World Applied Sciences Journal 8 (10): 1183-1192, 2010
ISSN 1818-4952

A Fuzzy-Random Utility Model of Consumer Choice; Uncertainty Effect

[1]Hossein Ali Momeni and [2]Jamshid Nazemi

[1]Management Department, Islamic Azad University, Karaj Branch, Iran
[2]Management Department, Islamic Azad University, Research and Science Branch, Iran

Abstract: Consumers in many choice contexts are facing with uncertainty relates to understanding of attributes of the choice set with ambiguity. However, the way in which this uncertainty affects choice alternatives may consistently vary considering randomness and fuzziness. In vehicle choice problems, there are some variables affects on the choice set which are potentially sources of ambiguity in the decision process. This paper discusses the implement of a Fuzzy-Random approach for choice modeling of family's behavior in choice contexts that involve conditions of uncertainty and ambiguity and especially investigates the effect of uncertainty on choice probability, which derives from fuzzy approach compare to traditional multinomial logit. It is concluded that the higher the uncertainty versus average uncertainty in attribute is, the higher change is the alternative choice probability in fuzzy approach versus traditional multinomial logit. Moreover, three standard defuzzification methods are used to compare the results of fuzzy versus traditional approach, which produces reordering of choice probabilities on the basis of typical summation of three interval fuzzy choice probabilities. This approach can be applied besides the traditional approach in order to capture the uncertainty of decision makers. An application of the methodology to a vehicle choice context considering two attributes presented.

Key words: Vehicle Choice · Uncertainty modeling · Fuzzy sets · Utility theory

INTRODUCTION

The issue of how to deal with uncertainty has become one of the major topics in consumer behavior especially choice problems. Uncertainty usually affects choice alternatives in many different choice contexts distinguishing by two main types of uncertainties, randomness and fuzziness, especially in vehicle choice problems there are subjective factors potentially are sources of vagueness which influence on car choice by decision makers or households.

Randomness is associated to the effective variability of the alternative attributes and is strictly related to the main characteristics of the alternative attributes as well as the local conditions in which choice set alternatives are provided.

Fuzziness can be identified in the vagueness with which individuals perceive the attributes of choice alternatives and is related to human perceptions and usu-ally influenced by the familiarity individuals have with the available choices as in vehicle choice problems, there are factors influence human perceptions considering vagueness of attributes which affect on decision making among alternatives.

Several approaches have been proposed by researchers in order to simulate consumer behavior in different choice contexts especially considering one or both of the two sources of uncertainty [1].

From the other side, people's rationality is restricted because of their cognitive limitations [2] especially on understanding of some attributes, which are originally cognitive or qualitative not quantitative in the choice context.

In marketing models especially in consumer choice models, in situations where the average affects are not of primary interest then the impact of parameter uncertainty needs to be considered [3].

Vehicle choice is one of the critical decisions needed to be taken by consumers in an uncertain environment. Consumer's choices among the available choices reflect their perception of the utilities associated with each alternative. If the utilities are perceived to be uncertain then the choice is influenced by the consumer's attitude to that uncertainty.

Therefore, in problems such as vehicle choice there is a necessity to consider subjective or qualitative factors beside quantitative factors which are common to be studied. So, in this study, a fuzzy-random approach is

Corresponding Author: Hossein Ali Momeni, Management Department, Islamic Azad University, Karaj Branch, Iran.
Tel: +982144528532-3

implemented in the vehicle choice problem with three alternatives and two attributes with a two-term modeling of alternative attributes, aiming at providing more robust information on people choice behavior in those contexts that involve high vagueness or uncertainty and discusses the uncertainty effects on choice probability.

Background: The choice theories are mainly based on some basic assumptions on human behavior.

