World Applied Sciences Journal 8 (9): 1136-1143, 2010
ISSN 1818-4952

Real Time Study of a k-out-of-n System: n Identical Elements with Constant Fuzzy Failure Rates

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Abstract: Reliability models based on Markov chain (Except in queuing systems) have extensive applications in electrical and electronic devices. In this paper we consider a system with n parallel and identical elements with constant failure rates (failure rates are exponentially distributed) and the elements are non repairable. The failure rates increase when some elements are failed. The system works until at least k elements work. Since the data for the failure rates are either based on historical data or the judgment of the experts, therefore uncertainty is inherent in the information. These uncertainties can be expressed as fuzzy rates. For simplicity we consider a triangular fuzzy number either to quantify a linguistic expression or if estimated quantitative data is available, this fuzzy numbers can be calculated by point estimation and % (1-β) confidence interval of the parameters of the failure rates. The system of equations are established and the exact equations are sought for the parameters like MTTF and the probability that system working at the time t. A numerical example has been solved to demonstrate the procedure which clarifies the theoretical development. Use of fuzzy parameters to model the uncertainties in the system causes this model to tackle more realistic situations.

Key words: Reliability . Markov chain . k-out-of-n models . exponential distribution . fuzzy failure rate

INTRODUCTION

k-out-of-n models, are one of the most useful models to calculate the reliability of electrical and electronically devices and systems. In the literature there are many studies in this area. We try to categorically classify them. At the first glance, they may be classified into two namely steady state and real time distinct classes. The elements in both classes may be repairable or non repairable. The failure rates of the elements can be considered constant, increasing or decreasing whereas the repair rate is constant. The Components may be parallel, series or combination of the two. Most of studies in this field are in steady state and a few studies are in real time conditions.

Gera [1] uses a matrix formulation and solution for these systems. Lam, Keong and Tony [2] provide a general model for consecutive k-out-of-n: F repairable system with exponential distribution and (k-1) step Markov dependence. Sarhan, Abouammoh [3] work on nonrepairable system with no independent components subject to common shocks. Arumozhi [4] calculates the exact equation and suggests an algorithm for reliability evaluation of k-out-of-n systems. Koucky [5] computes exact reliability formula and bounds for general systems. Flynn and Chung [6] use a heuristic algorithm for determining replacement policies in consecutive k-out-of-n systems. Guan and Wu [7] work on a repairable consecutive system with fuzzy states. Li, Zho and Yam [8] determine a system with some components being suspended when the system is down. Sharifi and Moosakhani [9] establish a system containing two elements with constant and increasable fuzzy failure rates in real time situations.

In this paper we work on a system with n parallel and identical elements with fuzzy failure rates for real time conditions. The system works until more than (n-k) elements fail (less than k elements work). The paper is divided into five parts. The second part explores the models. In the third part, the calculations of the model with fuzzy failure rates are carried out. Numerical examples are presented in the fourth part and the final section deals with the conclusion.

MODELING

Assume a system with n parallel and identical elements. The system works until at least k elements

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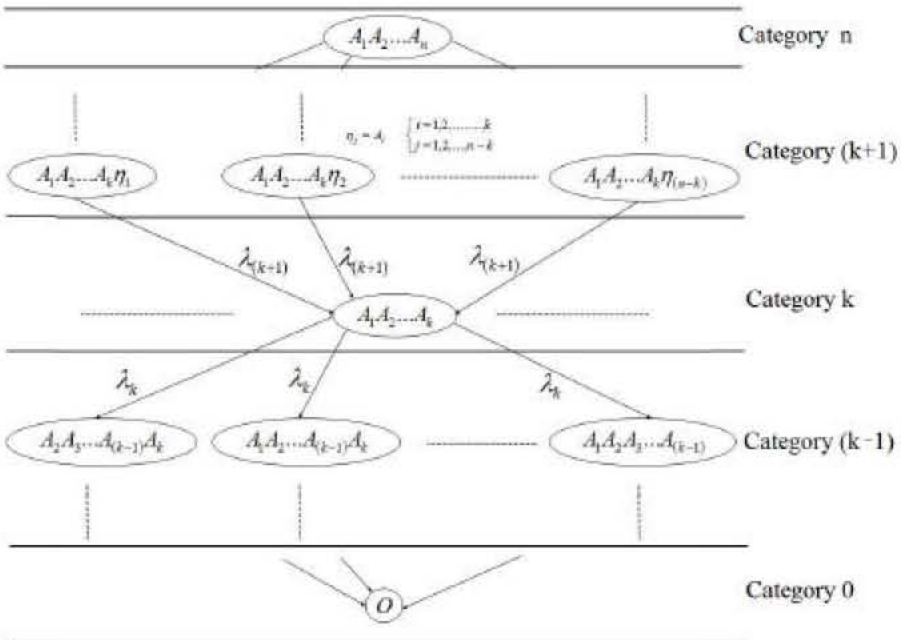

