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One Dimensional Flow and Sediment Transport Fully Coupled Model Applicable to Sandy River Streams

[1]*Mehdi Fuladipanah,* [2]*S.H. Musavi-Jahromi,* [2]*M. Shafai-Bajestan and* [3]*A. Khosrojerdi*

[1]PhD Student, Department of Agriculture and Natural Resources,
Islamic Azad University, Science and Research Branch of Tehran, Iran
[2]Water Engineering, Shahid Chamran University, Ahwaz, Iran
[2]Water Engineering, Shahid Chamran University, Ahwaz, Iran
[3]Water Engineering, Islamic Azad University, Science and Research Branchof Tehran, Iran

Abstract: A new one dimensional (1D) fully coupled numerical model is developed based on mass and momentum conservation for calculating flow and suspended load. This model which stands for sandy rivers is applicable to unsteady flow condition. The Reynolds Transport Theory was used to convert system analysis to control volume analysis. Implicit finite difference was used to equation discretization. The model predicts the flow depth, velocity and sediment concentration at the end of each time step for all nodes. It was calibrated and validated with measured flow and suspended load data from a reach between Ahwaz and Mollasani stations, Karoon River, Iran. NS coefficient was used for estimating model accuracy. The model prediction in mean flow condition was superior to high flow condition. According to NS coefficients, there was not significant difference between model prediction and measured data.

Key words: 1D fully coupled numerical model · Sandy rivers · Conservation law · Diffusion equation · Suspended load

INTRODUCTION

Fluvial sediment transport is the most important process in river engineering. River flow and sediment transport are two phenomena which interact with each other in an alluvial river system, simultaneously. Hence, providing a model to simulate this phenomenon will be very useful. But, river flow and sediment transport are among the most complex and least understood processes or phenomena in nature. It's very difficult to find analytical solutions for most problems in river engineering.

Fluvial models have taken precedence in river engineering. These range from the three dimensional modeling of circulation surrounding a confluence, detailed two dimensional finite element grids of water surface profiles and the more classic one dimensional approach of calculating over cross sections. For more engineering applications, cross sectional properties of sediment and flow is important. 1-D models require the least amount of field data and the numerical schemes used for solving the water and sediment governing equations are more stable and offer order of magnitude gains in computational time over 2-D and 3-D models. 1-D models simulate the flow and sediment transport in the stream wise direction of a channel without solving the details over the cross section. Barry *et al.* studied sediment transport using of numerical modeling [1]. Minh and Rutschmann studied sediment transport modeling in a curved channel [2]. Qiao *et al.,* developed a model to study transport and distribution of suspended load in Yellow River mouth and the nearby Bohai Sea. Their model has good results [3]. Shen *et al.,* simulated sediment transport in channels with vegetation with a k-ε turbulent model which the simulated results were in good agreement with available experimental data. They used finite element method for their model [4]. Abderrezzak developed 1-D model based on common di SantVenant-Exner approaches using one layer and assuming clear water [5]. Cao *et al.* have modeled suspended load transport in alluvial rivers by mathematical modeling [6]. Huybrechts and Verbanck studied sediment transport by numerical modeling.

Corresponding Author: Mehdi Fuladipanah, Department of Agriculture and Natural Resources, Islamic Azad University, Science and Research Branch of Tehran, Iran, E-mail: fuladipanah@gmail.com.

They found that fully dynamic model has the best fitness with measured data [7]. Luong and Verbanck, Nanson and Huang, Van Maren, Verbanck, Larcy *et al.,* Zhang and Verbanck have studied similar models in recent years, wary numerical model [8-13]. The purpose of this paper is to present a mathematical model which couples water flow and sediment transport dynamics. The development of this model was motivated by the following four main factors. First, many scientific problems cannot be solved by neglecting flow dynamics (i.e. assuming the balance existence between exerted forces to flow, which are leaded to the normal flow). Thus, this is a fully dynamic model. Second, flow and sediment transport are time dependent processes. Empirical relationships often do not satisfy this dependence. Hence, calculation of flow and sediment properties at arbitrary time, is the model's grant. Third, as mentioned aforesaid, the complexity of interaction between flow and sediment transport, makes it difficult to describe this coupled phenomena. In this model based on conservation rules, this phenomenon has been modeled explicitly. Forth, in many early models, it is assumed that the actual sediment transport rate is equal to the capacity of flow sediment at equilibrium condition at each cross section (these models are called equilibrium models). However, alluvial river systems always change in time and space due to many reasons; therefore, absolute equilibrium states rarely exist in natural condition. The local equilibrium assumption is no realistic, particularly in cases of strong erosion and deposition. Non-equilibrium sediment transport models renounce this assumption and adopt transport equations to determine the actual sediment load rate. The presented model is a non-equilibrium one.

