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Variation of Parameters Method Using Laplace Transformation

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Abstract: This paper is related to the study of elegant coupling of variation of parameters method (VPM) and Laplace transformation to solve partial differential equations related to foam drainage. The proposed scheme is fully capable of handling such problems. Numerical results clearly reveal the reliability of this elegant coupling.

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Key words: Variation of parameters method · Partial differential equations · Foam drainage equations · Laplace transformation

INTRODUCTION

The rapid development of nonlinear sciences witnesses a wide range of new analytical and numerical techniques including decomposition, homotopy perturbation, homotopy analysis, polynomial spline, sink Glarkin, B-spline, perturbation, differential transform, exp-function and variational iteration to cope with the nonlinearities arising in physical problems, see [1-15] and the references therein. Most of these used schemes are coupled with the inbuilt deficiencies like calculation of the so-called Adomian's polynomials, linearization, perturbation, limited convergence and non compatibility with the physical nature of the problems. Moreover, these techniques involve very lengthy calculations coupled with a complicated computational procedure. In the similar context there was a dire need to develop an appropriate reliable, efficient and simple technique which could be fully compatible with the physical nature of the nonlinear problems without compromising their basic physics coupled with the suitable level of accuracy. Ma *et al.* [6-9] used an exceptionally simple but very accurate technique which is called the variation of parameters method (VPM) for solving involved non-homogeneous partial differential equations and obtained solution formulas helpful in constructing the existing solutions coupled with a number of other new solutions including rational solutions, solitons, positions, negatons, breathers, complextions and interaction solutions of the KdV equations. It has been observed that VPM [6-9, 11-14] is much better as compare to the above mentioned algorithms. Firstly, it does not require the small parameter assumption which is a major draw back in the traditional perturbation methods. No discretization or linearization is required and hence the scenario of getting some ill posed problems is avoided successfully. The VPM does not require the calculation of so-called Adomian's polynomials and hence is a better option as compare to decomposition method. Moreover, variation of parameters method (VPM) is brief, concise and more generalized than the above mentioned technique and does not even require any unrealistic assumptions which ruin the basic physical structure of the nonlinear problems. The basic inspiration of the present paper is the elegant coupling of VPM and Laplace transformation. It is observed that the proposed combination is very efficient and easier to implement. The suggested coupling is applied without any discretization, perturbation, linearization and is independent of the complexities arising in the calculation of the so-called Adomian's polynomials. The proposed technique has been tested on two partial differential equations which are related with foam drainage and arise frequently in physics, fluid mechanics and nonlinear sciences. Numerical results explicitly reveal the complete reliability of the suggested scheme.

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Variation of Parameters Method (VPM): Consider the following second-order partial differential equation

$$y_{tt} = f(t, x, y, z, y_x, y_y, y_z, y_{xx}, y_{yy}, y_{zz}), \qquad (1)$$

Where:
t such that $(-\infty < t < \infty)$ is time and f is linear or non linear function of $y, y_x, y_y, y_z, y_{xx}, y_{yy}, y_{zz}$

The homogeneous solution of (1) is

$$y(t,x,y,z) = A + Bt,$$

Where:
A and B are functions of x, y, z and t. Using Variation of parameters method we have following system of equations

$$\frac{\partial A}{\partial t} + \frac{\partial B}{\partial t} = 0, \qquad \frac{\partial B}{\partial t} = f,$$

and hence

$$A(x,y,z,t) = D(x,y,z) - \int_0^t sf ds, \qquad B(x,y,z,t) = C(x,y,z) - \int_0^t f ds,$$

therefore,

$$y(x,y,z,t) = y(x,y,z,0) + ty_t(x,y,z,0) + \int_0^t (t-s) f(s,x,y,z,y_x,y_y,y_z,y_{xx},y_{yy},y_{zz}) ds,$$

which can be solved iteratively as [6-9, 11-14]

$$y^{k+1}(x,y,z,t) = y(x,y,z,0) + ty_t(x,y,z,0) + \int_0^t (t-s) f(s,x,y,z,y^k,y^k_x,y^k_y,y^k_z,y^k_{xx},y^k_{yy},y^k_{zz}) ds,$$

$k = 0,1,2,....$

Example 3.1 Consider the following foam drainage equation

$$u_t + 2u^2 u_x - u_x^2 - \frac{1}{2} u_{xx} u = 0,$$

with initial conditions

$$u(x,0) = -\sqrt{c} \tan h\left(\sqrt{c}\, x\right).$$

Applying Laplace transformation

$$u(x,s) = \frac{-\sqrt{c} \tan h\left(\sqrt{c}\, x\right)}{s} - \frac{1}{s} L\left(2u^2 u_x - u_x^2 - \frac{1}{2} u_{xx} u\right).$$