Fig. 1: The model of three-wheel robot

fail. Therefore each element has two states and consequently the system will have 2^n states. Let $A_1A_2..A_k$ be the state where the elements $A_1, A_2,...$ and A_k are working and other (n-k) elements are failed. Also $A_1A_2..A_k\eta_j$ is the state that the element η_j works in addition to other k elements. The state O indicates that all the elements are failed. The state structure of the system is shown in Fig. 1.

In this figure, the category i, i = 0,1,2,..., n indicates that the states in this category are with i elements working and (n-i) elements are failed. Each state is closely related to the states in the antecedent and precedent category. i.e., if an element is failed in any state, then the state is transferred to the next category. In other words, if the system is in any states in category k, with the failure of one element, the state will be in category (k-1).

When some elements are failed, other elements work with more load and therefore the failure rate is increased for the remaining elements. Then we have $\lambda_n<\lambda_{(n-1)}<...<\lambda_1$. The system works if at least k elements work. Therefore we have:

$$R_p(t) = \sum_{\substack{i=k \\ 1\le A_1<A_2<..\le A_i\le n}}^{n} P_{A_1A_2..A_i}(t) \qquad (1)$$

We know that:

$$\sum_{\substack{i=1 \\ 1\le A_1<A_2<..\le A_i\le n}}^{k-1} P_{A_1A_2..A_i}(t) + \sum_{\substack{i=k \\ 1\le A_1<A_2<...<A_i\le n}}^{n} P_{A_1A_2..A_i}(t) = 1 \qquad (2)$$

The first part of equation (02) is related to states with less than k elements working and the second part deals with two states with at least k elements are working. In order to find $R_p(t)$ from equation (02), we must calculate $P_i(t)$ for each states. From the state $A_1A_2..A_n$ through O in Fig. 1 we have:

$$P_{A_1A_2..A_k}(t+\Delta t) = P_{A_1A_2..A_k}(t) - \sum_{i=1}^{k}\lambda_k \times \Delta t \times P_{A_1A_2...A_k}(t) + \sum_{\substack{j=1 \\ \eta_j\ne A_i \quad i=1,2,...,k}}^{n} \lambda_{(k+1)} \times \Delta t \times P_{A_1A_2..A_k\eta_j}(t) \qquad (3)$$

$P_{A_1A_2..A_k}(t)$ is the probability that the system be in the state $A_1A_2..A_k$ at time t. Also

$$\sum_{\substack{j=1 \\ \eta_j\ne A_i \quad i=1,2,...,k}}^{n} \lambda_{(k+1)} \times \Delta t \times P_{A_1A_2..A_k\eta_j}(t)$$

is the rate of transfer from category (k+1) to this state and

$$\sum_{i=1}^{k}\lambda_k \times \Delta t \times P_{A_1A_2..A_k}(t)$$

is the rate of transfer from this state to the states of category (k-1) at time Δt. By solving equation (03) (the solution provided in Appendix 1) we can calculate the values of $P_i(t)$ as follows:

$$\begin{cases} P_1(t) = e^{-n\times\lambda_n\times t} & k = n \\ P_{A_1A_2\ldots A_k}(t) = (n-k)!\times\left(\prod_{v=k+1}^{n}\lambda_v\right)\times & \\ \sum_{j=k}^{n}\left[\left(\prod_{\substack{\theta=k\\ \theta\neq j}}^{n}\frac{1}{\theta\times\lambda_\theta - j\times\lambda_j}\right)\times e^{-j\times\lambda_j\times t}\right] & k < n \end{cases} \quad (4)$$

