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Implementation of FFT by Using Discrete Wavelet Packet Transform (DWPT)

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Abstract: In this paper, a Discrete Wavelet Packet Transform (DWPT) based Fast Fourier transform (FFT) is proposed for OFDM system. The system uses the channel estimation in receiver after the demodulation. A mathematical derivation of the new DWPT (FFT)-OFDM system is also presented. The performance comparison between of the new DWPT(FFT)-OFDM system by having a pilot carrier operation now is used, Discrete Fourier Transform (DFT)-OFDM Modulation and DWPT-OFDM Modulation. Against the FFT-OFDM system over different channels is obtained. From through this paper will get a low Bit Error Rate (BER) compared with the high Signal to Noise Ratio (SNR) at the same point. Simulation results show that the DWPT (FFT)-OFDM system produces better BER performance than FFT-OFDM system in AWGN channel and Multipath fading channels.

Key words: FFT · OFDM · Discrete Wavelet Packet Transform

INTRODUCTION

The wavelet packet modulation (WPM) has been proposed as one of the multicarrier transmission methods using discrete wavelet transform (DWT). The wavelet transform can analyze a signal not only in the frequency domain but also in the time domain, enabling a wide use in the field of signal analysis. The Frequency Domain Equalizer (FDE) can be applied to WPM with a guard interval like OFDM and it was shown that a high-quality mobile communication is expected using WPM because the performance of equalization was better than OFDM [1].

The idea of using wavelet transform instead of Fourier transform, because of its high spectral containment properties compared with Fourier transform, was proposed by a number of researchers [2-4]. The vast majority of these papers use the term 'wavelet orthogonal frequency division multiplexing' to describe wavelet-based multicarrier systems. Furthermore and to the best of our knowledge, all previous research has been performed using discrete wavelet transform (DWT), sometimes also referred to as the discrete-time wavelet transform (DTWT). In [4] they presented a mathematical framework for wavelet based multicarrier systems using truly discrete wavelet transform using Mallat's algorithm [5] and they derived a formula to represent convolution's counterpart in wavelet domain. In addition, as the DWT does not have a fixed definition, no work has been done on the choice and type of wavelet filters or on the effects of different decomposition levels. The work in [6] gives the performance comparison of conventional Discrete Fourier Transform (DFT) with Discrete Wavelet Packet Transform (DWPT) in an OFDM transceiver.

In another work, an algorithm that uses DWT as a tool to compute the discrete Fourier transform (DFT) is presented in [7]. In [8] presents a wavelet packet based FFT using the butterfly operation and its application to SNR estimation based on [7] for OFDM system. In this paper, a Discrete Wavelet packet based FFT for OFDM system is presented. However, a pilot carrier operation now is used instead of a butterfly operation. Thus, this results the new OFDM system to offer the same good time and frequency localization as compared to the work in [7] and [8].

The rest of the paper is organized as follows; section 2 gives the DWPT (FFT)-OFDM Modulation, section 3 present the Mathematical of DWPT-OFDM, section 4 present the Computational Complexity and section 5, 6 presents the simulation results and Conclusions respectively.

