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Numerical Solution of Modified Equal Width Wave Equation

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Abstract: Homotopy perturbation method is presented to solve the modified equal width equation. This method provides remarkable accuracy in comparison with the analytical solution. Three conservation quantities are reported. Numerical results demonstrate that this method is a promising and powerful tool for solving the modified equal width equation.

Key words: Homotopy perturbation method . The Modified Equal Width (MEW) equation . Motion of single solitary wave . Maxwellian initial condition

INTRODUCTION

The objective of this paper is to extend the application of the homotopy perturbation method (HPM) to obtain numerical solution of the modified equal width equation. The homotopy perturbation method was first proposed by the Chinese mathematician Ji-Huan He [1-6]. The essential idea of this method is to introduce a homotopy parameter, say p, which takes values from 0 to 1. When $p = 0$, the system of equations usually reduces to a sufficiently simplied form, which normally admits a rather simple solution. As p gradually increases to 1, the system goes through a sequence of deformations, the solution for each of which is close to that at the previous stage of deformation. Eventually at $p =1$, the system takes the original form of the equation and the final stage of deformation gives the desired solution. One of the most remarkable features of the HPM is that usually just few perturbation terms are sufficient for obtaining a reasonably accurate solution. Considerable research works have been conducted recently in applying this method to a class of linear and non-linear equations [7-15]. The interested reader can see the [16-19] for last development of HPM.

The equal width (EW) equation was suggested by Morrison *et al*. [20]. to use as a model partial differential equation for the simulation of one-dimensional wave propagation in nonlinear media with dispersion processes. Many methods have been proposed to solve the EW equation [21-24]. Based on the EW equation, Zaki [25] considered the solitary wave interactions for the Modified Equal Width (MEW) equation by Petrov-Galerkin method using quintic B-spline finite elements, Wazwaz [26] investigated the modified equal width equation and two of its variants by the tanh and the sine-cosine methods and Saka [27] proposed algorithms for the numerical solution of the modified equal width wave equation using collocation method. Reviewing these improvements, we can find that it's inevitable to involve the quintic B-splines, the computations of time dependent parameters, linearization and discretization of the MEW equation, while the tanh and sine-cosine methods are based on the solutions that can be expressed in terms of the tanh, sech or csch functions, therefore, these methods for solving the MEW equation narrow down their applications. A nature question kept in our mind is that whether we can solve the MEW equation without quintic B-splines, linearization and discretization. Also Lu [28] applied variational iteration method for solving the modified equal width wave equation.

The aim of this paper is to extended the homotopy perturbation method for solving the modified equal width wave equation. The homotopy perturbation method provides a new approach to solve the MEW equation without linearization and discretization. Numerical examples are presented to illustrate the efficiency of the homotopy perturbation method. The rest of paper is organized as follows. In Section 2, we give the analysis of the homotopy perturbation method. In Section 3, the modified equal width equation is introduced and solved by the homotopy perturbation method, also we present numerical results to demonstrate the efficiency of the homotopy perturbation method with the help of motion of single solitary wave and Maxwellian initial condition for the MEW equation.

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ANALYSIS OF THE HOMOTOPY PERTURBATION METHOD (HPM)

To illustrate the basic idea of He's homotopy perturbation method, consider the following general nonlinear differential equation;

$$A(u) - f(r) = 0,\ r\in\Omega \quad (1)$$

with boundary conditions;

$$B(u, \partial u/\partial n) = 0, r\in\Gamma \quad (2)$$

where A is a general differential operator, B is a boundary operator, *f(r)* is a known analytic function, Γ is the boundary of the domain Ω.