and

$$\begin{cases} R_p(t) = e^{-n\times\lambda_n\times t} & k = n \\ R_p(t) = \sum_{\substack{i=k\\ 1\leq A_1<A_2<\ldots<A_i\leq n}}^{n} P_{A_1A_2\ldots A_i}(t) = \left(\prod_{j=k}^{n}\lambda_j\right)\times & \\ \sum_{i=k}^{n}\left[\frac{n!}{i}\left(\prod_{\substack{\theta=k\\ \theta\neq i}}^{n}\frac{1}{\theta\times\lambda_\theta - i\times\lambda_i}\right)\times\frac{e^{-i\times\lambda_i\times t}}{\lambda_i}\right] & k < n \end{cases} \quad (5)$$

The MTTF of the system is also calculated as follows:

$$MTTF = \int_{t=0}^{+\infty} P(t)\,dt = \begin{cases} \frac{1}{n\times\lambda_n} & k = n \\ \left(\prod_{j=k}^{n}\lambda_j\right)\times\sum_{i=k}^{n}\left[\frac{n!}{i^2\times\lambda_i^2}\left(\prod_{\substack{\theta=k\\ \theta\neq i}}^{n}\frac{1}{\theta\times\lambda_\theta - i\times\lambda_i}\right)\right] & k < n \end{cases} \quad (6)$$

When a component fails in each category, the other component must work harder and the failure rate of these components increase. We can calculate the failure rate of each category as follows:

$$\lambda_k = \frac{n}{n-\gamma(n-k)}\lambda_n \quad (7)$$

where $0\leq\gamma\leq 1$. If $\gamma = 0$, then the failure rates are equal and constant and if $\gamma = 1$ then

$$\lambda_k = \frac{n}{k}\lambda_n$$

CALCULATIONS WITH FUZZY FAILURE RATES

Most important consideration is that the values of λ_n are not fixed. Since they are driven from collected data or the opinions of the experts, uncertainty of the value is an undeniable fact. For example, based on an independent sample, the intervals between consequent failures are measured and the result is {35, 235, 105, 140, 125}. Then λ_n = 5/(35+235+105+140+125) = 0.0078125. But if we have a new observation of failure like 26 hour, then λ_n = 6/(35+235+105+140+125+26) = $0.00\overline{900}$ which is very different from 0.0078125. We know that calculation of exact value of λ_n, depends on a large number of observations which is expensive to collect. So we can model this uncertainty of the value of failure rates as fuzzy numbers. The failure rates are constant but not fixed. So the intervals between two consequent failures are exponentially distributed. In this condition, λ_n is considered in the form of a triangular fuzzy number as follows:

$$\tilde{\lambda}_n = (L, M, U) \quad (8)$$

where L is the lower limit of %(1-β) confidence interval, M is the point estimation and U is the upper limit of %(1-β) confidence interval of λ_n [10].

The α-cut of these failure rates can be calculated as follows:

$$\tilde{\lambda}_\alpha = [L + \alpha(M-L), U - \alpha(U-M)] \quad (9)$$

Since failure rates have the exponential p.d.f, we can calculate the point estimation and the %100(1-β) confidence interval of λ_n as follows:

$$M = \frac{1}{\overline{X}} \quad (10)$$

$$L = \frac{\chi^2_{(1-\beta/2),(2n)}}{2n\times\overline{X}} \quad (11)$$

$$U = \frac{\chi^2_{(\beta/2),(2n)}}{2n\times\overline{X}} \quad (12)$$

In equations (10), (11) and (12), n is number of independent sample, $\overline{X}$ is the expected value of the failures which is calculated by:

$$\overline{X} = \frac{1}{n}\sum_{j=1}^{n} X_i \quad (13)$$

and $\chi^2_{\theta,2n}$ is the area under the chi square distribution function and can be calculated as follows:

$$\chi^2_{\theta,2n} = \int_{\theta}^{+\infty} f_{\chi^2_{2n}}(t)\,dt \quad (14)$$