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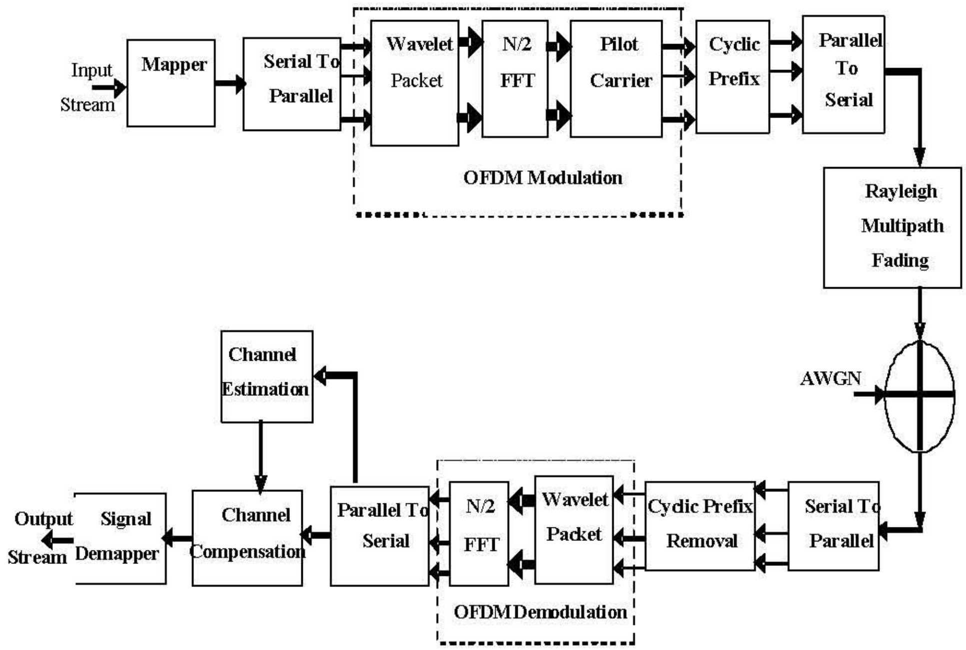

Fig. 1: Block diagram of DWPT-OFDM Modulation

In this paper, we proposed the Discrete Wavelet Packet Transform (DWPT) based Fast Fourier transform (FFT) for OFDM system.

DWPT (FFT)-OFDM Modulation: The design of the proposed WPT (FFT) first to perform the Wavelet Packet decomposition, followed by reduced size FFT and Pilot Carrier operation as shown in Figure 1.

Pilot Carrier is appropriately designed with the number of sub-carrier and the analysis level for WP. The DWPT (FFT) is represented by equation (1).

The structure of the DWT based FFT algorithm can be exploited to generalize the classical pruning idea for arbitrary signals. From the input data side, the signals are made sparse by the wavelet transform thus approximation can be made to speed up the algorithm by dropping the insignificant data. Although the input signals are normally not sparse, DWPT creates the sparse inputs for the Pilot Carrier operations stages of the FFT. OFDM training/synchronization data of length 'N' is sent from the transmitter (T_x). To avoid intersymbol interference (ISI) cyclic prefix (CP) is added. After removing cyclic prefix at receiver, OFDM data is divided into 2^n sub-bands using periodic wavelet packets where 'n' shows the number of levels. The length of each sub-band is $N_{sub}=N/2^n$. It inherits the two identical halves property of synchronization preamble.

$$F_N = \begin{bmatrix} A_{N/2} & B_{N/2} \\ C_{N/2} & D_{N/2} \end{bmatrix} \left[FFT\right]_{N/2} \left[PC_N\right] \tag{1}$$

The F_N is the DFT matrix which is the ordinary matrix used in the ordinary FFT-OFDM so that the system will use the property of DFT as presented in [8].

Wavelet packet allows a finer and adjustable resolution of frequencies at high frequency. Input data are first filtered by pair of filters h and g (low pass and high pass respectively) and then down sampled. The same analysis is further iterated on both low and high frequency bands. Where we image the real part of DFT matrices and the magnitude of the matrices for Pilot Carrier operations and the one-scale DWPT using Haar wavelets.

Mathematical of DWPT-OFDM: Suppose the data are transmitted by blocks of size N: $s(k) = [s_1(k), \ldots, s_i(k), \ldots, s_N(k)]$ where index k is the OFDM block symbol number and the subscript i is the carrier index. The block OFDM symbol is precoded by an inverse FFT matrix $F_N^H = F_N^{-1}$ that produces the so-called time domain block vector $x(k) = [x_1(k), \ldots, x_i(k), \ldots, x_N(k)]$.

$$\begin{bmatrix} x_1(k) \\ \cdots \\ x_N(k) \end{bmatrix}_{Nx1} = F_N^H \begin{bmatrix} s_1(k) \\ \vdots \\ \vdots \\ s_N(k) \end{bmatrix}_{Nx1} \tag{2}$$