The operator A can, generally speaking, be divided in to two parts L and N, where L is linear and N is nonlinear, therefore Eq.(1) can be written as,

$$L(u) + N(u) - f(r) = 0 \quad (3)$$

By using homotopy technique, one can construct a homotopy $v(r,p): \Omega\times[0,1]\to R$ which satisfies

$$\begin{aligned} H(v,p) = (1-p)[L(v) - L(u_0)] \\ + p[A(v) - f(r)] = 0, \quad p\in[0,1] \end{aligned} \quad (4a)$$

or

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \quad (4b)$$

where $p\in[0,1]$ is an embedding parameter and u_0 is the initial approximation of Eq.(1) which satisfies the boundary conditions. Clearly, we have

$$H(v,0) = L(v) - L(u_0) = 0 \quad (5)$$

$$H(v,1) = A(v) - f(r) = 0 \quad (6)$$

the changing process of p from zero to unity is just that of $v(r, p)$ changing from $u_0(r)$ to $u(r)$. This is called deformation and also, $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopic in topology. If, the embedding parameter p; ($0\le p\le 1$) is considered as a "small parameter", applying the classical perturbation technique, we can naturally assume that the solution of Eqs.(5) and (6) can be given as a power series in p, i.e.,

$$v = v_0 + pv_1 + p^2 v_2 + ... \quad (7)$$

and setting p = 1 results in the approximate solution of Eq. (4) as;

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + ... \quad (8)$$

The convergence of series (8) has been proved by He in his paper [5]. It is worth to note that the major advantage of He's homotopy perturbation method is that the perturbation equation can be freely constructed in many ways (therefore is problem dependent) by homotopy in topology and the initial approximation can also be freely selected.

HOMOTOPY PERTURBATION METHOD FOR THE MEW EQUATION

Consider the MEW equation that has the normalized form [20, 25],

$$u_t + 3u^2 u_x - \mu u_{xxt} = 0 \quad (9)$$

where μ is a positive parameter and the subscripts *x* and *t* denote differentiation. When solved analytically, within an infinite region with physical boundary conditions $u\to 0$ as $|x|\to\infty$, the MEW (Eq.(9)) has a solitary wave solution of the form [25]

$$u(x,t) = A\operatorname{sech}[k(x - ct - x_0)] \quad (10)$$

where

$$k^2 = \frac{1}{\mu},\ c = \frac{A^2}{2} \quad (11)$$

In this section, we consider the following two test problems: the motion of a single solitary wave and the birth of solitons from a given initial condition, to verify the performance of the homotopy perturbation method. For the MEW equation, it is important to discuss the following three invariant conditions given as [25]:

$$I_1 = \int_{-\infty}^{\infty} u\,dx \quad (12)$$

$$I_2 = \int_{-\infty}^{\infty} \left(u^2 + \mu(u_x)^2\right)dx \quad (13)$$

$$I_3 = \int_{-\infty}^{\infty} u^4\,dx \quad (14)$$

which, respectively correspond to conservation of mass, momentum and energy and will be used to check the conservation properties of the numerical solutions. Moreover, in order to show how good the numerical solutions in comparison with the analytical solutions, we use the absolute error to verify the efficiency of the homotopy perturbation method.

Motion of single solitary wave: We consider the MEW Eq. (9) with the boundary condition $u\to 0$ as $|x|\to 0$ and the initial condition

$$u(x,0)=A\operatorname{sech}\left[k\left(x-x_0\right)\right] \tag{15}$$

Note that the analytical solution of this problem is given by [25]

$$u(x,t)=A\operatorname{sech}\left[k\left(x-ct-x_0\right)\right] \tag{16}$$

where $k,\ c$ are the same as Eq. (11). The analytical values of the invariants are [25]

$$I_1=\frac{A\pi}{k},I_2=\frac{2A^2}{k}+\frac{2\mu kA^2}{3},I_3=\frac{4A^4}{3k} \tag{17}$$

We consider the MEW Eq. (9) for the case $x_0 = 0$, we begin with an initial approximation given by

$$u(x,0)=A\operatorname{sech}(kx)$$

To solve Eqs. (9) by homotopy perturbation method, we construct the following homotopy:

$$\left(\frac{\partial u}{\partial t}-\frac{\partial u_0}{\partial t}\right)=p\left(-3u^2\frac{\partial u}{\partial x}-\mu\frac{\partial^3 u}{\partial x^2\partial t}-\frac{\partial u_0}{\partial t}\right) \tag{18}$$