Applying inverse Laplace transformation

$$u(x,t) = -\sqrt{c} \tan h\left(\sqrt{c}\, x\right) - L^{-1}\left(\frac{1}{s} L\left(2u^2 u_x - u_x^2 - \frac{1}{2} u_{xx} u\right)\right).$$

Applying variation of parameters method (VPM), we get

$$u_{n+1}(x,t) = -\sqrt{c} \tanh\left(\sqrt{c}\, x\right) - \int_0^t L^{-1}\left(\frac{1}{s} L\left(2u_n^2 (u_n)_x - \left(\frac{\partial u_n}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial^2 u_n}{\partial x^2}\right) u_n\right)\right) dt.$$

Following approximants are obtained

$$u_0(x,t) = -\sqrt{c} \tan h\left(\sqrt{c}\, x\right),$$
$$u_1(x,t) = t \sec h^2 x,$$
$$u_2(x,t) = t^2 \sec h^2 x \tanh x,$$
$$u_3(x,t) = -\frac{2}{3} t^3 \sec h^4 x + \frac{1}{3} t^3 \cos h 2x \sec h^4 x,$$
$$u_4(x,t) = \frac{1}{12} t^4 \sec h^5 x \sinh 3x - \frac{11}{12} t^4 \sec^4 x \tanh x,$$
$$\vdots$$

The series solution is given by

$$u(x,t) = -\sqrt{c} \tanh\left(\sqrt{c}\, x\right) + t \sec h^2 x + t^2 \sec h^2 x \tanh x - \frac{2}{3} t^3 \sec h^4 x + \frac{1}{3} t^3 \cos h 2x \sec h^4 x + \frac{1}{12} t^4 \sec h^5 x \sinh 3x - \frac{11}{12} t^4 \sec^4 x \tanh x + \cdots.$$

Table 1: shows the compatibility of proposed algorithm (LVPM) and the decomposition method (ADM)

x	LVPM	ADM
0.0	0.666667	0.666667
0.2	0.551378	0.551378
0.4	0.469241	0.469241
0.6	0.376361	0.376361
0.8	0.17331	0.17331
1.0	-0.54328	-0.54328
2.0	-142.488	-142.488

Table 2: shows the compatibility of proposed algorithm (LVPM) and the decomposition method (ADM)

x	LVPM	ADM
0.0	-0.108989	-0.108989
0.2	-0.155444	-0.155444
0.4	-0.199108	-0.199108
0.6	-0.23944	-0.23944
0.8	-0.276098	-0.276098
1.0	-0.308932	-0.308932
2.0	-0.420061	-0.420061

Example 3.2 Consider the following foam drainage equation

$$u_t + 2u^2 u_x - u_x^2 - \frac{1}{2} u_{xx} u = 0,$$

with initial conditions

$$u(x, 0) = -\frac{1}{2} + \frac{1}{1+e^x}.$$

Applying Laplace transformation

$$u(x, s) = \frac{1}{s}\left(-\frac{1}{2} + \frac{1}{1+e^x}\right) - \frac{1}{s} L\left(2u^2 u_x - u_x^2 - \frac{1}{2} u_{xx} u\right).$$

Applying inverse Laplace transformation

$$u(x, t) = \left(-\frac{1}{2} + \frac{1}{1+e^x}\right) - L^{-1}\left(\frac{1}{s} L\left(2u^2 u_x - u_x^2 - \frac{1}{2} u_{xx} u\right)\right).$$

Applying variation of parameters method (VPM)

$$u_{n+1}(x, t) = \left(-\frac{1}{2} + \frac{1}{1+e^x}\right) - \int_0^t L^{-1}\left(\frac{1}{s} L\left(2u_n^2 (u_n)_x - \left(\frac{\partial u_n}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial^2 u_n}{\partial x^2}\right) u_n\right)\right).$$

Following approximants are obtained

$$u_0(x, t) = \left(-\frac{1}{2} + \frac{1}{1+e^x}\right),$$

$$u_1(x,t) = \frac{4e^x t}{(1+2e^x)^4} + 8\frac{e^{2x} t}{(1+2e^x)^4} - \frac{5e^x t}{(1+2e^x)^3} - \frac{2e^{2x} t}{(1+2e^x)^3} + \frac{3e^x t}{2(1+2e^x)^2} - \frac{e^x t^2}{16(1+2e^x)^3} + \frac{e^{2x} t}{8(1+2e^x)^3},$$

$\vdots$

The series solution is given by

CONCLUSION

In this paper, we developed and implemented the elegant coupling of variation of parameters method (VPM) and Laplace transformation for solving foam drainage equations. It is observed that proposed algorithm is more user friendly and is easier to implement as compare to the traditional decomposition method.

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