NUMERICAL EXAMPLE

In this Example, we consider a k-out-of-3 system. Assume that λ_3 is the failure rates of all elements in category 3. Let also $\gamma = 0.5$ and $\beta = 0.05$ and based on an independent sample, the intervals between consequent failures are measured and the result is {135, 605, 195, 160, 325} and the expected value of the failure time intervals is $\overline{X} = (135+605+195+160+325)/5 = 284$ hours. We know that $\lambda = 1/\overline{X} = 0.00352113$. The states of the system are shown in Fig. 2:

If k = 2, the $R_p(t)$ and MTTF in crisp condition is calculated as follows:

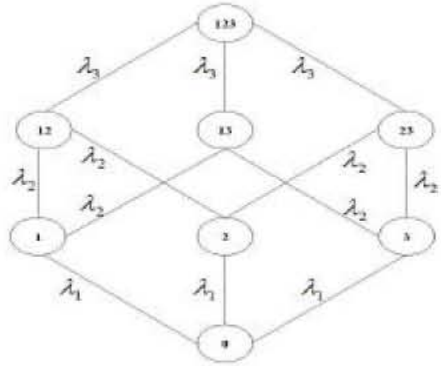

Fig. 2: The states of the example

$$R_p(t) = \sum_{\substack{i=2 \\ 1\le A_1 < A_2 < \dots < A_i \le n}}^{5} P_{A_1A_2\dots A_i}(t) = \left(\prod_{j=2}^{3}\lambda_j\right)\times\sum_{i=2}^{3}\left[\frac{3!}{i}\left(\prod_{\substack{\theta=2\\ \theta\ne i}}^{3}\frac{1}{\theta\times\lambda_\theta - i\times\lambda_i}\right)\times\frac{e^{-i\times\lambda_i\times t}}{\lambda_i}\right] =$$

$$(\lambda_2\times\lambda_3)\times\left[\frac{3!}{2}\left(\prod_{\substack{\theta=2\\ \theta\ne i}}^{3}\frac{1}{\theta\times\lambda_\theta - 2\times\lambda_2}\right)\times\frac{e^{-2\times\lambda_2\times t}}{\lambda_2} + \frac{3!}{3}\left(\prod_{\substack{\theta=2\\ \theta\ne i}}^{3}\frac{1}{\theta\times\lambda_\theta - 3\times\lambda_3}\right)\times\frac{e^{-3\times\lambda_3\times t}}{\lambda_3}\right] = \frac{3\lambda_3}{3\lambda_3 - 2\lambda_2}e^{-2\times\lambda_2\times t} + \frac{2\lambda_2}{2\lambda_2 - 3\lambda_3}e^{-3\times\lambda_3\times t} \tag{15}$$

$$MTTF = \int_{t=0}^{+\infty} P(t)dt = \left(\prod_{j=2}^{3}\lambda_j\right)\times\sum_{i=2}^{3}\left[\frac{n!}{i^2\times\lambda_i^2}\left(\prod_{\substack{\theta=2\\ \theta\ne i}}^{3}\frac{1}{\theta\times\lambda_\theta - i\times\lambda_i}\right)\right] =$$

$$(\lambda_2\times\lambda_3)\times\left[\frac{3!}{2^2\times\lambda_2^2}\times\left(\frac{1}{3\lambda_3 - 2\lambda_2}\right) + \frac{3!}{3^2\times\lambda_3^2}\times\left(\frac{1}{2\lambda_2 - 3\lambda_3}\right)\right] = \frac{3}{2}\times\frac{\lambda_3}{\lambda_2}\times\frac{1}{3\lambda_3 - 2\lambda_2} + \frac{2}{3}\times\frac{\lambda_2}{\lambda_3}\times\frac{1}{2\lambda_2 - 3\lambda_3} = \frac{1}{6}\times\frac{3\lambda_3 + 2\lambda_2}{\lambda_2\times\lambda_3} \tag{16}$$