Assume the solution of Eq. (18) to be in the form:

$$u=u_0+pu_1+p^2u_2+p^3u_3+\ldots\ldots \tag{19}$$

Substituting Eq. (19) into Eq. (18) and collecting terms of the same power of p give:

$$p^0:\ \frac{\partial u_0}{\partial t}-\frac{\partial u_0}{\partial t}=0 \tag{20}$$

$$p^1:\ \frac{\partial u_1}{\partial t}=-3u_0^2\frac{\partial u_0}{\partial x}-\mu\frac{\partial^3 u_0}{\partial x^2\partial t}-\frac{\partial u_0}{\partial t} \tag{21}$$

$$p^2:\ \frac{\partial u_2}{\partial t}=-6u_0u_1\frac{\partial u_0}{\partial x}-3u_0^2\frac{\partial u_1}{\partial x}-\mu\frac{\partial^3 u_1}{\partial x^2\partial t} \tag{22}$$

$$\ldots$$

The given initial value admits the use of

$$u_0(x,t)=A\operatorname{sech}(kx) \tag{23}$$

The solution reads

$$u_1(x,t)=3A^3k\operatorname{sech}^3(kx)\tanh(kx) \tag{24}$$

$$\begin{aligned}u_2(x,t)&=3A^2\operatorname{sech}^2(kx)\\&\quad\left(\begin{array}{l}-9A^3\operatorname{sech}^3(kx)\tanh^2(kx)k^2t\\+3A^3\operatorname{sech}^3(kx)\end{array}\right)\\&\quad\left(1-\tanh^2(kx)k^2t\right)\\&\quad-18A^5\operatorname{sech}^5(kx)\tanh^2(kx)k^2t\\&\quad-\mu\left(\begin{array}{l}27A^3\operatorname{sech}^3(kx)\tanh^3(kx)k^3\\-33A^3\operatorname{sech}^3(kx)\tanh(kx)k^3\end{array}\right)\\&\quad\left(1-\tanh^2(kx)\right)\\&\quad\ldots\end{aligned} \tag{25}$$

and so on, in this manner the rest of components of the homotopy perturbation series can be obtained.

The solution of Eqs. (9) can be obtained by setting $p = 1$ in Eq. (19):

$$u=u_0+u_1+u_2+u_3+\ldots\ldots \tag{26}$$

Thus, we have approximate solution

$$u_{app}=u_0+u_1+u_2 \tag{27}$$

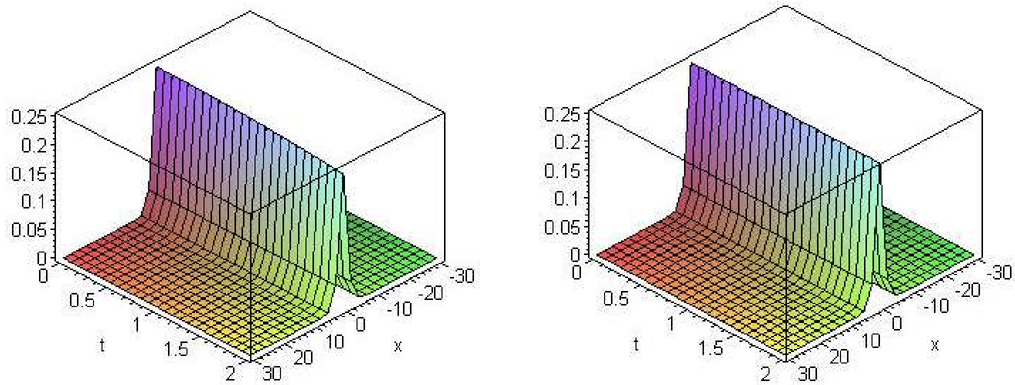
For the numerical simulation of the motion of a single solitary wave, we choose $\mu = 1$, $x_0 = 0$. From the numerical results of Table 1, we can conclude that the homotopy perturbation method for this MEW equation gives remarkable accuracy in comparison with the analytical solution especially for small values of time t. The Fig. 1 shows the behavior of the approximate solution obtained by homotopy perturbation method and analytical solution. Figure 2 shows the evolution results for the approximate solution and the analytical solution. The invariants I_1, I_2, I_3 given in Table 2 are in good agreement with the corresponding analytical values $I_1 = 0.1570796327$, $I_2 = 0.0066666667$, $I_3 = 8.333333333E-6$. Besides, the invariants of the same problem with $A = 0.1,\ 0.25$ are recorded in Table 3 and 4, respectively.

