian meteorological department (IMD).The statistical error tests-Mean Absolute Percentage Error (MAPE), Mean Bias Error (MBE), Root Mean Square Error (RMSE) and t-stat are used for comparison. The results show that for global radiation, the highest values of MAPE, MBE, RMSE and t-stat are -9.57%, 0.5kWh/m^2-day, 1.61 kWh/m^2-day and 1.08 respectively, while in case of tilted global radiation these errors are -11.93%, 0.66 kWh/m^2-day, 1.66 kWh/m^2-day and 1.40.

Key words: Solar radiation · Tilted Global radiation · Validation · Statistical errors · Clearness index

INTRODUCTION

Out of all renewable energy resources, solar energy is one of the most feasible alternative and sustainable energy resources in the world. It is omnipresent, safe, abundant, freely available and environment friendly [1, 2] A drawback, common to the solar energy systems is their unpredictable nature and their output cannot be accurately predicted, because, these systems are dependent on weather and climatic conditions [3, 4]. Research and development efforts are required to improve the performance of solar energy systems and to establish new techniques for accurate prediction of their output from available environmental and climatic conditions solar radiation data are the best source of information for estimating average incident radiation necessary for proper design and the assessment of solar energy conversion systems [5]. The availability of more comprehensive solar radiation data is invaluable for the design and evaluation of solar-based conversion systems. Particularly, the basic solar radiation data for the surfaces of interests are not readily available in most developing countries [6, 7]. Generally, the meteorological stations measure global and diffuse solar radiation intensities mostly on horizontal surfaces only [8]. Whereas, the stationary solar conversion systems are tilted towards the sun in order to maximize the amount of solar radiation incident on the collector or module surface. But, the availability of required data on tilted surfaces is very rare [9, 10].

Therefore, the tilted surface irradiation in most cases is calculated from measured global horizontal irradiation by means of empirical models. There are several forms of solar radiation data, which could be used for a variety of purposes in the design and development of solar energy systems. Daily data is often available and hourly radiation can be estimated from available daily data. Monthly total solar radiation on a horizontal surface can be used in some process design methods. However, the process performance is generally not linear with solar radiation. The use of averages may lead to serious errors if non-linearities are not taken into account [11]. So that, the measurements of solar radiation on tilted surfaces are important for determination of accurate input to solar photovoltaic (PV) systems or collectors [12].

Numerous researchers [13-16] have predicted the importance of solar radiation data for design and efficient operation of solar energy systems. Proper design and performance of these solar appliances require accurate

Corresponding Author: K.N. Shukla, Electrical &Electronics, Lakshmi Narain College of Technology, Bhopal-462021, India.

information on solar radiation availability [17]. Hence many studies have been made on solar climates of specific regions for the development of solar technologies [18].

In this study estimated, global solar radiation and tilted global solar radiation at Bhopal climatic condition compare with the ground measured data from Indian metrological department (IMD) to build a comprehensive solar radiation data base.

MATERIALS AND METHODS

Study Location: The geographical location of the Bhopal City lies within North Latitude 23°16' and East Longitude 77°36'. The location of Bhopal falls in the north western portion of Madhya Pradesh. It seen in the Map of India, Bhopal occupies the central most region of the country. The total area of the city of Bhopal is 697 sq km and the total number of inhabitants roughly equals 30,00,000. The climate of Bhopal is subtropical, with hot and humid summer and a cool but dry winter. The average temperature during the day is around 30°, whereas in the month of May, it rises to 40°. Humidity always remains high during this time and hence the atmosphere remains sweaty. Monsoons usually start from June and last till September end. The total rainfall of the city does not exceed 1200 mm, accompanied by frequent thunderstorms and occasional floods. In brief about the Bhopal district:

- On average, the temperatures are always high.
- Most rainfall is seen in June, July, August and September.
- It has dry periods in January, February, March April and December.
- The warmest month is May.
- The coolest month is December.
- The wettest month is August.

Solar Radiation on Horizontal Surface:

Estimation of Monthly Average Daily Extraterrestrial Solar Radiation ($\overline{H}_o$): The monthly mean daily extraterrestrial solar radiation ($\overline{H}_o$) on the horizontal surface is computed by taking the values of single day (closed to monthly mean values) for every month of the year by using days suggested by Klein [19] which are representing the individual month. The proposed days were 17th of January and July, 16th of February, March and August, 15th of April, May, September and October, 14th of November, 11th of June and 10th of December [2, 18, 35]. The monthly average daily extraterrestrial solar radiation ($\overline{H}_o$) on the horizontal surface is determined by the following empirical relationship:

Table 1: Days on which extraterrestrial radiation is equal to monthly mean value [19].