The value of λ_2 is:

$$\lambda_2 = \frac{3}{3 - 0.5(3-2)}\lambda_3 = 1.2\lambda_3$$

then:

$$MTTF = \frac{1}{6}\times\frac{3\lambda_3 + 2\times 1.2\lambda_3}{1.2\lambda_3\times\lambda_3} = \frac{0.75}{\lambda_3} = \frac{0.75}{1/284} = 213 \tag{17}$$

and R_p (250) is:

$$R_p(250) = \frac{3\lambda_3}{3\lambda_3 - 2\lambda_2}e^{-2\times\lambda_2\times t} + \frac{2\lambda_2}{2\lambda_2 - 3\lambda_3}e^{-3\times\lambda_3\times t} = 5e^{-2.4\times 0.00352113\times 250} - 3.3e^{-3\times 0.00352113\times 250} = 0.3669 \tag{18}$$

Now for the fuzzy form of λ_n, the parameters can be determined as:

$$M = \frac{1}{284} = 0.0035211 \tag{19}$$

$$L = \frac{\chi^2_{(0.975)(10)}}{10\times 284} = 0.0011433 \tag{20}$$

$$U = \frac{\chi^2_{(0.025)(10)}}{10\times 284} = 0.0072124 \tag{21}$$

Hence $\tilde{\lambda} = (L, M, U)$ and: $\tilde{\lambda}_{3\alpha} = [\lambda_{31}(\alpha), \lambda_{32}(\alpha)] = [0.0011433 + 0.0023778\alpha, 0.0072124 - 0.0036913\alpha]$

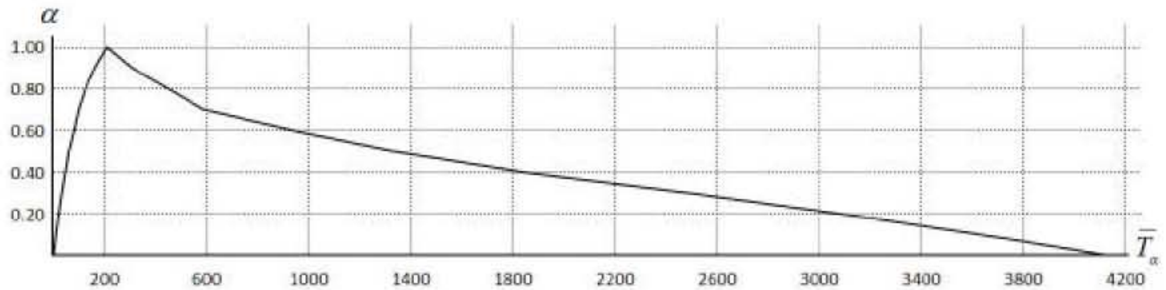

Fig. 3: The exact shape of $\overline{T}_\alpha$ For different values of α between 0 to 1 with interval 0.2

Table 1: The result of T for β = 0.1 and different values of γ

β	The values of T					
0.10	γ = 0.0	γ = 0.1	γ = 0.2	γ = 0.3	γ = 0.4	γ = 0.5
	627.51	614.96	602.41	589.86	577.31	564.76
	γ = 0.6	γ = 0.7	γ = 0.8	γ = 0.9	γ = 1.0	
	552.21	539.66	527.11	514.56	502.01	

Table 2: Result of T for γ = 0.6 and different values of β

			The values of T						
γ	λ_2/λ_3	MTTF	β = 0.01	β = 0.02	β = 0.05	β = 0.10	β = 0.25	β = 0.40	β = 0.50
0.6	1.25	208.27	2021.18	1371.49	815.92	552.21	339.27	272.92	249.66

$$\tilde{\lambda}_{3\alpha} = 1.2\lambda_{2\alpha} = [1.2\lambda_{31}(\alpha), 1.2\lambda_{32}(\alpha)] \quad (22)$$