MATERIALS AND METHODS

Hydrodynamic Model

Model Equations: FSC1D[2]model is developed on the following considerations: 1) Suspended load in sandy rivers has significant segment, 2) Cross sections in the reach which is undertaken for this study are regular, 3) Pressure distribution is hydrostatics and 4) Diffusion model can be used for suspended load transport [14]. Under these assumptions, the laws of conservation of mass and momentum and diffusion model were used to derivate FSC1D model equations.

All the laws of mechanics are written for a system which is defined as an arbitrary quantity of mass of fixed identity. Every thing external to system is defined by theterm Surroundings and the system is separated from its surroundings by its boundaries. The laws of mechanics then state what happens when there is an interaction between the system and its surrounding. In fluid mechanics, fluid forms the environment whose effect on a product must be known. This requires that the basic laws be written to apply to a specific region neighborhood the product. This region is called control volume. In analyzing a control volume, the system laws are converted to apply a specific region which the system may occupy for only an instant. To convert a system analysis to control volume analysis, system mathematics must be converted to apply a specific region rather than to individual masses. This conversion, called the Reynolds transport theorem, can be applied to all the basic laws. The compact form of the Reynolds transport theorem for 1-D flow and a fixed control volume is [14].

$$\frac{d}{dt}(N_{SYS}) = \frac{d}{dt}\left(\int_{c.v.} \eta\, p dv\right) + \int_{c.v.} \eta\, p(\vec{V} \tag{1}$$

Which N is any property of the fluid (mass, momentum, etc), $\eta=$ is the amount of N per unit mass in any small portion of the fluid, is fluid density, v is fluid volume element, V is fluid velocity, dA is surface element, $\vec{V}, d\vec{A} = |\vec{V}||d\vec{A}|$ is inner product of velocity and section area vectors, is angle between two vectors and subscripts sys, C.V. and C.S. stands for system, control volume and control surface, respectively. is a vector quantity that its direction is surface element outward.

Continuity Equation: Figure 1 shows a control volume between cross sections 1 and 2. Its length along the flow direction is dx. Depth, velocity and suspended load concentration (volumetric) is illustrated by y, v and c, respectively. Water and sediment enter it at section 1 and leave at section 2. There is no lateral flow between sections 1 and 2. For conservation of mass, N=m and . Equation (1) becomes:

$$\frac{d}{dt}\left(m_{sys}\right) = \frac{d}{dt}\left(\int_{c.v.} pdV\right) + \int_{c.v.} p(\vec{V}.d\vec{A}): \tag{2}$$

But, the system mass is constant and does not change. This called conservation of mass which is defined as:

$$m_{system} = constant - \frac{dm}{dt})system$$

The density of the water and sediment mixture is determined by [15]:

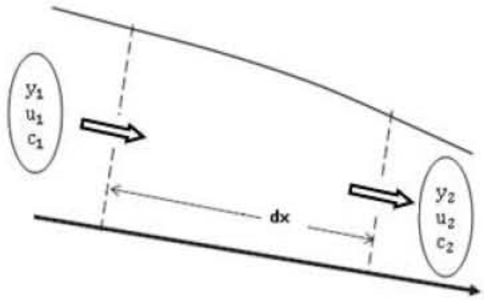

Fig. 1: Control volume in a alluvial channel

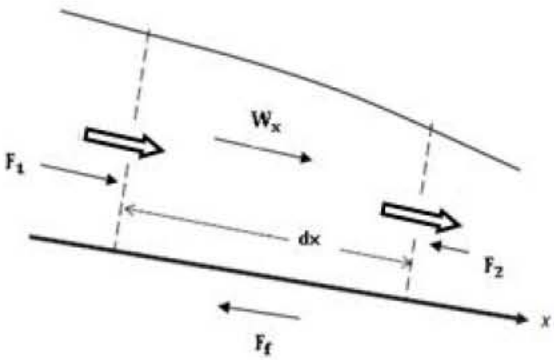

Fig. 2: External forces on the acting on control volume

$$p_m = p_w + (p_s - p \tag{3}$$

Which is mixture density, is water density, is sediment density and c is sediment volumetric concentration. According to equation (3), equation (2) is written as:

$$\frac{d}{dt}\left(\int_{c.v.}\left[P_w+(P_3-P_w)c\right]dV\right)+\int_{c.v.}\left[P_w+(P_3-P_w)c\right](\bar{V}, d\bar{A}) \tag{4}$$