Month	Day	Day of the year (N)
Jan.	17	17
Feb.	16	47
Mar.	16	75
Apri.	15	105
May	15	135
Jun.	11	162
Jul.	17	198
Aug.	16	228
Sept.	15	258
Oct.	15	288
Nov.	14	318
Dec.	10	344

$$\overline{H}_o = \frac{24}{\pi} I_{SC}\left(1+.033\cos\frac{360N}{365}\right)\left(\frac{\pi\omega_S}{180}\sin\varnothing\sin\delta+\cos\varnothing\cos\delta\sin\omega_S\right)\text{kWh/m}^2\text{-day} \quad (1)$$

Where, I_{SC} is solar constant 1.367 kW/m^2, N is day of the year (N=1 for 1st January and N=365 for 31st December), ω_s is sunshine hour angle for the mean day of the month (degrees), δ is latitude angle (degrees) and δ is declination angle (degrees).

The declination angle (δ) can be mathematically presented by the equation [20]:

$$\delta = 23.34\ \frac{360}{365}\ (284+N) \quad (2)$$

Where, N is the Nth day of the year starting from January as shown in Table 1.

The sunshine hour angle (ω_s) for a location is a function of solar declination angle and the latitude [21] is given by:

$$\omega_s = \cos^{-1}(-\tan\delta\ \tan\varnothing) \quad (3)$$

Estimation of Monthly Average Daily Global Solar Radiation ($\overline{H}_g$): There are many methods, which have been devised to predict the amount of solar radiation reaching the Earth's surfaces at a given location. The method used in this paper is the one developed by Angstrom [22].The monthly average daily global solar radiation on a horizontal surface $\overline{H}_g$ is given by:

$$\frac{\overline{H}_g}{\overline{H}_o} = a + b\left(\frac{S}{S_{max}}\right) \quad (4)$$

Where, S is monthly average daily hours of bright sunshine, $^S max$ is monthly average of the maximum possible daily hours of bright sun shine,which is given by the equation

$$^S max = \left(\frac{2}{15}\right)\omega_S \quad (5)$$

a,b are constants known as angstrom constants and they are empirical and obtained by the curve fitting data:

$$\left.\begin{array}{l} a = 0.409 + 0.5016\sin(\omega_s - 60) \\ b = 0.6609 + 0.4767\sin(\omega_s - 60) \end{array}\right\} \quad (6)$$

The value of constants a and b are given [23] for many cities of the world and [24, 34] for many Indian cities. (Bhopal a =0.26, b = 0.5)

Estimation of Monthly Average Daily Diffuse Radiation ($\overline{H}_d$): Based on a study of data for a few countries [25] the daily diffuse-to-global radiation ratio could be correlated against the daily global-to-extraterrestrial radiation ratio. The correlation was expressed by the following cubic equation

$$\frac{\overline{H}_d}{\overline{H}_g} = 1.390 - 4.027\left(\frac{\overline{H}_g}{\overline{H}_o}\right) + 5.531\left(\frac{\overline{H}_g}{\overline{H}_o}\right)^2 - 3.108\left(\frac{\overline{H}_g}{\overline{H}_o}\right)^3 \quad (7)$$

Where, $\overline{H}_d$ is Monthly average of the daily diffuse radiation on a horizontal surface (kWh/m^2-day) the other symbols have the same meaning as given earlier. The ratio $\left(\frac{\overline{H}g}{\overline{H}o}\right)$ is often denoted by the symbol K_T and is called the monthly average clearness index. As in the case of monthly average daily global radiation, many investigators have developed empirical equations for estimating the diffuse-to-global radiation ratio for various parts of the world. Gopinathan and Soler [26] examined radiation data for 40 widely spread locations all over the world in the latitude range 36° S to 36° N. They have proposed the following equation involving the clearness index and the sunshine ratio:

$$\frac{\overline{H}_d}{\overline{H}_g} = 0.87813 - 0.33280\,K_T - 0.53039\left(\frac{S}{S_{max}}\right) \quad (8)$$

Equation (8) is based on more recent data then that available [25] and is recommended for use for predicting the daily diffuse radiation at location across the world.

