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A Comparison of Three Mathematical Models of Response to Applied Nitrogen Using Lettuce

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Abstract: Modern fertilization recommendation must optimize crop yield and quality and minimize chances of negative environmental effects due to over fertilization. Data from fertilizer studies can be fitted to several mathematical models to help determine optimum fertilizer rates, but resulting recommendations can vary depending on the model chosen. In this study, lettuce (*Lactuca sativa L.*) was used as a case study vegetable crop to compare models for estimating fertilizer N requirements. Field studies were conducted to measure yield response to applied N. The area was located between 25°,21′ E longitude and 51°,38′ N latitude in the North of Varamin city, (Tehran province, Iran) in the alluvial plain of Varamin. Soils family were fine, mixed, active, thermic, typic haplocambids based on Soil Taxonomic system. Plants were grown in Central Research Station of Varamin and received five rates of N (0.0, 150, 200, 250 and 300 kg ha^{-1}) as a urea in split applications. Data for plant fresh mass and N uptake were recorded. The logistic model described the data for cultivar quite well, with correlation coefficients of 0.90 and above. Logistic, linear-plateau and quadratic models were compared for the field data. Coefficients for the linear-plateau model were derived from the logistic model. All three models for lettuce production were compared graphically and analytically. The model coefficients were then used to make improved estimates of fertilizer recommendations for field production of lettuce.

Key words: *Lactuca sativa* · Logistic equation · Nitrogen

INTRODUCTION

Recommendations for fertilization of crops are derived from field studies in which crop yield and quality responses to a range of fertilizer rates are measured. Responses are often modeled to determine optimum fertilizer rate. Today, the relationship of nutrient management to environmental pollution also is an important aspect of any fertilization recommendation. There are many mathematical models for fitting crop response data. The research seeks to find a model that describes the data well and aids in defining reasonable fertilization recommendations that result in optimum crop yield and quality without risking over fertilization.

Quadratic models have been very popular for describing crop response to fertilization but tend to overestimate response if the maximum point on the curve is taken as the best fertilization rate. Often, fertilization rates less than the function maximizing rate statistically similar to the single function maximizing rate [1,2]. Models other than quadratic functions have been used to describe crop response to fertilizer. Plateau models, such as linear-plateau [3,4], have been used with agronomic crops [1,5,6] and vegetables [2,7,8,9] and logistic models with agronomic crop [10,11,12]. Other studies with vegetable crops to test functions such as the logistics model were need. Vegetables such as lettuce that require fertilization for optimum yield and quality are ideal candidates for such study.

Lettuce is an important vegetable crop that is grown widely throughout the Iran, with much of the commercial production in Varamin, Tehran, Gilan and Mazandaran regions. Most of Varamin's lettuce is produced on Aridisols soils of northern Varamin. Varamin is a major supplier of lettuce for Tehran.

Because the proportion of leaf tissue in lettuce, yields are greatly impacted by N fertilization. Research in Varamin with lettuce grown on loamy soils showed that N fertilization requirements were from 150 to 200 kg ha^{-1}. Sources of N fertilizer did not differ in their effects on

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lettuce yield or head quality [13]. Low levels of N resul in small head size and poor yields. Even short periods of N deficiency can have a long-lasting negative effect on lettuce yield [14]. Current N recommendation is 200 kg ha^{-1} for lettuce grown on loamy soils in Varamin. Yield and N uptake tend to increase linearly with N application rate. At high levels of N, plant yields and N uptake asymptotically approach maximum values. Decisions concerning optimum rates of fertilization usually involve fitting some type of model to yield data in response to several rates of fertilizer application. Regression analyses have been conducted on numerous data sets for response of agronomic forage crops to applied nutrients [11,12,15-21]. In all these studies, the logistic equation accurately described data for dry-matter yields of forages and corn. In several studies, the extended logistic model also described plant N uptake as well as yield [16,17,20,21]. In the latter case, a common N response coefficient, c, existed between yield and plant N uptake. As a consequence, yield could be expressed as a hyperbolic function of plant N uptake. Willcutts *et al*. [22] study models of response to applied nitrogen using lettuce. They found the logistic model offers a useful tool for evaluation of lettuce response to applied N.

The objective of this study was to demonstrate the utility of the logistic model to describe response of lettuce to applied N. A comparison was made with the linear-plateau and quadratic models for data obtained in field. Coefficients of the linear-plateau model were obtained as approximations from the logistic model. Both the linear-plateau and quadratic model predicted negative yields at very low N levels, whereas the logistic equation shows asymptotic approach to zero. The general characteristic and a rational basis for the logistic equation have been given by Overman [17]. Output (yield or plant N uptake) remains positive for all applied N, which must be true of the system by definition. Linear-plateau and quadratic models do not meet this constraint.