Choice predictions given by the Random Utility Theory [4] are based on the hypothesis of rational behavior. This theory asserts that the utility U_{in} associated by the user i to the option j is a random variable, given by the sum of a systematic utility V_{in}, which is a linear func-tion of the attribute values and of a random error termε_{in}:

$$U_{in} = V_{in} + \varepsilon_{in} \tag{1}$$

Several sources of randomness of the utility lead to the introduction of the random residualε_{in}.The probability distribution associated to the random term define the different models belonging to the group of the random util-ity models. For these models, it is assumed that the alternative with highest utility is chosen. The choice probability of alternative i by decision maker n is expressed in equation (2):

$$P_{in} = P[U_{in} \geq U_{jn} \forall j \neq i, j \in C_n] = P[U_{in} = MAX_{j \in C_n} U_{jn}] \tag{2}$$

These choice models have been used to predict consumer's behavior. However, several criticisms have been moved on the capabilities of such models in predicting choices, especially in the contexts under uncertainty or vagueness [5].

In most real-world problems, data have a certain degree of imprecision. Sometimes, this imprecision is small enough so that it can be safely ignored. On other occasions, the uncertainty of the data can be modeled by a probability distribution (e.g., additive random noise). Lastly, there is a third kind of problems where the imprecision is significant and a probability distribution is not a natural model [6] and although it has long been recognized that most purchase decisions are made with incomplete information, we still know very little about the effect of missing information on consumer choice [7].

In this new context, fuzzy logic may be viewed as the capability to make rational decisions in an environment of imprecision, uncertainty, incompleteness of information, conflicting information, partiality of truth and partiality of possibility-in short, in an environment of imperfect information [8]. So, fuzziness is considered as a basic type of subjective uncertainty which is initiated by Zadeh [9]. In some cases, the use of possibility theory was applied as an alternate way to probability theory in order to represent or measure uncertainty in decision makers [10] and as well as choice predictions were defined by the use of possibility and necessity measures [11].

In addition, the fuzzy set theory was usefully applied to cope with the vagueness of the informa-tion, giving the basis for fuzzy decision rules. Choice behavior was modeled using concepts from fuzzy sets and approximates reasoning which assumes that individuals make their choices based on simple rules relating perceptions to preferences which are modeled using fuzzy sets [12].

As Liu [13] stated, fuzziness and randomness simultaneously appeared in a system whereas a fuzzy-random variable was proposed as a tool to describe the quantities with fuzziness and randomness.

Therefore, probability and possibility theories showed interest-ing capabilities in understanding different features of human behavior and it was concluded that the treatment of uncertainty in decision-making is going through a paradigm shift from a probabilistic framework to a generalized framework that includes both probabilistic and non-probabilistic methods [14]. In this regards, a hybrid approach was proposed to combine fuzziness and randomness in travel choice prediction [1].

However, in consumer behavior context especially in vehicle choice problem there are some factors affects vehicle choices, which are potential sources of ambiguity and uncertainty so the fuzzy-random consideration and investigating uncertainty effect on probability choice is necessity.

Perception of Vehicle Choice Attributes-cognitive Variables: As two main types of uncertainty that may affect choice alternatives, which are randomness and fuzziness, the random component of uncertainty is related to the conditions in which vehicle choices are provided, affecting all choices, even if in quite different ways. The ran-domness of the choice alternatives is mainly influenced by the way the consumer's characteristics are organized. For example, sources of randomness for cognitive variables can be found in variances of consumer's characteristics such as age, gender, education level.

In addition, uncer-tainty affects consumer choice alternatives in many different choice contexts. This is particularly evident in the estimation of cognitive variables such as prestige-which affects people in order to choose vehicle type-, which are not in a crisp way.

This uncertainty in choice processes is related to the vagueness with which consumers perceive the attributes of choice alternatives. This type of uncertainty is mainly associated to human perceptions and it is usually influ-enced by the familiarity consumers have with the available choice alternatives. In fact, the perception of the effective choice attribute is indeed influenced by the level of confidence consumers have with choice alternatives. Consequently, it is realistic to assert that consumers have a more vague perception for those choice alternatives that they do not know very well. In these cases, consumers' perception is mainly influenced by the way the alternative looks like rather than by the real characteristics of the alternatives.

MATERIALS AND METHODS

The particular methodology for fuzzification we utilize in this work is based on the methodology studied by Dell'orco *et al*. [1] while some defuzzification methods are added in order to investigate the uncertainty effect on choice probability.

Therefore, the fuzzy-random approach to the descrip-tion of cognitive variables is considered aiming at combining both effects of randomness and fuzziness, in order to deal with choice predictions in complex contexts. The methodology shares the basic axioms of traditional choice mod-els, assuming that consumers are rational decision makers, who choose the alternative with the highest value of utility among the available choice alternatives.