When available Indian data was analyzed, the following linear equation was obtained that is [24]

$$\frac{\overline{H}_d}{\overline{H}_g} = 1.411 - 1.696\left(\frac{\overline{H}_g}{\overline{H}_o}\right) \quad (9)$$

After examination of solar radiation data for 11 Indian cities and proposed the equation [27]:

$$\frac{\overline{H}_d}{\overline{H}_g} = 0.8677 - 0.7365\left(\frac{S}{S_{max}}\right) \quad (10)$$

Equation (9) and Equation (10) agree well with each other. Either one of the equation may be used for Indian locations

Solar Radiation on the Tilted Surface

Estimation of Monthly Average Daily Incident Solar Radiation ($\overline{H}_T$): The incident solar radiation on a tilted surface is the sum of the set of radiation streams including direct or beam radiation, the three components of diffused radiation from the sky and the radiation reflected from the various surfaces seen by the tilted surface. The total incident solar radiation on the tilted surface ($\overline{H}_T$) can be written as in the following forms:

$$\overline{H}_T = \overline{H}_{T,b} + \overline{H}_{T,d} + \overline{H}_{T,r} \quad (11)$$

Where, $\overline{H}_T$ is monthly total daily incident solar radiation, $\overline{H}_{T,b}$ is beam radiation, $\overline{H}_{T,d}$ is diffused radiation and $\overline{H}_{T,r}$ is ground reflected radiation on tilted surface.

Beam radiation on tilted surface is given by:

$$\overline{H}_{T,b} = \overline{H}_b\overline{R}_b$$

Where, $\overline{H}_b$ is monthly average daily beam radiation on horizontal surface and $\overline{R}_b$ is the ratio of mean daily beam radiation on the tilted surface to that on horizontal.

Basically $\overline{R}_b$ is a function of transmittance of atmosphere, which is equal to $\left(\frac{\overline{H}_{T,b}}{\overline{H}_b}\right)$ and be determined by the following expression for the surface that are sloped towards the equator in the northern hemisphere or 180° in the southern hemisphere (most favorable azimuth angle $\gamma = 0$, for collector of PV module) [33]. Therefore the value of $\overline{R}_b$ is computed by:

$$\overline{R}_b = \frac{\cos\theta}{\cos\theta z} = \frac{\sin\delta\,\sin(\emptyset-\beta)+\cos\delta\cos(\emptyset-\beta)}{\sin\delta\sin\emptyset+\cos\delta\cos\emptyset\cos\omega} \quad (12)$$

Where, δ is declination angle, $\varnothing$ is Latitude angle, β is inclination of tilted surface and ω is the hour angle. (All is in degrees)

Diffused radiation ($\overline{H}_d$) is that fraction of total solar radiation which is received from the sun when its direction has been changed by atmospheric scattering [28]. The direction of diffused radiation is highly variable and difficult to determine [2]. It is function of condition of cloudiness and atmospheric clearness which are extremely unpredictable. The diffused radiation fraction is also the combination of three components namely isotropic, circumsolar and horizon brightening [2]. The isotropic diffuse radiation component is received evenly from the entire sky dome. The circumsolar diffuse part received from onward dispersion of solar radiation and concentrated in the section of the sky around the sun [29]. The horizon brightening component is concentrated near the horizon and it is most obvious in the clear skies [30]. In general the diffuse fraction of radiation on inclined surface is composed of isotropic, circumsolar and horizon brightening factors as given by:

$$\overline{H}_{T,b} = \overline{H}_{d,iso}\overline{R}_d + \overline{H}_{d,cs}\overline{R}_b + \overline{H}_{d\,hz}\overline{R}_{dz} \quad (13)$$

If the diffuse radiation is considered to be only isotropic. Then it is the ratio of diffuse on the tilted surface with slop (β) to that on the horizontal surface denoted by $\overline{R}_d$ since:

$$\overline{R}_d = \frac{(1+\cos\beta)}{2} \quad (14)$$

and $\overline{H}_d$ is computed from equation (10)

$$\overline{H}_{T,d} = \overline{H}_d\overline{R}_d$$
$$\overline{H}_{T,d} = \overline{H}_d\frac{(1+\cos\beta)}{2} \quad (15)$$

Reflected radiation is the part of total solar radiation that is reflected by the surface of the earth and by any other surface intercepting object such as trees, terrain or buildings on to a surface exposed to the sky is termed as ground reflected radiation or Albedo [2] so reflected radiation on tilted surface is:

$$\overline{H}_{T,r} = \overline{H}_T\overline{R}_r \quad (16)$$

Where, $\overline{H}_r$ is monthly average daily reflected radiation on horizontal surface, $\overline{R}_r$ is the ratio of mean daily reflected radiation on the tilted surface to that on horizontal surface. $\overline{R}_r$ is given by:

$$\overline{R}_r = \rho\frac{(1+\cos\beta)}{2} \quad (17)$$

Where, β is slop of tilted surface, ρ is reflectivity of the surrounding in which the collector is located. Normally a value ρ = 0.2 is taken in a condition that the mean monthly temperature is greater than 0°C and the measuring station is located on a roof top with low reflectance. Its value could be taken as 0.7 if the temperature is less than -5°C [30].