The value of $\tilde{T} = (l, m, u)$ is also obtained by:

$$\tilde{T}_\alpha = [T_1(\alpha), T_2(\alpha)] = \frac{1}{6}\times\frac{3\tilde{\lambda}_3 + 2\tilde{\lambda}_2}{\tilde{\lambda}_2\times\tilde{\lambda}_3} = \frac{1}{6}\times\frac{3[\lambda_{31}(\alpha),\lambda_{32}(\alpha)] + 2\times 1.2[\lambda_{31}(\alpha),\lambda_{32}(\alpha)]}{[\lambda_{31}(\alpha),\lambda_{32}(\alpha)]\times 1.2[\lambda_{31}(\alpha),\lambda_{32}(\alpha)]} = \frac{5.4[\lambda_{31}(\alpha),\lambda_{32}(\alpha)]}{7.2[\lambda_{31}^2(\alpha),\lambda_{32}^2(\alpha)]} \quad (23)$$

$$\frac{3}{4}\times\left[\frac{\lambda_{31}(\alpha)}{\lambda_{32}^2(\alpha)},\frac{\lambda_{32}(\alpha)}{\lambda_{31}^2(\alpha)}\right] = \frac{3}{4}\times\left[\frac{0.0011433 + 0.0023778\,\alpha}{(0.0072124 - 0.0036913\,\alpha)^2},\frac{0.0072124 - 0.0036913\,\alpha}{(0.0011433 + 0.0023778\,\alpha)^2}\right]$$

$$l = T_1(0) = 16.4840 \quad (24)$$

$$u = T_2(0) = 4138.2875 \quad (25)$$

$$m = T_1(1) = T_2(1) = 213 \quad (26)$$

$$\tilde{T} = (l, m, u) = (16.4840, 213.0000, 4138.2875) \quad (27)$$

For different values of α from 0 to 1 with interval 0.2 in equation (23), the exact shape of $\overline{T}_\alpha$ is illustrated in Fig. 3.

In order to compare the result in fuzzy form with the crisp one, the value of $\tilde{T}$ is defuzzified by:

$$T = \frac{(l + 4m + u)}{6} = 834.46 \quad (28)$$

We can notice that it increases by 213 which may be due to the tolerances that have been relaxed in the case of fuzzy failure rates. The result of T for β = 0.1 and different values of γ is shown in Table 1.

Also the result of T for γ = 0.6 and different values of β is shown in Table 2.

In these tables we can observe that by considering any parameter and increasing the other one, the value of T decreases.

CONCLUSION

A real time k-out-of-n system has been studied in which the failure rate of the elements is increasing. That is by failure of one element, the load on the remaining elements, increases the chance of failure of the remaining elements. Necessary relations have been developed for the failure rates. The ambiguity of the data for failure rates is demonstrated by fuzzy triangular numbers. Therefore, the MTTF parameter has been estimated in the form of fuzzy triangular number. In order to construct the fuzzy numbers, we use point estimation and confidence intervals as the tolerances. Because using fuzzy parameters to modelling the uncertainties in the system, it seems that this model provides more reliable solutions.

This model can be applied to a wide variety of complex industrial problems like the engines of an airplane. As an extension to this work, the failure rates may be considered different for each element. In that case either a system of k-out-of-n model should be taken into consideration or for the k-out-of-n model, a certain policy should be developed when whole system fails. Moreover, the elements may be repairable. Hence, the cost for procurement, repair of the elements and the failure of the whole system may be taken into account in such a way that the optimum number of elements with minimum cost and maximum reliability will be determined. These variations could be studied for the systems with increasing failure rate of the elements too.

Appendix 1: Solution of the equation 03

In a k-out-of-n system, for the state in category n we have:

$$P_{12\ldots n}(t+\Delta t) = P_{12\ldots n}(t) - n\times\lambda_n\times\Delta t\times P_1(t) \Rightarrow \tag{29}$$

$$\mathrm{Lim}_{\Delta t\to 0}\frac{P_{12\ldots n}(t+\Delta t)-P_{12\ldots n}(t)}{\Delta t} = \frac{P'_{12\ldots n}(t)}{P_{12\ldots n}(t)} = -n\times\lambda_n \Rightarrow \begin{cases} P_1(t) = e^{-n\times\lambda_n\times t}+c \\ P_1(0)=1 \end{cases} \Rightarrow P_{12\ldots n}(t) = e^{-n\times\lambda_n\times t} \tag{30}$$

