World Journal of Agricultural Sciences 6 (3): 249-251, 2010
ISSN 1817-3047

Treatment of a Hetroscedastic Complexity in Estimation of a Consumption-income Relationship for Egyptian Onions

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Abstract: The hetrosceadastic problem revealed through least squares estimation of consumption-income relationship for fresh onions in Egyptian rural districts may be explained by increasing influence of tastes diversity and auto-consumption conditions An attempt to treat such problem involved estimation of the error variance-covariance matrix,data transformation and reapplication of the least squares method. The relationship estimation efficiency improved as indicated by both drop of the regression coefficient's variance to nearly 1% of its initial estimate and rise of the determination coefficient to almost three folds.

Key words: Econometric complexities · Heteroscedasticity · Error correction models · Demand-Income functions · Food consumption pattern

INTRODUCTION

Hetroscedasticity is an econometric problem which violates one of the conditions for obtaining best linear estimates (BLUE), i.e. constant variance of the disturbance error (homoscedasticity). This problem results in inconsistent regression estimates inflicting their viability as repetition on different samples produce dispersed estimates or even end with false statistical significance of regression coefficients [1]. The hetroscedastic complexity is widely detected in consumption studies, where taste differences impact increases for higher income levels as access to substitutes becomes easier. Likewise, for rural regions exaggerated consumption of a specific commodity may occur for its producers while others-relying on the market-keep reasonable levels of consumption. In general, existence of this problem is initially detected by observed change in dispersion of quantities consumed (or demanded) with the level of income.

The study tends to suggest a specific treatment of the hetroscedastic problem in consumption-income relationships estimation. The case considered in this study is of fresh onions consumption in the Egyptian rural sector, based on data of "The Family Budget Survey 1964-65" [2].

Methodology: The method used for testing prevalence of the hetroscedastic problem and treatment is suggested by Goldfield and Quandt [3]. It is based on dividing the variables concerned into three groups after arranging the explanatory variable in an ascending manner and applying regression for both the lowest and highest groups and testing for the statistical difference between the estimated error variance of both groups using "F" test.

$F_1 = S^2\hat{e}_H / S^2\hat{e}_L$, $F_2 = S^2\hat{e}_L / S^2\hat{e}_H$ for $(n_H - 2)$ and $(n_L - 2)$ degrees of freedom

If either F_1 or F_2 is significant, prevalence of a hetroscedastic scheme is accepted[4].

For treatment, both the dependent and explanatory variables are transformed such as to neutralize the problem. Treatment is based on estimating the hetroscedastic scheme[5] (m) relying on regression of the explanatory variable's impact upon the dependent variable.

$$\begin{aligned} y_t &= A + B\,x_t + Ut \\ y_t &= A' + B'\,x_t \\ \hat{U}^2_t &= (y_t - \hat{y}_t)^2 \\ m'_t &= (\hat{U}^2_t / S^2\hat{e}) = a' + b'\,x_t. \end{aligned} \quad (1)$$

Where:

$S^2\hat{e}$ = the estimate of the error variance ($\sigma^2 e$), estimated from eq. (1).

The homoscedastic variance-covariance matrix which is

$$E\,(U'U) = \sigma^2 . [I] = \sigma^2 \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} \quad (2)$$

Becomes under hetroscedasticity

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$$E(U'U) = \sigma^2 [m] = \sigma^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_n \end{bmatrix} \quad (3)$$

To re-establish homoscedasticity, eq. (3) is multiplied by matrices: [1/√m]' [1/√m]:

$$\begin{bmatrix} 1/\sqrt{m_1} & 0 \\ 0 & 1/\sqrt{m_n} \end{bmatrix}' \begin{bmatrix} 1/\sqrt{m_1} & 0 \\ 0 & 1/\sqrt{m_n} \end{bmatrix}$$

Which is automatically obtained through conversion of variables y and x to y√m and x√m, respectively.

To match economic logic, the semi-log mathematical form was chosen to estimate eq. (1) [6].

Data resembled per capita consumption of fresh onions in average for the rural regions of Egypt (1964-65), obtained from the "Family Budget Survey 1964/65". It is worth mentioning that data of recent surveys, up to the last (2000), which have met some difficulties hindering maximum possible accuracy, have hardly shown any indications of the problem concerned.

RESULTS

Test for Hetroscedasticity and Initial Estimation of the Consumption-income Relationship: The scatter diagram of Fig. 1, of per capita shares of fresh onions consumption plotted against natural logarithm of per capita share of total expenditure (as a proxy for income),permitted a preliminary detection of hetroscedasticity prevalence. It was statistically approved at by use of the "Goldfield-Quandt" test with repetition of regression estimation for both groups of highest lowest explanatory variable's values (ln x), as $F1_{(7,7)} = S^2\hat{e}_H / S^2\hat{e}_L$ was significant at.05 level of significance.