Then total incident radiation on a tilted surface $\overline{H}_T$ rewritten as:

$$\overline{H}_T = \overline{H}_b\overline{R}_b + \overline{H}_d\overline{R}_d + \overline{H}_r\overline{R}_r$$
$$\overline{H}_T = \overline{H}_b\overline{R}_b + \overline{H}_d\left(\frac{1+\cos\beta}{2}\right) + \overline{H}_r?\left(\frac{1-\cos\beta}{2}\right) \quad (18)$$

The ratio of daily radiation falling on tilted surface ($\overline{H}_T$) to the daily global radiation on a horizontal surface is given by [25]:

$$\frac{\overline{H}_T}{\overline{H}_g} = \left(1 - \frac{\overline{H}_d}{\overline{H}_g}\right)\overline{R}_b + \frac{\overline{H}_d}{\overline{H}_g}\overline{R}_d + \overline{R}_r \quad (19)$$

Where,

$\overline{R}_b$ = R_b on the representative day,

$$\overline{R}_d = R_d = \frac{(1+\cos\beta)}{2}$$
$$\overline{R}_r = R_r = \rho\frac{(1-\cos\beta)}{2}$$

Equation (19) is used for calculating the monthly average daily radiation falling on a tilted surface if the values required are calculated for the representative day of the month.

Comparison Methods: In this study, estimated global solar radiation data and tilted global solar radiation data at Bhopal climatic conditions is compared with the data measured by Indian meteorological department. The comparison is done by four statistical tests:

- Mean Absolute Percentage Error (MAPE),
- Mean Bias Error (MBE),
- Root Mean Square Error (RMSE),
- t-statistic

These tests to evaluate the accuracy of the correlations described above.

Mean Absolute Percentage Error (MAPE): Mean7 absolute percentage error is a indicator of accuracy in which usually expresses accuracy as a percentage of the data. It may be expressed as:

$$MAPE = \frac{1}{n}\sum_{i=i}^{n}\left(\frac{H - H_P}{H}\right) \times 100 \quad (20)$$

Where, H is Real Value, HP is Predicted Value and n is the total number of observations. The ideal value of Mean absolute percentage error is zero.

Mean Bias Error (MBE): The mean bias error provides information on the long-term performances of the correlations by allowing a comparison of the actual deviation between calculated and measured values term by term. In other words, it is an indicator for the average deviation of the predicted values from the measured data. Mean bias error is given by:

$$MBE = \frac{1}{n}\sum_{i=1}^{n}(H_{Pi} - H_i) \quad (21)$$

Where, Hi is ith Real Value, Hpi is ith Predicted Value and n is the total number of observations.

The mean bias error provides information on the long term performance. A low MBE is desired. Ideally a zero value of MBE should be obtained. A positive value gives the average amount of over-estimation in the calculated value and vice versa. A drawback of this test is that over estimation of an individual observation will cancel under estimation in a separate observation.

Root Mean Square Error (RMSE): Provides information on the short-term performance and is a measure the variation of the predicted values around the measured data. The Root Mean Square Error may be computed from the following equation:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(H_{Pi} - H_i)^2} \quad (22)$$

Where, Hi is ith Real Value, Hpi is ith Predicted Value and n is the total number of observations.

The RMSE is always positive, a zero value is ideal. This test provides information on short- terms performance of the correlation by arranging a term by term comparison of the actual deviation between the calculated value and the measured value. The smaller the value, the better the model's performance.

T-statistic (t-stat): MBE and RMSE separated do not represent a reliable assessment of the models performance and can lead to the false selection of the best model from a set of candidates [31].

$$t - stat = \sqrt{\frac{(n-1)MBE^2}{RMSE^2 - MBE^2}} \quad (23)$$

It is obvious that each test by itself may not be an adequate indicator of a model's performance. It is possible to have a large RMSE value and at the same time a small MBE (a large scatter about the line of perfect estimation). On the other hand, it is also possible to have a relatively small RMSE and a relatively large MBE (a consistently small over-or under-estimation).

However, although these statistical indicators generally provide a reasonable procedure to compare models, they do not objectively indicate whether a model's estimates are statistically significant, i.e. not significantly different from their measured counterparts. In this article an additional statistical indicator, the t-statistic was used. This statistical indicator allows models to be compared and at the same time indicate whether or not a model's estimates are statistically significant at a particular confidence level [31, 18]. It was seen that the t-statistic used in addition to the RMSE and MBE gave more reliable and explanatory results [32].