We can write concisely the glossary of the methodology as follows and detail explanations hereafter:

Defining fuzzy-random attributes
↓
Defining fuzzy utility function
↓
Selecting choice model: Multinomial Logit (MNL) as a base
↓
Calculating fuzzy probability using fuzzy arithmetic
↓
Interpreting fuzzy results considering uncertainty effects
↓
Converting fuzzy results to crisp values using uncertainty concept
↓
Converting fuzzy results using standard defuzzification methods
↓
Comparing MNL result with defuzzified results and interpretation

The definition of the alternatives differs from tra-ditional approach in the description of cognitive variables. The possibility theory (fuzzy component) is applied to describe alternatives in terms of approximate val-ues (or intervals of values) [9], which represent people perception of cognitive variables. A triangular shaped membership function, defined by the center and by the spread, may be a practical assumption, commonly used in the application of fuzzy logic.

A systematic utility is defined based on the intervals of values of the alternative cognitive variables. Referring to the m attributes of an alternative, the systematic utility is a vector computed as a linear combination of fuzzy numbers as follows:

$$\tilde{V}_j = f(\tilde{x}_1^j, \tilde{x}_2^j, \ldots, \tilde{x}_m^j) = b_0^j + \sum_{i=1}^{m} b_i^j \tilde{x}_i^j \tag{3}$$

The perceived utility of the alternative *j* is, therefore, defined by the sum of the systematic utility and of the random termε_{in}:

$$\tilde{U}_j = \tilde{V}_j + \varepsilon_j \tag{4}$$

In other words, the fuzzy-random utility function can be considered as following expression:

$$\tilde{U}_j = \tilde{V}_j [+] L \tag{5}$$

Where *Vj* represents the fuzzy systematic utility and *L* is the probability function with which the random term ε_j is assumed to be distributed. The notation [+] ex-presses the combination between a fuzzy number and the probability function *L*. Consequently, the fuzzy-random Utility *Uj* can be completely individuated through the definition of the interval of values and of the probability distribution f(L) whereas the fuzzy nature of the fuzzy-random utility is represented by three values, l_{uj} ,u_j,r_{uj} which correspond to the values whose possibility measure is respectively 0 (left limit value), 1 (central value) and 0 (right limit value).

Therefore, the probability of choosing the alternative j assuming as a multinomial logit model is given by [4]:

$$\tilde{P}_j = \frac{e^{-\alpha \tilde{V}_j}}{\sum_k e^{-\alpha \tilde{V}_k}} \tag{6}$$

Arithmetical operations required to calculate the choice probability p_j are performed applying the extension principle of fuzzy logic, in dependence of the fuzzy values of the systematic utility. Since choice probabilities are calculated in dependence of the fuzzy values of the systematic utility of the alternatives, they are fuzzy values, too. They can be expressed in terms of intervals of values (l_{uj},u_j,r_{uj}).

In this representation of choice probabilities, the effects of uncertainty that affects cognitive variables are still present in the results of the application of the model. In fact, choice probabilities are expressed in a fuzzy way, conveying not a single value but a range of values. The lower the uncertainty of an alternative is, the narrower the interval of values. In other words, the interval of values expresses somehow the uncertainty associated to the choice probability, in dependence of the uncertainty related to consumers' perception of the attributes of the alternative. The amount of uncertainty can be calculated through the formula of Uncertainty [15]:

$$U(A) = \frac{1}{h(A)} \int_0^{h(A)} \log\left[1 + \mu({}^{\alpha}A)\right] d\alpha \qquad (7)$$

Where A is a finite nonempty set, U(A) is a measure of predictive uncertainty, h (A) is the height of A, ${}^{\alpha}A$is a measurable and Lebesgue-integrable function, $\mu({}^{\alpha}A)$ is the measure of ${}^{\alpha}A$ defined by Lebesgue integral of the characteristic function of ${}^{\alpha}A$.