Homotopy perturbation method provides the approximate solution without the complicated quintic B-splines and discretization, therefore, it's promising and readily implemented. It's important to note that we get the high accuracy only by two iterations and the accuracy can be further improved by considering more components of the iteration.

Maxwellian initial condition: We consider the birth of solitons from the initial condition

$$u(x,0)=e^{-x^2} \tag{28}$$

with the boundary conditions



(a) (b)

Fig. 1: The evolution results for the approximate solution u2 and analytical solution to MEW Equation for $0 \le t \le 2$ and $-30 \le x \le 30$

Table 1: The numerical results for the approximate solutions u2 obtained by HPM in comparison with the analytical solutions of Eqs. (9,15) for A = 0.25

x	Anal. Time	Approx. solution	Absolute solution	Error
-10	0.01	0,0000226929	0,0000227000	-0,0000000071
	0.02	0,0000226858	0,0000227000	-0,0000000142
	0.03	0,0000226787	0,0000227000	-0,0000000213
	0.04	0,0000226716	0,0000227000	-0,0000000284
	0.05	0,0000226645	0,0000227000	-0,0000000354
	0.07	0,0000226504	0,0000227000	-0,0000000496
	0.09	0,0000226362	0,0000227000	-0,0000000638
	0.10	0,0000226291	0,0000227000	-0,0000000708
-5	0.01	0,0033677680	0,0033688091	-0,0000010411
	0.02	0,0033667158	0,0033687976	-0,0000020818
	0.03	0,0033656640	0,0033687862	-0,0000031222
	0.04	0,0033646125	0,0033687747	-0,0000041622
	0.05	0,0033635613	0,0033687632	-0,0000052019
	0.07	0,0033614599	0,0033687403	-0,0000072804
	0.09	0,0033593599	0,0033687174	-0,0000093575
	0.10	0,0033583103	0,0033687059	-0,0000103956
0	0.01	0,2499999877	0,2499995605	0,0000004272
	0.02	0,2499999511	0,2499982421	0,0000017090
	0.03	0,2499998901	0,2499960449	0,0000038452
	0.04	0,2499998046	0,2499929688	0,0000068358
	0.05	0,2499996948	0,2499890136	0,0000106812
	0.07	0,2499994018	0,2499784668	0,0000209350
	0.09	0,2499990112	0,2499644042	0,0000346070
	0.10	0,2499987793	0,2499560546	0,0000427247
5	0.01	0,0033698734	0,0033688320	0,0000010414
	0.02	0,0033709265	0,0033688435	0,0000020830
	0.03	0,0033719800	0,0033688550	0,0000031250
	0.04	0,0033730338	0,0033688664	0,0000041674
	0.05	0,0033740879	0,0033688779	0,0000052100
	0.07	0,0033761972	0,0033689008	0,0000072964
	0.09	0,0033783078	0,0033689237	0,0000093841
	0.10	0,0033793636	0,0033689352	0,0000104284
10	0.01	0,0000227071	0,0000227000	0,0000000071
	0.02	0,0000227142	0,0000227000	0,0000000142
	0.03	0,0000227213	0,0000227000	0,0000000213
	0.04	0,0000227284	0,0000227000	0,0000000284
	0.05	0,0000227355	0,0000227000	0,0000000355
	0.07	0,0000227497	0,0000227000	0,0000000497
	0.09	0,0000227639	0,0000227000	0,0000000639
	0.10	0,0000227710	0,0000227000	0,0000000710

Table 2: Invariants for single solitary wave with A = 0.05, $\mu = 1$, $x_0 = 0$ and [-30, 30]