RESULT AND DISCUSSION

Regression Constant for Bhopal: Input parameters for estimation of solar radiation on horizontal surface and tilted surface are shown in Table 2. From this it is observed that declination angle (δ) varies according to the cooper's model [20], -23.04° (December solstice) and +23.08° (June solstice). Twice in year the value of declination angle becomes zero on two equinoxes (in March and September). Sunrise and sunset hour angle varies according to the latitude and both will be the same due to symmetry. For Bhopal location average sunshine hour angle (ω_s) is observed approximately 87° which is very good for estimation of solar radiation in this location. From the same Table 2, it is found that percentage sunshine duration (s/s_{max}) is about 79% thought the year. Employing these parameters the regression constant a and b are obtained from the Angstrom equation (8) [22] as a=0.27 and b=0.50 for Bhopal.

Table 2: Input parameters for estimation of monthly average daily global solar radiation at Bhopal, Madhya Pradesh, India.

Month	δDegree	ω_sDegree	s Hours	s_{max} Hours	$\frac{S}{S_{max}}$%
Jan.	-20.81	80.54	10	10.24	0.87
Feb.	-12.95	84.60	11	11.28	0.83
Mar.	- 2.41	88.96	11	11.86	0.79
Apr.	9.41	135.15	12	11.45	0.82
May	18.79	98.39	13	13.11	0.71
Jun.	23.08	100.55	10	13.40	0.70
Jul.	21.18	80.41	5	10.72	0.87
Aug.	13.45	84.10	5	11.21	0.84
Sept.	2.21	89.07	8	11.78	0.79
Oct.	-9.54	85.84	10	11.44	0.82
Nov.	-18.91	81.53	9	10.87	0.86
Dec.	-23.04	79.47	9	10.59	0.88

Fig. 1: Input parameters S,S_{max} and S/S_{max} for estimation of global solar radiation.

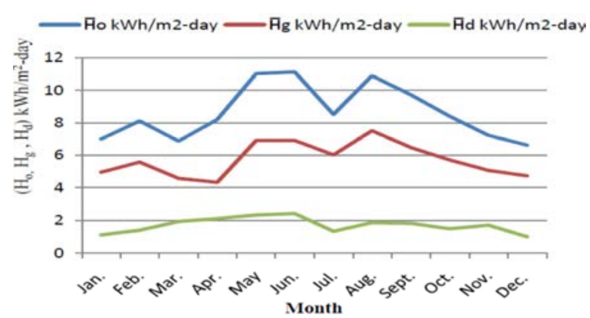

Fig. 2: Monthly average daily solar radiation (H_O,H_g,H_d) on horizontal surface at Bhopal.

Variation of Extraterrestrial Solar Radiation: Input parameters like declination angle δ, sunshine hour angle ω_s and day length ^{S}max are inserted in equation(1) and to estimate the extraterrestrial solar radiation in daily basis ($\overline{H}_o$) as shown in Fig. 2. ($\overline{H}_o$) is observed to be maximum in May 11.00kWh/m^2-day and minimum in December 6.62 kWh/m^2-day.

Variation of Global Solar Radiation for Horizontal Surface: Global solar radiation ($\overline{H}_g$) on monthly average daily basis is estimated with the help of Angstrom equation (8) [22] and regression constant for Bhopal (a=0.27 and b=0.50). Estimated value of $\overline{H}_g$ are compared with the $\overline{H}_o$. It is found that $\overline{H}_g$ is lower than $\overline{H}_o$ as shown in Fig. 2. Monthly average daily global radiation $\overline{H}_g$ is estimated as 5.72kWh/m^2-day.

Variation of Diffused Solar Radiation on Horizontal Surface: The diffused solar radiation based on Liu and Jordan is commonly recommended for predicting daily diffused radiation at location across the world. However in Indian context Modi and Sukhtame, Garg and Garg [24, 27] proposed the modified equation. Hence for estimation of monthly average hourly diffused radiation on daily basis Garg and Garg model [27] is adopted in this study. From the estimated result it is seen that $\overline{H}_d$ is 1.70 kWh/m^2-day which is 30% of total global radiation. That much availability of average monthly solar radiation is encouraging from utilization point of view as shown in Fig. 2.