However, in experimental analysis and planning processes, probability expressed as a fuzzy number is scarcely useful: for this reason, the amount of information I_j, conveyed by the triangle of fuzzy probability should be summarized into a unique term. To do this, in this approach we calcu-late a crisp value of probability as the average of central values of fuzzy probabilities, p_j, weighted with the information I_j:

$$P_{crisp,j} = \frac{P_j.I_j}{\sum_i P_i.I_i} \qquad (8)$$

Whereas information could be equivalent to uncertainty, therefore, replacing uncertainty instead of information as below:

$$P_{crisp,j} = \frac{P_j.I_j}{\sum_i P_i.I_i} = \frac{P_j.U_j}{\sum_i P_i.U_i} \qquad (9)$$

One of the main concerns after reaching to the crisp values in fuzzy approach is investigating the uncertainty effects on choice probability of alternatives.
We can write the equation (12) as below:

$$P_{UCF,j} = \frac{P_j.U_j}{\sum_i P_i.U_i} = P_j.\frac{U_j}{\tilde{U}} \qquad (10)$$

Whereas $P_{UCF,j}$ is considered as uncertainty-based crisp fuzzy (UCF) indication which explains a crisp value obtained from a fuzzy value.

Or we can write:

$$\frac{P_{UCF,j}}{P_j} = \frac{U_j}{\tilde{U}}; \qquad (11)$$

Which states the change of choice probability (uncertainty based crisp fuzzy (UCF) compare to traditional MNL) for each alternative depends on the relationship between uncertainty of the alternative and the average uncertainty, which the latter is obtained over all the alternatives. Thus, the proportion of uncertainty of each alternative to average uncertainty reveals the position of the choice probability in UCF approach compare to MNL.

This means:

If $\frac{U_j}{\tilde{U}} > 1$, then $\frac{P_{UCF,j}}{P_j} > 1$ and wise versa.

We may interpret as the uncertainty of the alternative is higher than average uncertainty; the choice probability of that alternative would be higher in UCF approach compare to MNL and wise versa.

In addition to investigating the uncertainty effect on choice probability, comparing choice probability based on traditional MNL and defuzzified output of the fuzzy approach is needed.

For this purpose, in order to keep the fuzziness limited, it is necessary to standardize the fuzzy vector with a defuzzified method before further calculation, so some defuzzification methods are used to be able to compare with traditional approach.

A most prevalent and physically appealing of all the defuzzification methods, center of gravity (COG) [16] is used. The algebraic expression of COG can be stated as follows:

$$F(\tilde{A}) = \frac{\int_A \mu f_A(\mu) d\mu}{\int_A f_A(\mu) d\mu} \qquad (12)$$

Where $f_A(\mu)$is the membership function of μ on the fuzzy set A.

When the fuzzy number A is triangular, i.e. A (*l, m, u*), the formula can be rewritten as below:

$$F_{\mu}(\tilde{A}) = (l + m + n)/3 \tag{13}$$

The other method for comparing fuzzy numbers is using Minkowski Coefficient, which in triangular fuzzy numbers case is expressed as follows [17]:

$$MC = (l + 4m + n)/6 \tag{14}$$

The other general approach to the problem of comparison of fuzzy numbers is to associate with a fuzzy number *F* some representative value, *Val* (*F*) and to compare the fuzzy subsets using these single representative values.

Yager and Filev, basing their work upon the transformation of a fuzzy subset into an associated probability distribution, extended this formulation and developed a generalized formulation for a class of valuation functions [18].

$$Val(F) = \frac{\int_0^1 Average(F_\alpha).f(\alpha).d\alpha}{\int_0^1 f(\alpha).d\alpha} \tag{15}$$

In the above f is a mapping from [0, 1] to [0, 1].

Which for a trapezoidal fuzzy set F (a,b,c,d), the valuation formula becomes:

$$Val(F) = \frac{\frac{1}{2}\int_0^1 [(b+c).\alpha + (1-\alpha).(a+d)].f(\alpha).d\alpha}{\int_0^1 f(\alpha).d\alpha} \tag{16}$$

Which it can be as following:

$$Val(F) = \left(\frac{b+c}{2}.w\right) + \left(\frac{a+d}{2}.(1-w)\right) \tag{17}$$

Where w is computed by:

$$w = \frac{\int_0^1 \alpha.f(\alpha).d\alpha}{\int_0^1 f(\alpha).d\alpha} \tag{18}$$

Considering above-mentioned methods of comparing fuzzy numbers through defuzzification and applying them to the choice problems, it is possible to compare the crisp fuzzy choice probabilities with the traditional MNL to see the effects of defuzzification on reordering of choice probabilities. It is obvious that the defuzzification methods produce one crisp number as a representative of each fuzzy numbers (as a choice probability) and the summation of these numbers will not be necessarily 100% needed to be reproduced in order to reach to 100% as well as MNL traditional output.