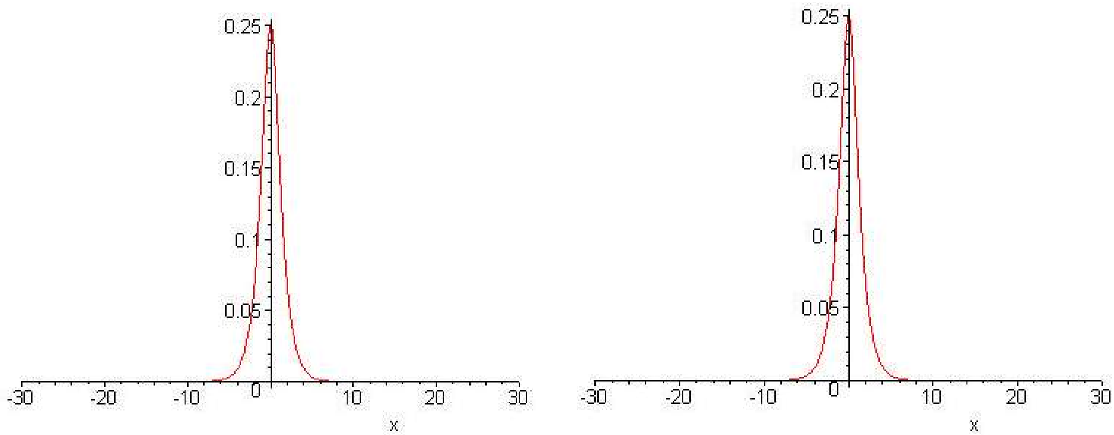
Time	I1	I2	I3
0	0,157079633	0,6666666667e-2	0,8333333333e-5
0.01	0,1570796328	0,6666667529e-2	0,8333334049e-5
0.02	0,1570796328	0,6666670069e-2	0,8333336097e-5
0.03	0,1570796328	0,6666674301e-2	0,8333339512e-5
0.04	0,1570796328	0,6666680225e-2	0,8333344292e-5
0.05	0,1570796328	0,6666687849e-2	0,8333350437e-5
0.07	0,1570796328	0,6666708167e-2	0,8333366827e-5
0.09	0,1570796328	0,6666735256e-2	0,8333388675e-5
0.10	0,1570796328	0,6666751340e-2	0,8333401654e-5

Table 3: Invariants for single solitary wave with A = 0.1, $\mu = 1$, $x_0 = 0$ and [-30, 30]

Time	I1	I2	I3
0	0.3141592654	0.2666666667e-1	0.1333333333e-3
0.01	0.3141592656	0.2666672090e-1	0.1333335087e-3
0.02	0.3141592655	0.2666688345e-1	0.1333340331e-3
0.03	0.3141592656	0.2666715434e-1	0.1333349072e-3
0.04	0.3141592655	0.2666753361e-1	0.1333361309e-3
0.05	0.3141592655	0.2666802126e-1	0.1333377042e-3
0.07	0.3141592656	0.2666932157e-1	0.1333418997e-3
0.09	0.3141592656	0.2667105536e-1	0.1333474939e-3
0.10	0.3141592656	0.2667208480e-1	0.1333508154e-3

Table 4: Invariants for single solitary wave with A = 0.25, $\mu = 1$, $x_0 = 0$ and [-30, 30]

Time	I1	I2	I3
0	0.7853981635	0.1666666667	0.5208333333e-2
0.01	0.7853981638	0.1666798946	0.5208600102e-2
0.02	0.7853981639	0.1667195779	0.5209400340e-2
0.03	0.7853981638	0.1667857165	0.5210734098e-2
0.04	0.7853981640	0.1668783108	0.5212601393e-2
0.05	0.7853981633	0.1669973608	0.5215002269e-2
0.07	0.7853981638	0.1673148283	0.5221404978e-2
0.09	0.7853981638	0.1677381211	0.5229942760e-2
0.10	0.7853981638	0.1679894526	0.5235012520e-2



(a) (b)

Fig. 2: The evolution results for the approximate solution u2 and analytical solution to MEW Equation for t = 1 and $-30 \le x \le 30$

Table 5: Invariants for Maxwellian initial condition with $\mu = 0.0025$ and [-20, 20]