Sky Condition of Bhopal: Clearness index is the parameter which indicates the transparency of the atmosphere and indicated by fraction of extraterrestrial radiation that reaches the earth surface as global solar radiation. It is measurement of the degree of clearness of the sky. Clearness Index K_T is defined as $K_T = \frac{\overline{H}_g}{\overline{H}_o}$. From the estimated value of $\overline{H}_o$ and $\overline{H}_g$ for Bhopal, K_T is calculated and it is very encouraging to note that the sky over Bhopal is very clear almost throughout the year ($K_T > 0.66$). The transmission through atmosphere K_T along with the diffused radiation and global radiation is shown in Figure 3. The dip in the value of K_T is accordance with the high value of $\overline{H}_d/\overline{H}_g$ for the same month. The sky is fairly clear during winter months when the solar radiation is demand for utilization purpose for photovoltaic application. Empirical coefficients K_T, $\frac{\overline{H}_d}{\overline{H}_o}$ and $\frac{\overline{H}_d}{\overline{H}_g}$ for Bhopal are shown in Table 3.

Variation of Total Solar Radiation on Tilted Surface: Liu and Jordan model [25] is adopted for estimation of total solar radiation on the tilted surfaces, equal to latitude (23.26°), latitude+15(38.26°) and latitude-15(8.26°). View factors $\overline{R}_b$ and $\overline{R}_r$ for estimation of tilted global solar radiation at Bhopal shown in Table 4. Maximum daily average of monthly solar radiation available on the titled surface at Bhopal varies between 4.0 kWh/m^2-day to

Table 3: The estimation of monthly average daily solar radiation on horizontal surface at Bhopal Madhya Pradesh, India.

Month	$\overline{H}_o$ kWh/m^2-day	$\overline{H}_g$ kWh/m^2-day	$\overline{H}_d$ kWh/m^2-day	$K_T = \frac{\overline{H}_g}{\overline{H}_o}$	$\frac{\overline{H}_d}{\overline{H}_o}$	$\frac{\overline{H}_d}{\overline{H}_g}$
Jan.	7.00	4.95	1.10	0.70	0.15	0.22
Feb.	8.10	5.57	1.40	0.68	0.17	0.25
Mar.	6.86	4.57	1.94	0.66	0.28	0.42
Apr.	8.20	4.33	2.10	0.53	0.25	0.48
May	11.00	6.91	2.34	0.62	0.21	0.33
Jun.	11.12	6.91	2.42	0.62	0.21	0.35
Jul.	8.51	6.03	1.33	0.70	0.15	0.22
Aug.	10.87	7.51	1.87	0.69	0.17	0.24
Sept.	9.7	6.47	1.83	0.66	0.18	0.28
Oct.	8.4	5.71	1.48	0.68	0.17	0.25
Nov.	7.22	5.07	1.70	0.70	0.16	0.22
Dec.	6.62	4.73	1.00	0.71	0.15	0.21

Table 4: View factors, $\overline{R}_d$ and $\overline{R}_r$ for tilted surface at Bhopal, Madhya Pradesh, India.

Latitude ⌀ =23.26°N		Ground Reflectivity ρ = 0.2	
View factor	β = latitude	β = latitude + 15	β = latitude - 15
Diffused radiation $\overline{R}_d = \frac{(1+\cos\beta)}{2}$	0.9593	0.8926	0.9948
Reflected radiation $\overline{R}_r = \rho\frac{(1+\cos\beta)}{2}$	0.00813	0.02148	0.00104

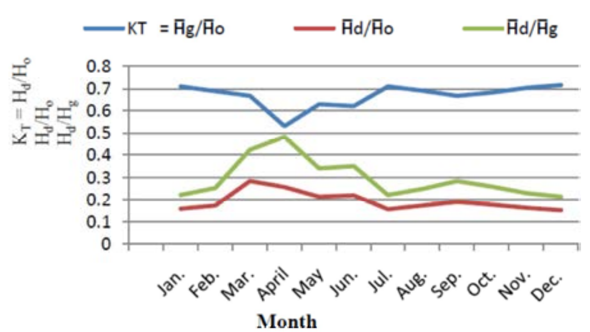

Fig. 3: Monthly variation of clearness index $K_T = \frac{\overline{H}_g}{\overline{H}_o}, \frac{\overline{H}_d}{\overline{H}_o}$, and $\frac{\overline{H}_d}{\overline{H}_g}$ at Bhopal

7.41kWh/m^2- day throughout the year listed in Table 5. It is revealed from the results, that solar energy incident on tilted surface is more than on the horizontal surface due to low incident angle of solar radiation in these months.