Also for the states in category (n-1) we have: (31)

$$P_{A_1A_2\ldots A_{n-1}}(t+\Delta t) = P_{A_1A_2\ldots A_{n-1}}(t) + \lambda_n\times\Delta t\times P_{12\ldots n}(t) - (n-1)\times\Delta t\times P_{A_1A_2\ldots A_{n-1}}(t) \Rightarrow \tag{31}$$

$$\mathrm{Lim}_{\Delta t\to 0}\frac{P_{A_1A_2\ldots A_{n-1}}(t+\Delta t)-P_{A_1A_2\ldots A_{n-1}}(t)}{\Delta t} = P'_{A_1A_2\ldots A_{n-1}}(t) + (n-1)\times\lambda_{(n-1)}\times P_{A_1A_2\ldots A_{n-1}}(t) = \lambda_n\times P_{12\ldots n}(t) = \lambda_n\times e^{-n\times\lambda_n\times t} \tag{32}$$

$$e^{(n-1)\times\lambda_{(n-1)}\times t}\times P'_{A_1A_2\ldots A_{n-1}}(t) + e^{(n-1)\times\lambda_{(n-1)}\times t}\times(n-1)\times\lambda_{(n-1)}\times P_{A_1A_2\ldots A_{n-1}}(t) = \lambda_n\times e^{\left[-n\times\lambda_n+(n-1)\times\lambda_{(n-1)}\right]\times t} \tag{33}$$

$$\frac{d}{dt}\left[e^{(n-1)\times\lambda_{(n-1)}\times t}\times P_{A_1A_2\ldots A_{n-1}}(t)\right] = \lambda_n\times e^{\left[-n\times\lambda_n+(n-1)\times\lambda_{(n-1)}\right]\times t} \Rightarrow \tag{34}$$

$$e^{(n-1)\times\lambda_{(n-1)}\times t}\times P_{A_1A_2\ldots A_{n-1}}(t) = \frac{\lambda_n}{(n-1)\times\lambda_{(n-1)}-n\times\lambda_n}\times e^{\left[-n\times\lambda_n+(n-1)\times\lambda_{(n-1)}\right]\times t} + C \Rightarrow \tag{35}$$

$$\left.\begin{aligned} &P_{A_1A_2\ldots A_{n-1}}(t) = \frac{\lambda_n}{(n-1)\times\lambda_{(n-1)}-n\times\lambda_n}\times e^{-n\times\lambda_n\times t} + C\times e^{-(n-1)\times\lambda_{(n-1)}\times t} \\ &P_{A_1A_2\ldots A_{n-1}}(0) = 0 \end{aligned}\right\} \Rightarrow C = \frac{-\lambda_n}{(n-1)\times\lambda_{(n-1)}-n\times\lambda_n} \tag{35}$$

And the result is:

$$P_{A_1A_2\ldots A_{n-1}}(t) = \frac{\lambda_n}{(n-1)\times\lambda_{(n-1)}-n\times\lambda_n}\times\left[e^{-n\times\lambda_n\times t} - e^{-(n-1)\times\lambda_{(n-1)}\times t}\right] \tag{37}$$

Suppose that for the states in category (k+1) we know that:

$$P_{A_1A_2\ldots A_{k+1}}(t) = (n-k-1)!\times\left(\prod_{v=k+2}^{n}\lambda_v\right)\times\sum_{j=k}^{n}\left[\left(\prod_{\substack{\theta=k+1\\ \theta\neq j}}^{n}\frac{1}{\theta\times\lambda_\theta - j\times\lambda_j}\right)\times e^{-j\times\lambda_j\times t}\right] \tag{38}$$

That we can solve the differential equation of the states in category k as follows:

$$P_{A_1A_2..A_k}(t+\Delta t) = P_{A_1A_2..A_k}(t) - \sum_{i=1}^{k} \lambda_k \times \Delta t \times P_{A_1A_2..A_k}(t) + \sum_{\substack{j=1 \\ \eta_j \neq A_i \quad i=1,2,\ldots,k}}^{n} \lambda_{(k+1)} \times \Delta t \times P_{A_1A_2...A_k\eta_j}(t) \Rightarrow \quad (39)$$