Volume element of the control volume can be written as dV= which A is mean section area. At section 1 and 2, amount of is equal to 180° and 0°. Then equation (4) can be rewritten as:

$$\frac{\partial}{\partial t}\int_{x2}^{x2}(P_w\, Adx)+\frac{\partial}{\partial t}\int_{x2}^{x2}(P_3-P_w)-\int_{A_1}P_w u dA-\int_{A_1}(P_3-P_w)cudA + \int_{A_2}P_w u dA+\int_{A_1}(P_3-P_w)cudA=0 \tag{5}$$

Applying the Leibniz rule to two first terms in above equation and assumption 4 for the rest yields:

$$p_w\int_{x2}^{x2}\frac{\partial A}{\partial t}dx+(p_3-p_w)\int_{x2}^{x2}\frac{\partial(cA)}{\partial t}dx-p_w u_1 A_1+pu_2A_2+(p_2-p_w)cu_2A_2 \tag{6}$$

According to the Mean value Theorem for two first terms in equation (6) and also derivative definition, Mixture continuity equation is obtained as:

$$\frac{\partial A}{\partial t}+(S-1)\frac{\partial(cA)}{\partial t}+(S-1)\frac{\partial(Acu)}{\partial x}: \tag{7}$$

Which s is specific gravity of sediment particle.

Momentum Equation: If surrounding exerts net force on the system, according to Newton's second law: Then,

$$\bar{F}=m\bar{a}=\frac{d}{dt}(r.\ Then,\ N=m\bar{V}-\eta :$$

Relevant to the Reynolds transport theorem:

$$\frac{d}{dt}\left(m\bar{V}\right)_{sys}=\sum\bar{F}=\frac{d}{dt}\left(\int_{c.v.}VpdV\right)+\int_{c.v.}Vp(\bar{V},1 \tag{8}$$

Figure 2 shows a control volume and external forces acting on it in x-direction. According to assumption 3, pressure force at section 1 and 2 can be determined as:

$$F_{p1}=pg\bar{y}_1A_1=\lfloor p_w+(p_3-p_w)c\rfloor g\bar{y}_1A_1=y_w\bar{y}_1QA_1+(y_3-y_w)c_1\bar{y} \tag{9}$$

$$F_{p2}=pg\bar{y}_2A_2=\lfloor p_w+(p_3-p_w)c\rfloor g\bar{y}_2A_2=y_w\bar{y}_2QA_2+(y_3-y_w)c_2\bar{y} \tag{10}$$

Which g is gravity acceleration and is vertical distance from the free surface to the cross section centroid. Friction force is determined as following equation:

$$F_f=\int_{x2}^{x2}tpdx=\int_{x2}^{x2}pgRS_f\,pdx=\int_{x2}^{x2}pgAS_f dx=y_wAS_f(x_2-x_1)+(y_3-y_w)cAS_f(x_2- \tag{11}$$

Which is shear stress, P is wetted perimeter, is energy grade line,and are water and sediment specific weight, respectively. The weight component is determined as following:

$$W_x=\int pgs_oAdx=y_wAS_o(x_2-x_1)+(y_3-y_w)cAS_o(x_2- \tag{12}$$

Which is mean bed slope. Thus, the amount of can be written as:

$$\sum\bar{F}=y_w(x_2-x_1)\left[-\frac{\partial(Ay)}{\partial x}-(s-1)\frac{\partial(Acy)}{\partial x}+A(s_f-s_o)+(s-1)Ac(s_f-s\right. \tag{13}$$