This model predicted less amount of radiation in good weather condition due to high angle of incidence of solar radiation. For various tilt angles, average daily radiations obtained are as follows:

(a) β = 23.26°, $\overline{H}_T$ = 6.19 kWh/m^2-day
(b) β=38.26°, $\overline{H}_T$ = 6.05kWh/m^2-day
(c) β=8.26°, $\overline{H}_T$ = 6.08 kWh/m^2-day

These values are depicted in Fig. 4 and observed that at tilted angle equal to latitude of Bhopal (23.26^0), incident solar radiation is highest and more effective

Validation: Estimated and measured monthly average daily global solar radiation data on horizontal and tilted surfaces are listed in Table 6.FromTable 6 it is seen that the estimated values higher than the ground measured in all months of the year. Month of August and September ground measured values slightly exceed the estimated value. Averages of the estimated data in horizontal and tilted position are found 5.72kWh/m^2-day and 6.19kWh/m^2-day which are exceeding the respective average measured values 5.22kWh/m^2-day and 5.53kWh/m^2-day.

Statistical errors (MAPE, MBE,RMSE and t-state)for global radiation on horizontal and tilted global radiation are given in Table 7.These error are found for global solar radiation in horizontal surface are MAPE = -9.57%, MBE = 0.5 kWh/m^2-day, RMSE =1.61 kWh/m^2-day and

Table 5: Estimation Total solar radiations in Tilted Surface at Bhopal, Madhya Pradesh, India.

Latitude =23.26 Ground Reflectivity ρ =0.2

	β = latitude		β = latitude + 15		β = latitude - 15	
Month	$\overline{R}_b$	$\overline{H}_T$ (lat.)kWh/m^2-day	$\overline{R}_b$	$\overline{H}_T$ (lat.+15)kWh/m^2-day	$\overline{R}_b$	$\overline{H}_T$ (lat.-15)kWh/m^2-day
Jan.	1.37	6.36	1.49	6.82	1.15	5.52
Feb.	1.24	6.54	1.29	6.73	1.10	5.97
Mar.	1.11	4.18	1.09	4.70	1.06	5.69
Apr.	0.99	4.28	0.91	4.0	1.02	4.37
May	0.92	6.50	0.78	5.82	0.98	6.85
Jun.	0.88	6.35	0.73	5.59	0.97	6.80
Jul.	0.89	5.56	0.75	4.87	0.98	5.95
Aug.	0.97	7.28	0.85	6.65	1.00	7.5
Sep.	1.06	6.75	1.01	6.48	1.04	6.66
Oct.	1.38	7.30	1.41	7.41	1.25	6.77
Nov.	1.33	6.36	1.44	6.78	1.14	5.62
Dec.	1.41	6.26	1.56	6.81	1.17	5.36

Table 6: Comparison of global solar radiation and meteorological ground measured global solar radiation.

Month	Estimated Global Solar radiation $\overline{H}_g$ (kWh/m^2-day)	Ground Measured Global Solar Radiation $\overline{H}_{gm}$ (kWh/m^2-day)	Estimated Tilted Global Solar Radiation $\overline{H}_{gt}$ (kWh/m^2-day)	Ground Measured Tilted Global Solar Radiation $\overline{H}_{gmt}$ (kWh/m^2-day)
Jan.	4.95	4.38	6.36	5.59
Feb.	5.57	5.21	6.54	6.12
Mar.	4.57	6.62	4.81	6.75
Apri.	4.33	6.97	4.28	6.90
May	6.91	6.78	6.50	6.34
June	6.91	5.57	6.35	5.12
July	9.03	4.03	5.56	3.78
Aug.	7.51	3.91	7.28	3.77
Sept.	6.47	5.11	6.75	5.18
Oct.	5.71	5.33	7.30	5.97
Nov.	5.07	4.70	6.36	5.79
Dec.	4.73	4.49	6.26	5.07
Average	5.72	5.22	6.19	5.53

Table 7: statistics errors in estimated and meteorological ground measured solar global radiations.

S. No.	Name of Errors	Errors in Global Solar Radiation	Errors in Tilted Global Solar Radiation
1.	MAPE	-9.57%	-11.93%
2.	MBE	0.5kWh/m^2-day	0.66 kWh/m^2-day
3.	RMSE	1.61kWh/m^2-day	1.69 kWh/m^2-day
4.	t-stat	1.08	1.40

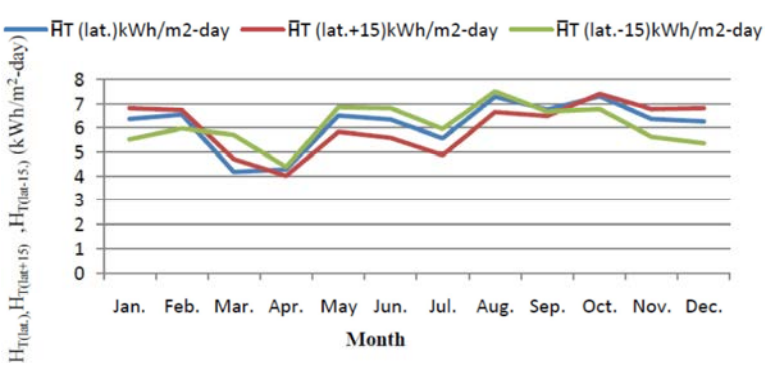

Fig. 4: Monthly average daily solar radiation on tilted surface at Bhopal.

t-stat 1.08 which is comparatively lower than the tilted global solar radiation errors which are MAPE = -11.93%, MBE = 0.66 kWh/m^2-day, RMSE = 1.69 kWh/m^2-day and t-stat 1.40.