$$\lim_{\Delta t \to \infty} \frac{P_{A_1A_2..A_k}(t+\Delta t) - P_{A_1A_2..A_k}(t)}{\Delta t} = P'_{A_1A_2..A_k}(t) = \sum_{i=1}^{k} \lambda_k \times P_{A_1A_2...A_k}(t) + \sum_{\substack{j=1 \\ \eta_j \neq A_i \quad i=1,2,\ldots,k}}^{n} \lambda_{(k+1)} \times P_{A_1A_2...A_k\eta_j}(t) \quad (40)$$

$$P'_{A_1A_2..A_k}(t) - \sum_{i=1}^{k} \lambda_k P_{A_1A_2..A_k}(t) = \sum_{\substack{j=1 \\ \eta_j \neq A_i \quad i=1,2,\ldots,k}}^{n} \lambda_{(k+1)} \times P_{A_1A_2..A_k\eta_j}(t) \Rightarrow \quad (41)$$

$$e^{\left(\sum_{i=1}^{k} \lambda_k\right) \times t} \times P'_{A_1A_2..A_k}(t) - e^{\left(\sum_{i=1}^{k} \lambda_k\right) \times t} \times \sum_{i=1}^{k} \lambda_k P_{A_1A_2..A_k}(t) = e^{\left(\sum_{i=1}^{k} \lambda_k\right) \times t} \times \sum_{\substack{j=1 \\ \eta_j \neq A_i \quad i=1,2,\ldots,k}}^{n} \lambda_{(k+1)} \times P_{A_1A_2..A_k\eta_j}(t) \quad (42)$$

Also for all values of η_j we know that $P_{A_1A_2...A_k\eta_j}(t) = P_{A_1A_2...A_{k+1}}(t)$, so we have:

$$\begin{cases} \frac{d}{dt}\left[e^{\left(\sum_{i=1}^{k} \lambda_k\right) \times t} \times P_{A_1A_2..A_k}(t) \right] = e^{\left(\sum_{i=1}^{k} \lambda_k\right) \times t} \times (n-k) \times P_{A_1A_2...A_{k+1}}(t) \\ P_{A_1A_2..A_{k+1}}(t) = (n-k-1)! \times \left(\prod_{v=k+2}^{n} \lambda_v \right) \times \sum_{j=k+1}^{n} \left[\left(\prod_{\substack{\theta=k+1 \\ \theta \neq j}}^{n} \frac{1}{\theta \times \lambda_\theta - j \times \lambda_j} \right) \times e^{-j \times \lambda_j \times t} \right] \end{cases} \quad (43)$$

and

$$e^{\left(\sum_{i=1}^{k} \lambda_k\right) \times t} \times P_{A_1A_2..A_k}(t) = \int e^{\left(\sum_{i=1}^{k} \lambda_k\right) \times t} \times (n-k) \times (n-k-1)! \times \left(\prod_{v=k+2}^{n} \lambda_v \right) \times \sum_{j=k+1}^{n} \left[\left(\prod_{\substack{\theta=k+1 \\ \theta \neq j}}^{n} \frac{1}{\theta \times \lambda_\theta - j \times \lambda_j} \right) \times e^{-j \times \lambda_j \times t} \right] dt \quad (44)$$

We sub situate the initial value of $P_{A_1A_2...A_k}(0) = 0$, so the result is:

$$P_{A_1A_2..A_k}(t) = (n-k)! \times \left(\prod_{v=k+1}^{n} \lambda_v \right) \times \sum_{j=k}^{n} \left[\left(\prod_{\substack{\theta=k \\ \theta \neq j}}^{n} \frac{1}{\theta \times \lambda_\theta - j \times \lambda_j} \right) \times e^{-j \times \lambda_j \times t} \right] \quad (45)$$

Nomenclature

The notations used in this paper are as follows:

- n: Number of elements,
- λ_i: Failure rate of the elements in ist category,
- $P_i(t)$: Probability that the system is in state i at the time t,
- $R_p(t)$: Probability that system works at time t,
- MTTF: Mean time to failure of the system,
- $\tilde{T}$: Fuzzy version of MTTF,
- T: Defuzzified version of $\tilde{T}$,
- X_j: Interval between (j-1) st and jst failure.

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