As we can see in these three standard defuzzification methods, the final choice probability for each alternative depends on all three elements of choice probabilities (left, central and right) and the typical summation of them. It means whatever the summation of these choice probabilities is higher, the final choice probability is higher and finally this fact will result in reordering of choice probabilities based on defuzzification methods versus MNL or even uncertainty based crisp fuzzy approach.

Experimental Study: The fuzzy-random methodology and defuzzification methods applied to a choice context in which three different alternatives are available in vehicle choice problem. This choice context recreates a typical choice situation for families or consumers in many large cities: the choice between several different car types, based on the analysis of the perceived cognitive variables of the types. The experimental context is geographically set in the Tehran, a mega city in Iran.

The three available vehicle types will be hereafter referred to as type Coupe, Hatchback 5 door and SUV. For each of them, the cognitive variables can be expressed by a fuzzy value to represent the consumers' uncertainty about the cognitive variables, which in this case are Prestige (P) and Roominess (R) that influence in vehicle choice. This value is expressed in terms of a numeric interval, individu-ated by the left limit value, the central value and the right limit value.

$$\begin{aligned} &Attributes: \\ &P(P_{jL}, P_{jM}, P_{jH}) \\ &R(R_{jL}, R_{jM}, R_{jH}) \end{aligned}$$

The intervals of cognitive variables used in this study are those corresponding to the perceived cognitive variables for the three vehicle choices, as arising from an experimental sur-vey carried out among consumers.

In the following paragraphs, choice predictions given by the fuzzy-random approach are compared to those obtained through a traditional Multinomial Logit Model.

The experimental survey: In the experimental survey, the average values of the attributes are summarized in Figure 1 and Figure2 as follows:

In the survey, the choice behavior of participants was observed and the choice rate for each alternative was thus calculated. The results of the survey were as follows: 14.53% of consumers preferred the type C; 41.88% chose the type H5 while the remaining 43.59% chose the type SUV.

The Fuzzy-random Approach: In the previous expression, systematic utility is ex-pressed in dependence of the unique relevant choice attribute, the perceived cognitive variables.

$$U_j = ASC_j + \beta_P P_j + \beta_R R_j \tag{19}$$

With the adoption of the fuzzy systematic utility, as previously explained, the choice probability of each alternative j can be valued as

$$\tilde{P}_j = \frac{e^{-\alpha \tilde{V}_j}}{\sum_k e^{-\alpha \tilde{V}_k}} \tag{20}$$

Considering fuzzy approach, the choice probability of each alternative j can be valued as

$$\begin{aligned} V_i &= ASC_i + \beta_P P_i + \beta_R R_i = ASC_i + \beta_P(P_{iL}, P_{iC}, P_{iR}) + \beta_R(R_{iL}, R_{iC}, R_{iR}) = \\ & ASC_i + (\beta_P P_{iL}, \beta_P P_{iC}, \beta_P P_{iR}) + (\beta_R R_{iL}, \beta_R R_{iC}, \beta_R R_{iR}) = \\ & ASC_i + (\beta_P P_{iL} + \beta_R R_{iL}, \beta_P P_{iC} + \beta_R R_{iC}, \beta_P P_{iR} + \beta_R R_{iR}) = \\ & (ASC_i + \beta_P P_{iL} + \beta_R R_{iL}, ASC_i + \beta_P P_{iC} + \beta_R R_{iC}, ASC_i + \beta_P P_{iR} + \beta_R R_{iR}) \\ exp(V_i) &= \exp((ASC_i + \beta_P P_{iL} + \beta_R R_{iL}, ASC_i + \beta_P P_{iC} + \beta_R R_{iC}, ASC_i + \beta_P P_{iR} + \beta_R R_{iR})) = \\ & (\exp(ASC_i + \beta_P P_{iL} + \beta_R R_{iL}), \exp(ASC_i + \beta_P P_{iC} + \beta_R R_{iC}), \exp(ASC_i + \beta_P P_{iR} + \beta_R R_{iR})) \\ exp(V_i) &= (\exp(V_{iL}), \exp(V_{iC}), \exp(V_{iR})) \\ P_i &= \exp(V_i) / \sum_j \exp(V_j) \end{aligned} \tag{21}$$