The decimation process continues until the desired number of samples, which can be used to calculate the DFT easily. There are many radices used in the decimation process. Among the numerous FFT algorithms, the radix-2 decimation in time (DIT) and decimation in frequency (DIF) algorithms are the most fundamental methods. In the radix-2 algorithm, the length of the data sequence, {x (n)}, $n = 0, 1... N-1$, is chosen to be a power of 2, i.e., $N = 2^p$, where p is a positive integer. Lets define two $(N/2)$-point sub-sequences $x_1(n)$ and $x_2(n)$ as even and odd index values of x(n), i.e.,

$$x_1(n)=x(2n), \quad n=0,1,......,\frac{N}{2}-1 \tag{3}$$

$$x_2(n)=x(2n+1), \quad n=0,1,......,\frac{N}{2}-1 \tag{4}$$

The summation is then split into two parts i.e. the even and odd n.

$$X(k)=\sum_{n}^{N-1} x(n)W_N^{kn} = \sum_{n=0}^{(N/2)-1} x(2n)W_N^{2kn} + \sum_{n=0}^{(N/2)-1} x(2n+1)W_N^{k(2n+1)} \tag{5}$$

As

$$W_N^2=[e^{j(2\pi/n)}]^2 = e^{j(2\pi/(n/2))}=W_{N/2} \tag{6}$$

Equation (5) now can be simplified as:

$$X(k)=\sum_{n=0}^{(N/2)-1} x_1(n)W_{N/2}^{kn} + W_N^k \sum_{n=0}^{(N/2)-1} x_2(n)W_{N/2}^{kn} \tag{7}$$

Or

$$X(k)=X_1(k)+W_N^k X_2(k) \tag{8}$$

Here the terms $X_1(k)$ and $X_2(k)$ are the $(N/2)$-point DFT of $x_1(n)$ and $x_2(n)$ respectively. Thus, the $X(k)$ N-point DFT can be decomposed into two (N/2)-point DFT of $X_1(k)$ and $X_2(k)$ for $0 \le k \le (N/2) -1$.

If the $(N/2)$-point DFT is calculated directly, each (N/2)-point DFT requires $(N/2)^2$ complex multiplications, in addition to (N/2) complex multiplications with W^k_N. Thus, the total number of complex multiplications required for computing X (k) is given as;

$$2(N/2)^2 + (N/2 = (N^2/2) + (N/2) \tag{9}$$

This results in the reduction of the number of complex multiplication from N^2 to $(N^2/2)+(N/2)$. The insight to this problem can be obtained by using matrix analysis. Let F_N be the $N\times N$ DFT matrix, i.e. $FN(m,n) = e^{-j2\pi mn/N}$ where $m,n \in \{0,1,......, N-1\}$. Let S_N be the $N\times N$ even-odd Separation matrix, e.g.

$$S_4=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

We use the matrix F_N is the DFT matrix as given $F_N(m,n)$ above (6), the same as OFDM, because we use the property DFT in the formation of DWPT.

Then, the DIT FFT is based on the following matrix factorization,

$$F_N=\begin{bmatrix} I_{N/2} & T_{N/2} \\ I_{N/2} & -T_{N/2} \end{bmatrix}\begin{bmatrix} F_{N/2} & 0 \\ 0 & F_{N/2} \end{bmatrix} S_N \tag{11}$$

Where:

$T_{N/2}$ is a diagonal matrix with $W^i_N, i\in\{0,1,.....,N/2-1\}$ on the diagonal.