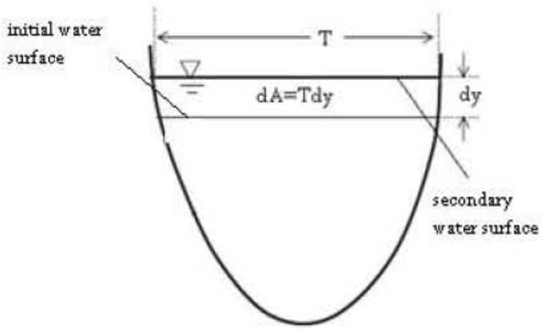

Fig. 3: Natural cross section of a river

The terms on the right-hand side of equation (8) can be written as:

$$\frac{d}{dt}\left(\int_{c.v.} V\rho dV\right)+\int_{c.v.} V\rho\left(\bar{V}.d\bar{A}\right)=\frac{d}{dt}\int_{x_1}^{x_2} V\rho_w A dx+\frac{d}{dt}\int_{x_1}^{x_2} V\left(\rho_s-\rho_w\right)cA dx+\int_{A_1} V\rho_w\left(\bar{V}.d\bar{A}\right)+$$
$$\int_{A_1} V\left(\rho_s-\rho_w\right)c\left(\bar{V}.d\bar{A}\right)+\int_{A_2} V\rho_w\left(\bar{V}d\bar{A}\right)+\int_{A_2} V\left(\rho_s-\rho_w\right)c\left(\bar{V}.d\bar{A}\right) \quad (14)$$

Applying the Leibniz rule and mean value theorem for two first terms on the right-hand side of equation (14) and also definition of 1D flow, equation (14) can be written as:

$$\frac{d}{dt}\left(\int_{c.v.} V\rho dV\right)+\int_{c.v.} V\rho\left(V.dA\right)=\rho_w\left(x_2-x_1\right)\left[\frac{\partial(VA)}{\partial t}+(s-1)\frac{\partial(VcA)}{\partial t}+\right.$$
$$\left.\frac{\partial(VcA)}{\partial x}+\frac{\partial\left(A\beta V^2\right)}{\partial x}+(s-1)\frac{\partial(A\beta c}{\partial x}\right. \quad (15)$$

According to equation (14) and assumption 2, mixture momentum equation is obtained as:

$$\frac{\partial(VA)}{\partial t}+(s-1)\frac{\partial(VcA)}{\partial t}+\frac{\partial\left(AV^2\right)}{\partial x}+(s-1)\frac{\partial\left(AcV^2\right)}{\partial x}+$$
$$g(s-1)\frac{\partial(A\bar{y})}{\partial x}=gA(s_t-s_o)(s-1 \quad (16)$$

According to Figure 3, it can be written:

$$\Delta(A\bar{y})=\left[A(\bar{y}+dy)+dA\frac{dy}{2}\right]-A\bar{y}=A\bar{y}+Ady+$$
$$dA\frac{dy}{2}-A\bar{y}=Ady+d_2 \quad (17)$$

Ignoring minuscule quantity, $\Delta(A\bar{y})\approx_2$ Thus, $\frac{\partial(Ay)}{\partial x}$ and $.\frac{\partial(Acy)}{\partial x}c_2$ Then, ultimate applicable form of momentum equation is as following:

$$\frac{\partial(AV)}{\partial t}+(s-1)\frac{\partial(VcA)}{\partial t}+\frac{\partial\left(AV^2\right)}{\partial x}+(s-1)\frac{\partial\left(AcV^2\right)}{\partial x}+gA\frac{\partial y}{\partial x}$$
$$+g(s-1)cA\frac{\partial y}{\partial x}=gA\left(s_f-s_o\right)\left(1+(s-1\right. \quad (18)$$

Diffusion Equation: To evaluate mixture properties, the amount of y, c and v be known. The third equation should be applied to evaluate sediment concentration. Diffusion equation can be used for suspended load description [14]. According Fick's law, diffusion equation is as following equation:

$$\frac{\partial c}{\partial t}+V\frac{\partial c}{\partial x}=D_x \quad (19)$$

Which is diffusion coefficient. Diffusion coefficient is as following [14].

$$\frac{D_x}{yV_*}=0.011\left(\frac{T}{y}\right)^2\left(- \quad (20)$$

Which T is water surface width and V_* is shear velocity. The set of equations (7), (18) and (19) which are basic equations of FSC1D model, should be solved using numerical methods. As it's clear, the set of equations (7), (18) and (19) isn't closed, due to the number of unknown variables that are more than equations. To close these equations, geometry relationships as following: A=A(y), P=P(y) and T=T(y) for each cross section were derived.