MATERIALS AND METHODS

Field Experiments: Field experiments were conducted in spring with Lettuce (*Lactuca sativa* L.) on Aridisols soils and soils family were fine, mixed, active, thermic, typic haplocambids based on Soil taxonomic system [23]. Soils family were fine, mixed, active, thermic, typic haplocambids based on Soil Taxonomic system [23]. The area was located between 25°,21′ E longitude and 51°,38′ N latitude in the North of Varamin city, (Tehran province, Iran) in the alluvial plain of Varamin. After soil was prepared by plowing and disking, plots were formed. Irrigation method was furrow irrigation. Lettuce sown on 9 Mar. Plots were 15 m long and 5 m wide and consisted of five rows on 40 cm spacing×20 cm between plants, for a total of 93 plants per plot (62000 plant ha^{-1}). Plants were grown in field of Central Research Station (Varamin Agricultural Research Center) and received five rates of N (0.0, 150, 200, 250 and 300 kg ha^{-1}) as a urea in split applications. Treatments were replicated three times, with irrigation and pest control following recommended cultural practices [24]. Lettuce heads were harvested on 4 June and fresh mass of marketable lettuce was recorded. N uptake with plants were measured in laboratory [25].

Model Description: Data were analyzed using several models for comparison. The logistic models for yield and plant uptake are given by equations [1] and [2].

$$Y = A/[1+\exp(b-cN)] \quad (1)$$

$$Nu = A'/[1+\exp(b'-cN)] \quad (2)$$

Where Y = yield in Lettuce fresh mass (kg fresh mass /plant); Nu = nitrogen uptake by lettuce (g N/plant); N = applied nitrogen (g N/plant) or kg ha^{-1}; A = maximum yield in fresh mass, kg/plant; b = intercept parameter for yield; b' = intercept parameter for nitrogen uptake; c = N response coefficient, plant/g or ha.kg^{-1}. Following Overman *et al*. [20], Eqs. [1] and [2] can be combined to give the hyperbolic phase relation between yield and plant uptake,

$$Y = Y_m N_u/(K'+N_u) \quad (3)$$

Where parameters Y_m and K' are defined in terms of the logistic parameters by,

$$Y_m = A/[1-\exp(b-b')] \quad (4)$$

$$K' = A'/[\exp(b'-b)-1] \quad (5)$$

Note that Y_m represents maximum potential yield and that N_u = K' produces Y = $Y_m/2$, or one- half of maximum potential yield. Calculus techniques show that maximum incremental response to applied N occurs at an application rate $N_{1/2}$ = b/c, where Y = A/2. This is the point of maximum slope T vs N. Similarly, maximum incremental response of plant N to applied N occurs at $N'_{1/2}$ = b'/c, with N_u = A'/2. The N response coefficient can be redefined as characteristic N given by N' = 1/c, which converts units to more familiar g/plant or kg ha^{-1}.

The linear-plateau model is given by,

$$Y_{lp} = B_{lp}+C_{lp}N \text{ for } N<N_x \quad (6)$$

$$Y_{lp} = A_{lp} \text{ for } N>N_x \quad (7)$$

Where for Y_{lp} = linear-plateau estimate of yield in fresh mass, kg/plant; A_{lp} = plateau or maximum freshyield, kg/plant; B_{lp} = intercept parameter, kg/plant; C_{lp} = slop parameter, ha/plant; and N_x = N application rate for intersection between Eqs. [6] and [7]. The linear-plateau parameters can be approximated from the logistic parameters as,

$$A_{lp} = A \quad (8)$$

$$B_{lp} = A/2(1-b/2) = A/4N'(2N'-N_{1/2}) \quad (9)$$

$$C_{lp} = Ac/4 = A/4N' \quad (10)$$

This occurs because the logistic model approximates a straight line in the midrange of response. It follows that the intersection of the linear and plateau portions occurs at,

$$N_x = (A_{lp}-B_{lp})/C_{lp} = (b+2)/c = N_{1/2}+2N' \quad (11)$$

The quadratic model can be written as,

$$Y_q = A_q+b_qN+C_qN^2 \quad (12)$$

Where Y_q = quadratic estimate of yield in fresh mass, kg/plant; A_q = intercept parameter, kg/plant; B_q = linear response coefficient, ha/plant; and C_q = quadratic response coefficient, ha^2.kg^{-1} per plant. Peak production can be estimated from the maximum where the derivative, $dY_q/dn = 0$, which occurs at,

$$N_{peak} = B_q/2c_q \quad (13)$$

And gives peak production of,

$$Y_{peak} = A_q+B_q^2/4C_q = A_q+B_q/2N_{peak} \quad (14)$$

Fertilization rates of N_{peak} may be optimal for production because of diminishing returns obtained as N approaches N_{peak}. Therefore, optimum applied N rates would tend to be below N_{peak} (i.e., $N_{opt}<N_{peak}$).