The operation can also be expressed in matrix form W_N, e.g. for Haar wavelet,

$$W_4^{Haar}=\frac{\sqrt{2}}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \tag{12}$$

The orthogonality conditions on h and g ensure $W'_N W_N = I_N$. The matrix for multiscale DWT is formed by W_N for different N. We could further iterate the building block on some of the high pass outputs. This generalization is called the wavelet packets. The key to the FFT is the factorization of F_N into several sparse matrices and one of the sparse matrices represents two DFTs of half the length. Similar to the DIT FFT, the following matrix factorization has been found,

$$\begin{bmatrix} A_{N/2} & B_{N/2} \\ C_{N/2} & D_{N/2} \end{bmatrix}=\begin{bmatrix} h(0) & g(0) \\ h(1) & g(1) \end{bmatrix} \tag{13}$$

Where:

$A_{N/2}$, $B_{N/2}$, $C_{N/2}$ and $D_{N/2}$ are all diagonal matrices. The values on the diagonal of $A_{N/2}$ and $C_{N/2}$ are the length-N DFT of h and the values on the diagonal of $B_{N/2}$ and $D_{N/2}$ are the length-N DFT of g. Now the twiddle factors are length-N DFT of h and g. The classical radix-2 DIT FFT is a special case of the above algorithm when h = [1,0] and g = [0, 1]. This can be seen using the polyphase. As for the PC structure is the pilot signal enables channel estimation at the receiver for coherent demodulation of the data. In OFDM systems, the pilot structure consists of the frequency and time placement of pilot symbols in the overall transmission waveform. The sampling frequency provided by the pilot subcarriers needs to be adequate to capture frequency domain variations of the channel and therefore is related to the channel coherence bandwidth. In the PC, we add the same number of bits for transmitter Data $W_{N,i}^{Pilot}$ and this PC is known in the receiver. By doing so, can we write equation (1) as follows:

$$F_N = \begin{bmatrix} A_{N/2} & B_{N/2} \\ C_{N/2} & D_{N/2} \end{bmatrix} [FFT]_{N/2} \left[W_{N,i}^{Pilot}\right] \tag{14}$$

Computational Complexity: The computational complexity of the FFT algorithm can be easily established. Let $C_{FFT}(N)$ be the complexity for a length-N FFT [8]. We can show.

$$C_{FFT}(N) = O(N) + 2C_{FFT}(N/2) \tag{15}$$

Where:
$O(N)$ denotes linear complexity,
The Eq. (15) is well known,

$$C_{FFT}(N) = O(N\log_2 N) \tag{16}$$

This is a classical case where the divide and conquer approach results in very effective solution. The matrix point of view gives us additional insight.

While, the computational complexity of the DWT algorithm can also be easily established.

Let $C_{DWT}(N)$ be the complexity for a length-N DWT. Since after each scale, we only further operate on half of the output data [8], we can show.