Model Performance: In this research, finite difference implicit scheme with disorganized mesh was utilized for FSC1D model application to evaluate flow and sediment properties in a reach of Karoon River, Iran. The reach is located between Mollasani and Ahwaz stations. It included 76 cross sections and 75 sub reaches. Flow and sediment transport was simulated using FSC1D model for 365 days from 22 September 2003 to 21 September 2004. The first 120 days were used as calibration period, with the remaining 245 days used for validation. All 120 days were used to compare flow rates. There were 113 days that sediment load were measured. Of the 113 days, 93 days were used to compare sediment concentration to measured values due to gaps in measured data. Manning coefficient was estimated in calibrated period. It's amount was obtained n=0.028 for the best conformity between model output and measured data.

The Nash-Sutcliffe coefficient (NS) of model efficiency was used as a statistical criterion for evaluating hydrodynamic goodness of fit between measured and calculated values for each variable tested. This statistic is recommended by American society of civil Engineers Watershed Management Committee for evaluating the performance of models that simulate continuous hydrographs. It is defined as below [16].

$$NS = 1\frac{\sum_{i=1}^{n}\left(Q_{O_i} - Q\right.}{\sum^{n}_{i=1}\left(_{O_i} - (\right.} \qquad (21)$$

Which are the observed values, are model predicted values and n is the number of data pairs. An NS value equal one indicates there is a perfect agreement between measured and calculated values and would plot as a 1:1. A value of zero suggests that the fit is as good as average value of all the measured data for each event, indicating a poor model fit. Negative NS values (having no lower limit), generally considered meaningless, indicate poor predictive value of model, with negative values indicating a poorer model fit.

RESULT AND DISCUSSION

A 1D fully coupled numerical model was developed to simulate flow and suspended load transport in sandy rivers. Model equations were fully hydrodynamic which were usable for unsteady and non equilibrium condition. They were based on momentum and mass conservation laws and diffusion equation. The model was tested both flow and sediment transport with field measured data on Karoon River, Iran. Model was used to simulate flow and sediment transport from 22 September to 21 September 2004. Steady uniform flow was assumed for initial condition. Flow rate and suspended load concentration were equaled to 249(m3/sec) and 483(m3/day), respectively. Various amount of Manning coefficient was tested for best agreement between measured and calculated flow and sediment concentration on calibration period. Its amount was obtained n=0.028 for the best conformity between model output and measured data. Figure 4 shows flow hydrograph in calibration and validation periods on Ahwaz station.

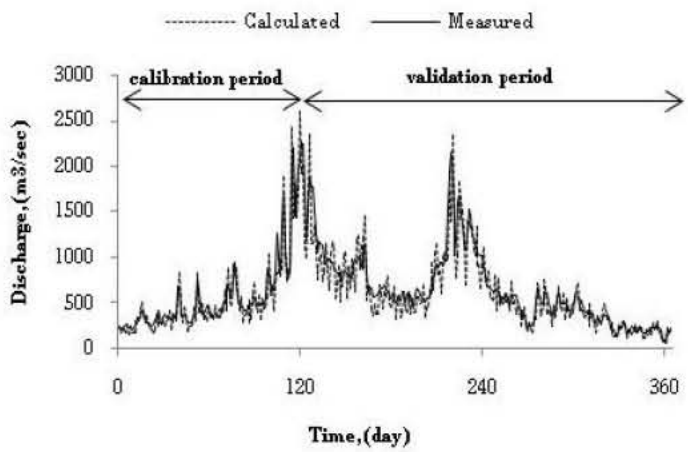

Fig. 4: Measured and calculated flow in calibration and validation periods on Ahwaz station

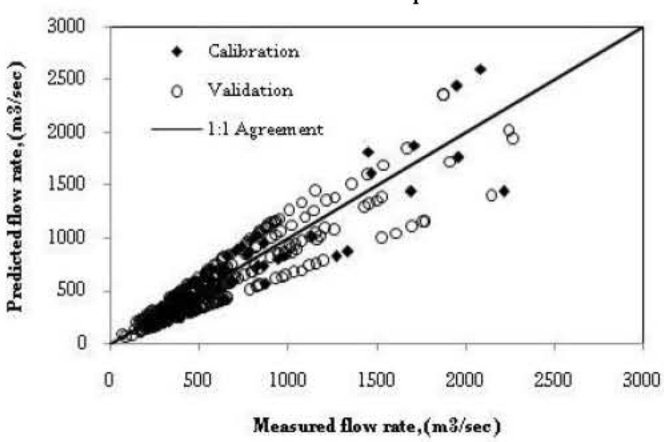

Fig. 5: Measured vs. predicted daily flow for calibration and validation model runs on Ahwaz station