RESULTS AND DISCUSSION

Field Experiments: Response of field lettuce to applied N is shown in Fig. 1. Logistic, linear-plateau and quadratic models were fitted to the data with parameters listed in Table 1. The logistic model provides a reasonable basis for the linear-plateau model (Fig. 1). The intersection point can be calculated from Eq. [11] and peak N values for the quadratic model were calculated from Eq. [13]. A summary of critical values of model parameters is listed in Table 2. At $N = N_{1/2}$yield is 505 of the plateau, whereas at $N = N_x$, yield is 88% of plateau. For $N = N_{peak}$ yields are well out on the plateau, beyond the region of significant response to applied N. Fig. 2 is shown dependence of fresh mass on plant N uptake for field lettuce. The results obtained were

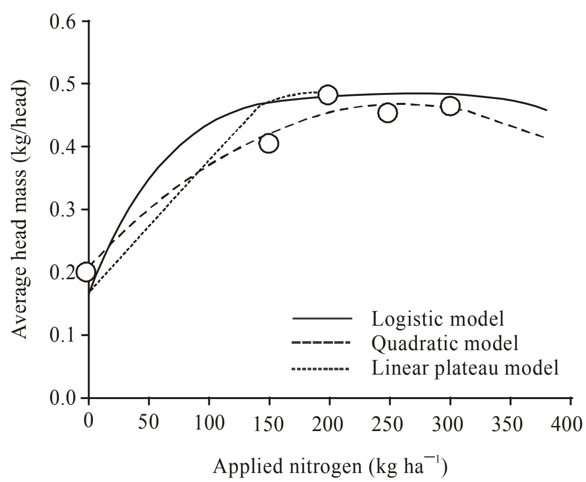

Fig. 1: Comparison of logistic, linear-plateau and quadratic models for response of field-grown lettuce to N application at Varamin, Iran. Models values calculated with parameters from Table 1

Table 1: Model parameters for field lettuce at Varamin, Iran

Model	Parameters	Value
Logistic		
$Y = A/[1+\exp(b-cN)]$	A, kg/plant	0.48
$Nu = A'/[1+\exp(b'-cN)]$	b	0.64
	c, ha.kg^{-1}	0.028
	A'	490.00
	B'	30.00
Linear-plateau		
$Y_{lp} = B_{lp}+C_{lp}N$ for $N<N_x$	A_{lp}, kg/plant	0.48
$Y_{lp} = A_{lp}$ for $N>N_x$	B_{lp}, kg/plant	0.163
	C_{lp}, ha/ plant	0.00336
Quadratic	A_q, kg/plant	0.1984
$Y_q = A_q+b_qN+C_qN^2$	B_q, ha/ plant	0.0021
	C_q, ha^2. kg^{-1}/plant	0.000004

Table 2: Critical N value (kg ha^{-1}) for the models for field-grown lettuce in Varamin, Iran

$N_{1/2}$	N_x	N_{peak}
23	94	202

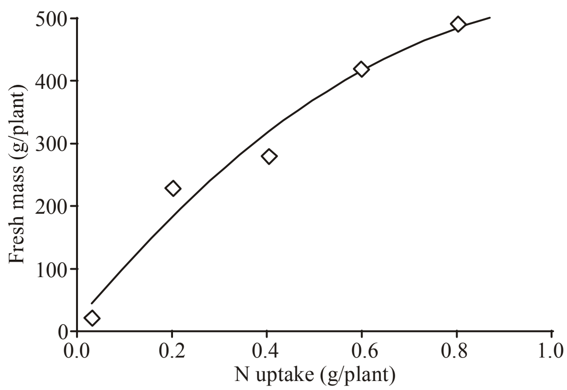

Fig. 2: Dependence of fresh mass on plant N uptake for field lettuce at Varamin, Iran. Curves draw n from Eq. 3 with parameters calculated by Eq. 4 and 5 using logistic parameters from Table 1

more than Willcutts *et al.* [22] but there were similar it. In Fig. 1, curve was not similar to Willcutts *et al.* [22] in intercept but trend it was similar to it.

From these results, the logistic model apparently provides an adequate description field results for response of lettuce to applied N. The obtained results were similar to Willcutts *et al*. [22]. Therefore obtained model were more accuracy than in University of Florida that obtained with Willcutts *et al.* [22]. In this research R^2 were obtained 0.02% more accuracy than Willcutts *et al*. [22]. It recommended for estimation of response to Lettuce applied nitrogen in Iran. In the future research, suggested comprised mathematical models of response to applied phosphorous in Lettuce and compare to nitrogen and in other vegetables, too.

Summary: From analysis of the field studies, N_x appears to give the most reasonable level for a nitrogen fertilizer recommendation, viz., 150 kg ha^{-1} for these conditions. This is considerably below the current Varamin recommendation of 200 kg ha^{-1}.

The logistic model offers a useful tool for evaluation of lettuce response to applied N. Parameters A, b and c in Eq. [1] can be estimated from data by nonlinear regression. One can also use the following simple alternative procedure. Parameter A (the plateau) can be estimated by visual inspection of the data for yield vs. applied N (such as Fig. 1). Then parameter b follows from,

$$B = \ln(A/Y_{\circ} - 1) \tag{15}$$

Where $Y_{\circ}$ = estimated intercept yield at N = 0. Finally, parameter c is calculated from,

$$C = b/N_{1/2} \tag{16}$$

Where $N_{1/2}$ is estimated as the value of N corresponding to y = A/2 (50% of the plateau) on the graph of yield response to applied N. With parameters b and c in hand, N_x can then be estimated from Eq. [11]. Estimates of yield at given applied N levels are easily made with Eq. [1] using a calculator with an equation writer.

The model contains the right characteristics to describe field data and is relatively simple to use in practice.

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