For the exponential of a fuzzy number we are referring to [19] which proof extraction of the exponential of a fuzzy number as:

If A was a two intervals fuzzy number as $A = (a,b)$ then the exponential of A would be as $Exp(A) = (Exp(a), Exp(b))$.

In equation 21, the interval for each attribute is defined as the attribute evaluation range.

Using the multinomial logit model (MNL) and data col-lected in the survey, the parameters of the attributes estimated and considering fuzzy approach to reach central choice probability as well as intervals choice probabilities (left and right).

The central probability in this fuzzy approach is equal to the calculated probability in traditional MNL; and for each alternative, there are two other probabilities based on the intervals considered in attribute evaluation range that they indicate the lower and upper possible probability.

Using the consumers' preferences, the alternative specific constants and parameters of prestige and roominess estimated as below:

The computing phase has been assisted by the use of MATLAB 8.0 software. Applying the described methodology and using the fuzzy arithmetic rules, fuzzy choice probabilities for all available types have been calculated. Each of them is described by an interval of values.

The probability of choosing each vehicle type is a fuzzy number indicates the range of acceptable probability. Thus, in this approach instead of having one precise choice probability, we have a fuzzy choice probability as an interval number.

As previously mentioned, the central values are not the only probability values acceptable for the three vehicle types. All values included in the support of fuzzy triangles are possible, even if a lower value of possi-bility is associated to them.

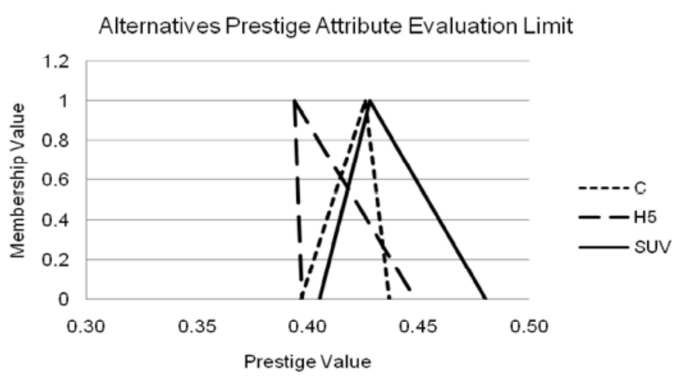


Fig. 1: family's perception of vehicle type Prestige factor

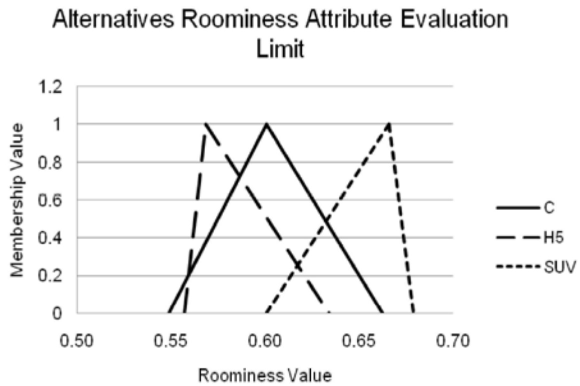


Fig. 2: family's perception of vehicle type Roominess factor

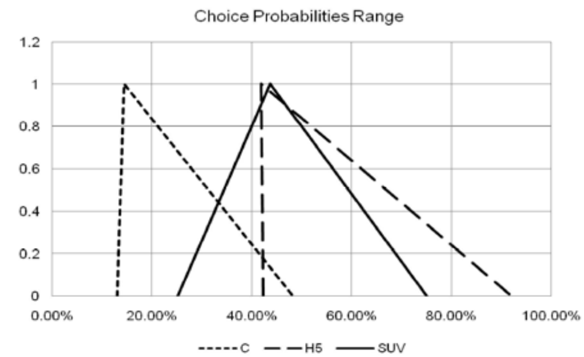


Fig. 3: Choice probabilities of the Vehicle Types

As we see, the right or upper limits for all of the alternatives are far from the central limits while the left or lower limits of two alternatives (e.g. C and H5) are close to the central limits.