CONCLUSIONS

From this Study Following Conclusions Are Drawn:

- Mathematical study of the estimation of solar radiation in central part of the India, Bhopal for horizontal and tilted solar collectors are carried out with considering different input parameters, inclination angle, sunshine hour and day length. Average of estimated solar radiation in all three tilted positions are found as $\bar{H}_T(lat)$ = 6.19 kWh/m^2-day, $\bar{H}_T(lat+15)$ = 6.05 kWh/m^2-day and $\bar{H}_T(lat-15)$ = 6.08 kWh/m^2-day in which tilted angle equal to latitude of the particular location is most effective tilted position for the collector to obtaining the highest solar radiation.
- The difference in estimated data and measured data should be smaller on clear sky or totally over cost days. When examining global solar radiation on horizontal surface, it is found that this dissimilarity is comparatively less mainly in January and February which is obvious from the fact that in these months sky is relatively clear. The variation between the data is higher in July august and September. All the four errors comparatively high in global solar radiation at tilted position.

Nomenclature:

$\bar{H}_o$	:	Monthly Average daily extraterrestrial solar radiation (kWh/m^2-day)
I_{SC}	:	Solar constant 1.367 kW/m^2
N	:	Day of the year
$\bar{H}_g$	:	Monthly average daily global solar radiation (kWh/m^2-day)
a, b	:	Angstrom constants (for Bhopal a = 0.26,b=0.05)
s	:	Monthly average daily hours of bright sunshine (hours)
s_{max}	:	Monthly average of the maximum possible daily hours (day length) of bright sunshine
$\bar{H}_d$	:	Monthly average daily defused radiation (kWh/m^2-day)
K_T	:	Monthly average clearness index
$\bar{H}_T$	:	Total incident solar radiation on tilted surface (kWh/m^2-day)
$\bar{H}_{T,b}$	:	Beam radiation on tilted surface (kWh/m^2-day)
$\bar{H}_{T,d}$	:	Defused radiation on tilted surface (kWh/m^2-day)
$\bar{H}_{T,r}$	:	Ground reflected radiation on tilted surface (kWh/m^2-day)
$\bar{H}_b$	:	Monthly average daily beam radiation on horizontal surface (kWh/m^2-day)
$\bar{R}_b$	:	View factor for beam radiation
$\bar{R}_d$	:	View factor for diffused radiation
$\bar{R}_r$	:	View factor for ground reflected radiation
$\bar{R}_{dz}$	:	Viewfactor for horizon brightening component of diffused solar radiation
$\bar{R}_{d,iso}$	:	Isotropic diffused radiation
$\bar{H}_{d,iso}$	:	Circumsolar component of diffused radiation
$\bar{H}_{d,hz}$	:	Horizon brightening component of diffused solar radiation
$\bar{H}_{T(lat)}$	:	Total solar radiation for tilted angle (latitude)
$\bar{H}_{T(lat-15)}$	:	Total solar radiation for tilted angle (latitude -15)
$\bar{H}_{T(lat+15)}$	:	Total solar radiation for tilted angle (latitude+15)
$\bar{H}_{gm}$	:	Metrological ground measured global solar radiation at horizontal surfaces (kWh/m^2-day)
$\bar{H}_{gt}$	:	Estimated tilted global solar radiation (kWh/m^2-day)
$\bar{H}_{gmt}$	:	Metrological ground measured tilted global solar radiation (kWh/m^2-day)
MAPE	:	Mean Absolute Percentage Error (%)
MBE	:	Mean Bias Error (kWh/m^2-day)
RMSE	:	Root Mean Square Error (kWh/m^2-day)
t-set	:	t-statistics error

Greek Symbols:

γ	:	Azimuth angle (degree)
β	:	Tilted angle (degree)
ω	:	Hour angle (degree)
ω_s	:	Sunset hour angle for mean day of month (degree)
□	:	Latitude angle (degree)
θ	:	Angle of incidence (degree)
θz	:	Zenith angle (degree)

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