$$C_{DWT}(N) = O(N) + C_{DWT}(N/2) \tag{17}$$

Which give rise to the solution

$$C_{DWT}(N) = O(N) \tag{18}$$

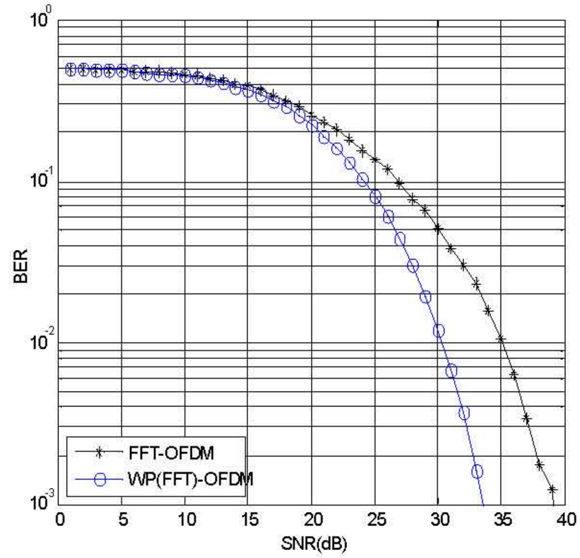

Fig. 2: BER performance for models in AWGN channel

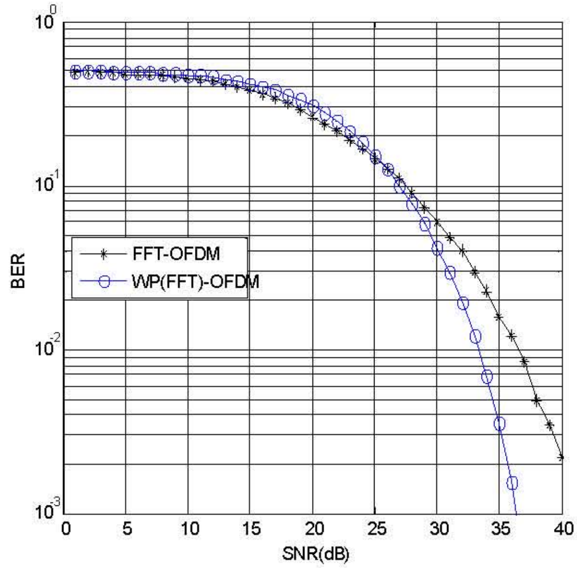

Fig. 3: BER performance for models in Flat fading channel

For the DWT based FFT algorithm, the computational complexity is also O(N $\log_2$N) as in [8]. Since the recursive relation in Equation 15 is again satisfied. However, the constant appears before N $\log_2$N depends on the wavelet filters used.

The wavelet coefficients are mostly sparse, so the inputs of the shorter DFTs are sparse.

Simulation Results: This section represents the results of simulating a conventional DWPT (FFT)- OFDM Modulation and comparing its performance with FFT-OFDM Modulatio. The performance results for such

Table 1: Simulation Parameters

	FFT-OFDM	DWPT (FFT)-OFDM
Modulation	QAM	QAM
Sampling Freq.	20 MHz	20 MHz
Doppler Freq.	5 Hz	5 Hz
Bit Rate	5 Mbps	5 Mbps
IFFT size	64	-
No. of WP points	-	64
Channel model	AWGN	AWGN
Flat fading + AWGN	Flat fading + AWGN	

system in three types of channels are obtained using the OFDM parameters listed in Table 1.

It is observed through simulations that DWPT (FFT)-OFDM outperform by an SNR margin of 30 dB, for the same BER of 10^{-2} on AWGN channel, as shown in Fig. 2. However in a flat fading multipath channel BER performance of FFT-OFDM deteriorate. For a BER of 10^{-2}wavelet packet transform based transceiver gives a gain of 33 dB over a wavelet based OFDM, as shown in Fig. 3.

From these results the DWPT (FFT)-OFDM system always out performs FFT-OFDM system. This is due to the fact the FFT-based OFDM system uses a rectangular pulse shaping technique for sinusoidal carrier that exhibits high side lobes. The high side-lobes in the transmitted signal increases OFDM system's sensitivity to ICI and introduce a Narrow-Band Interference (NBI).

WPM-based OFDM system provides better spectral shaping than FFT-based OFDM scheme. It offers much lower side lobes in the transmitted signal, which reduces ICI and NBI. In FFT-based OFDM, a large number of Fourier components are required to represent a sharp corner of the modulated signals, while in WPM-based OFDM system there are many different wavelets that can be used to represent the sharp corner.

CONCLUSION

In this paper, a wavelet packet based FFT using a pilot carrier operation and a channel estimation/compensation technique for OFDM system is proposed. Its performance has been evaluated via computer simulations using MatLab with various channels AWGN and flat fading channel. It can be seen that the proposed DWPT (FFT). OFDM system produces better BER performance than FFT-OFDM. The proposed DWPT to replace FFT in OFDM exploits a special property of wavelet transforms. In this design of transceiver, the analysis and synthesis filters are completely canceled the filter distortions and signal aliasing. This produces perfect reconstruction of the input signals with perfect extraction of the multiplexed inputs. The DWPT (FFT) algorithm, the computational complexity is also O(Nlog2N). However, the constant appears before Nlog2N depends on the wavelet filters used. The proposed DWPT-FFT computes the exact result. The efficient DWPT (FFT) based OFDM modulation design proposed here exploits a special property of wavelet transforms that is the analysis and synthesis filters together completely cancel the filter distortions and signal aliasing. However, equalization is required to improve the bit error rate performance in fading channels for FFT-OFDM.

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