Time	I1	I2	I3
0	1.772453851	1.256447422	0.8862269255
0.01	1.772453851	1.256440012	0.8861821173
0.02	1.772453851	1.256422125	0.8860710551
0.03	1.772453851	1.256406784	0.8859636500
0.04	1.772453851	1.256415697	0.8859758395
0.05	1.772453851	1.256479253	0.8862687249
0.07	1.772453851	1.256935270	0.8885594082
0.09	1.772453851	1.258191611	0.8949755207
0.10	1.772453851	1.259288133	0.9005702808

Table 6: Invariants for Maxwellian initial condition with $\mu = 0.01$ and [-20, 20]

Time	I1	I2	I3
0	1.772453851	1.265847278	0.8862269255
0.01	1.772453851	1.265814997	0.8860517977
0.02	1.772453851	1.265723262	0.8855497012
0.03	1.772453851	1.265587396	0.8847903135
0.04	1.772453851	1.265432936	0.8838891575
0.05	1.772453851	1.265295636	0.8830067022
0.07	1.772453851	1.265266599	0.8821567678
0.09	1.772453851	1.265990610	0.8843685477
0.10	1.772453851	1.266832927	0.8874563696

$$u(x,t) = 0 \text{ for } x \to \pm\infty,\ t > 0 \tag{29}$$

We use the same homotopy perturbation procedure which was used in Section 3.1. The given initial value admits the use of

$$u_0(x,t) = e^{-x^2} \tag{30}$$

Table 7: Invariants for Maxwellian initial condition with $\mu = 0.04$ and [-20, 20]

Time	I1	I2	I3
0	1.772453851	1.303446702	0.8862269255
0.01	1.772453851	1.303318316	0.8856798734
0.02	1.772453851	1.302941329	0.8840615971
0.03	1.772453851	1.302340258	0.8814405318
0.04	1.772453851	1.301555963	0.8779300586
0.05	1.772453851	1.300645648	0.8736874985
0.07	1.772453851	1.298757500	0.8638464279
0.09	1.772453851	1.297460333	0.8539932918
0.10	1.772453851	1.297350035	0.8498717614

Table 8: Invariants for Maxwellian initial condition with $\mu = 0.1$ and [-20, 20]

Time	I1	I2	I3
0	1.772453851	1.378645551	0.8862269255
0.01	1.772453851	1.378665195	0.8856528655
0.02	1.772453851	1.378738427	0.8839515370
0.03	1.772453851	1.378908150	0.8811852883
0.04	1.772453851	1.379245871	0.8774573495
0.05	1.772453851	1.379851698	0.8729108165
0.07	1.772453851	1.382411110	0.8621249255
0.09	1.772453851	1.387959295	0.8507077110
0.10	1.772453851	1.392408347	0.8454925694

The solution reads

$$u_1(x,t) = 6e^{-3x^2}tx \tag{31}$$

$$\begin{aligned} u_2(x,t) = {} & -9e^{-5x^2}t^2 - 108e^{-3x^2}tx - 72e^{-7x^2}t^3x \\ & + 90e^{-5x^2}t^2x^2 - 162e^{-9x^2}t^4x^2 + 216e^{-3x^2}tx^3 \\ & + 504e^{-7x^2}t^3x^3 + 972e^{-9x^2}t^4x^4 \end{aligned} \tag{32}$$

and so on, in this manner the rest of components of the homotopy perturbation series can be obtained.
Thus, we have approximate solution

$$u_{app} = u_0 + u_1 + u_2 \tag{33}$$

The invariants I_1, I_2, I_3 given in Table 5-8 are in good agreement with the corresponding analytical values.

CONCLUSIONS

In this study, the homotopy perturbation method has been successfully applied to find the solitary wave solution of the modified equal width equation. The solution obtained by the homotopy perturbation method can be expressed in a closed form of the analytical solution. The homotopy perturbation method is a successful approach to provide an analytical approximation to the modified equal width equation without linearization and discretization. Compared with the analytical solution, the approximate solution obtained by homotopy perturbation method provides remarkable accuracy and the accuracy can be further improved by considering more components of the iteration formula.

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