The initial estimation of eq.(1) was

$$\hat{y}_t = 2.08 + 1.53 \ln x_t \quad (4)$$
$$(1.11)\ (3.12)$$

Var A' = 3.53 Var B' = 0.25 $S^2\hat{e}$ = 1.09 adjusted R^2 = 0.37

Where:

$\hat{y}_t$ = estimated per capita consumption of fresh onions consumption (kg/year).

x_t = per capita of share of total expenditure (L.E./year)

Values between brackets are "t" ratios.

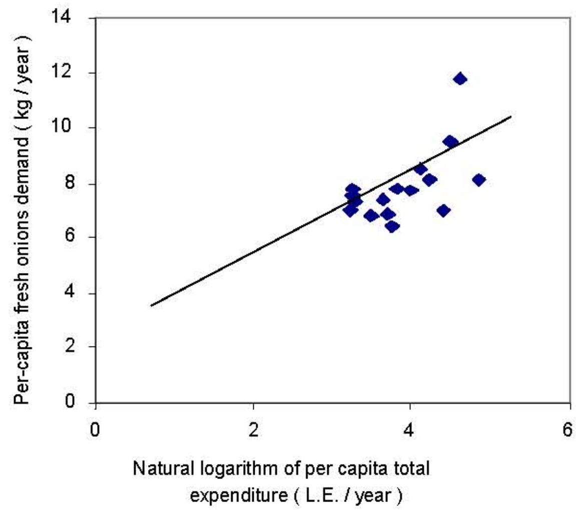

Fig. 1: Demand of onions and total expenditure in rural regions

Low efficiency of (4) is indicated by the relatively high coefficients' variances and low determination coefficient (adjusted R^2).

Estimation of the Hetroscedastic Scheme and Relationship Re-estimation: The estimated hetroscedastic scheme (m) is given in eq. (5).

$$m'_t = (\hat{U}^2_t / S^2\hat{e}) = -5.198 + 1.605 \ln x_t \quad (5)$$

Hence,

$$E(U'U) = S^2\hat{e} \begin{bmatrix} -5.198 + 1.605 \ln x_1 & 0 \\ 0 & -5.198 + 1.605 \ln x_n \end{bmatrix} \quad (6)$$

Accordingly, the estimated relationship is converted to (7), based on variables transformation.

$$(\hat{y}_t / \sqrt{-5.198 + 1.605 \ln x_1}) = -1.626 + 2.45 (\ln x_t / \sqrt{-5.198 + 1.605 \ln x_t}) \quad (7)$$
$$(4.29)\ (47.1)$$

Var A' = 0.144 Var B' = 0.0027 $S^2\hat{e}$ = 0.72 adjusted R^2 = 0.992

Comparing eq. (7) to eq. (4) indicates a remarkable improvement in estimation explicitly indicated by a drop of the regression coefficient estimated variance to nearly 1%, in addition to rise of the determination coefficient to almost 2.7 times its corresponding estimate of eq.(4). Hence, the coefficients obtained by eq. (7) have retained the consistency property of maximum likelihood estimates.

Eq. (7) may be alternatively presented by eq. (8) using the original variables.

$$\hat{y}_t = -1.626\ (\sqrt{-5.198 + 1.605 \ln x_t}) + 2.45 \ln x_t \qquad (8)$$

From which the average income-demand elasticity is estimated at about 0.34, which is not much different than that obtained from eq. (4), i.e. about 0.28, yet more statistically reliable.

CONCLUSION

Improvement in estimation efficiency resulted through the suggested solution of the hetroscedastic complexity maybe of strong validity for attainment of much more reliable consumption predictions demand-income elasticity estimates. For such treatment, though purely econometric wise, was found able to face the undesirable consequences of non-quantitative variables, such as taste differences especially dominant at high income levels, in addition to consumption extravagance as a possible result of dependence on own production (auto-consumption) in rural areas [7].

It is left to mention that the drastic improvement of estimation is compatible with conclusions of Glejser [8] concerning attainment of better estimates through solving the hetroscedastic problem whenever existent even though its assumed mathematical form may not be optimal. However, larger samples are more likely to enhance improvement of this defect in favor of better treatment for the complexity concerned.

Summary: The study aimed to present a realistic application of treatment of one of the most common econometric problems facing OLS estimation of economic variables interrelationships [9, 10], i.e. hetroscedasticity. The problem simply refers to cases where dispersion of values of the dependent variables changes with corresponding changes of the independent variable, leading as such to inconsistent estimates and hence low estimation efficiency. The solution relied on re-attainment of the random error's constant variance. It was accomplished through estimation of its response to the independent variable's changes and neutralizes such effect through variables adjustment as suggested by Goldfiield and Quand [3]. The suggested treatment was applied upon an estimation of the total expenditure impact upon per capita consumption of fresh onions in rural Egypt (1964-65). The problem was hardly detected in more recent surveys, which were in general less fortunate in accuracy fulfillment due to administration difficulties. Solving the econometric problem lead to drastic estimation improvement, featured by a drop of the regression coefficient's variance to nearly 1% its corresponding estimate before solution and rise of the determination coefficient to almost 2.7 folds.

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