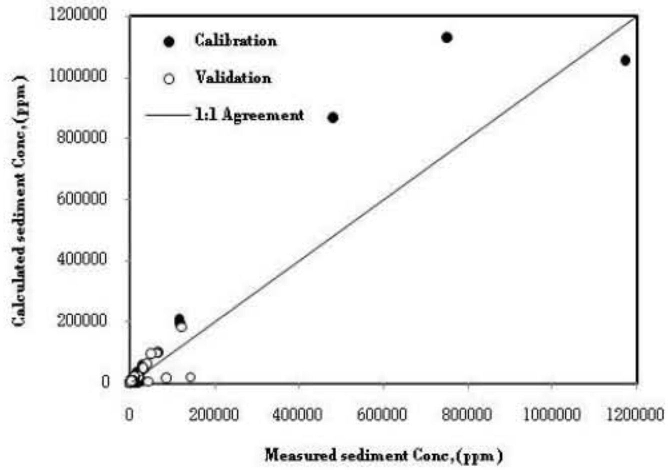

Fig. 6: Measured vs. predicted sediment concentration for calibration and validation model runs on Ahwaz station

Table 1: Summary statistics of measured and calculated daily runoff rates on Ahwaz station

	Calibration Run		Validation Run	
	Measured	Calculated	Measured	Calculated
Number of values	120	245		
Average(m3/sec)	562	543	668	629
Maximum	2200	2604	2273	2351
Minimum	214	160	77	60
Sum	67401	65215	163745	154008
Standard deviation	403.5	428.1	420.8	410.7
NS coefficient	0.18	0.25		

Table 2:. Measured and calculated values of suspended load concentration on Ahwaz station

	Calibration Run		Validation Run	
	Measured	Calculated	Measured	Calculated
Number of values	93		20	
Average(ton/day)	35486	45583	34083	33588
Maximum	1170556	1127771	140588	185131
Minimum	1260	204	1054	4506
Sum	3300187	4239198	671154	81656
Standard deviation	150676.7	181781.4	40966.5	4654.8
NS coefficient	0.15		0.09	

According to Tables 1 and 2, NS coefficient for flow and sediment concentration was 0.18 and 0.15, respectively. According to Table 1, both calibration and validation phases produced daily runoff rates, did not have considerable different from measured rates. Calculated values had comparable ranges to those observed. Thus, they are reflected in reasonable NS coefficient and have scattered firmly about 1:1 agreement line (Figure 5). According to Figure 4, model outputs have undulation about rising and falling limb of hydrograph. It can because of variation of flow resistance due to manning coefficient variation. But model output at normal flow condition, has consistent agreement with measured data. According to Table 1, the amount of NS coefficient in validation phase is more than calibration phase.

The calibration run produced daily suspended load concentration that did not have considerably different from observed data (Table 2). The validation run produced results that had significantly different from observed data (Table 2). Predicted values had comparable ranges to those observed but have significant scatter about the 1:1 agreement line (Figure 6) and thus led to lower NS coefficient (Table 2). Although model efficiency is relatively low, these results are typical of sediment transport models [16,17]. According to Tables 1 and 2, the model accuracy in flow prediction is superior to sediment concentration one.

CONCLUSION

The model presented here, provides a combination of mass and momentum conservation laws and diffusion equation which lead to Hydrodynamic-Suspended load transport equations. It can simulate flow and sediment distribution in sandy rives. Although model accuracy in high flow conditions reduces, but according to the amount of NS coefficient, it can be used to evaluate mean value of flow and sediment properties.

Although more testing and calibration of the model be necessary in order to improve its predictive ability, it is expected that the coupled hydrodynamic-sediment transport model would become a valuable predictive tool to river engineers and those who are interested in predicting sediment routing. It is expected that this model can be used as a preliminary design tool for river engineer projects and can facilitate quick computer runs for long steam reaches, particularly when long term simulations of long term effects are needed in a relatively short period.

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