Although in general, SUV has higher choice probability based on its central limit but in some cases, H5 has a higher choice probability rather than SUV because the upper limit of the H5 alternative is higher than other alternatives. This result can be obtained also by comparing the left limits of the H5 and SUV alternatives which state the left limit of the SUV alternative is lower than H5 alternative that results in higher choice probability of H5 rather than SUV in some conditions.

However, in order to provide practitioners with a unique value of proba-bility, the uncertainties related to the fuzzy proba-bilities are calculated:

Then, the values of crisp proba-bility for the three vehicle types have been calculated, as a function of the central values *pj* and the uncer-tainty values:

The Multinomial Logit (MNL) Model: A traditional Multinomial Logit model has been applied to the same choice context. The results of the application of the MNL model have been compared with the results of the

Table 1: Estimated Parameters

ASCs of Alternatives			Estimated parameters(Beta)	
C	H5	SUV	B_B-Prestige	B_B-Roominess
-1.16	0.00	-0.758	-5.82	10.1

Table 2: Uncertainty Index

	C	H5	SUV
Uncertainty Index	0.071	0.065	0.075

Table 3: Uncertainty based Crisp Fuzzy Choice Probability

	Alternatives		
Elements	C	H5	SUV
Uncertainty based Crisp Fuzzy (UCF) Choice Probability (%)	14.67%	38.47%	46.86%
Uncertainty Index (0-1)	0.071	0.065	0.075
Weighted uncertainty (0-1)			0.070

Table 4: MNL and Choice Rate

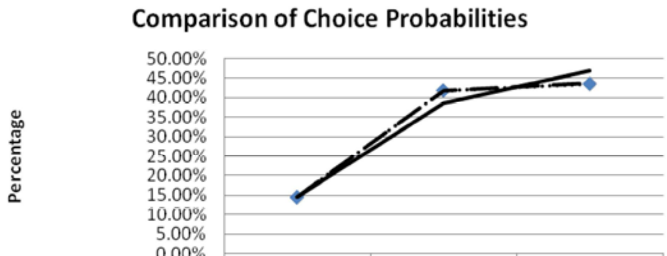
	Alternatives		
Elements	C	H5	SUV
Choice Rate	14.53%	41.88%	43.59%
MNL	14.48%	41.83%	43.69%

Table 5: Uncertainty of alternatives versus average uncertainty

	Average Uncertainty Calculation			
Alternatives	Uncertainty	Probability* Uncertainty	Uncertainty/Average Uncertainty	Average Uncertainty
C	0.071	0.010	1.01	0.070
H5	0.065	0.027	0.92	
SUV	0.075	0.033	1.07	

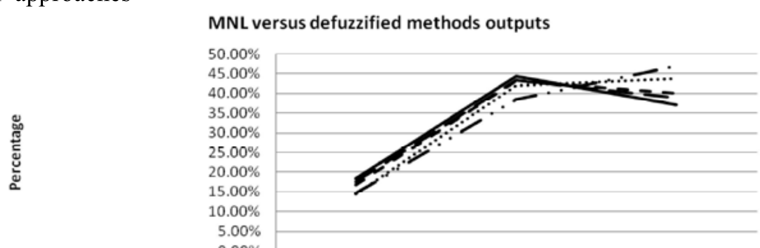
application of the fuzzy-random model. Choice predictions of the MNL model are based on the crisp values of cognitive variables of the alternatives. In the analyzed case study, the probability of choos-ing an alternative *j* is therefore expressed in depen-dence of the cognitive variables values in which the cognitive variables values are the central values of the fuzzy numbers, since they represent the most possible values obtained through the survey.

Likelihood Ratio was computed to test the null hypothesis that the coefficient is zero: the value resulted rather high (36.113), confirming that the coefficient is statis-tically different from zero. The Rho-square indicator for the presented model was 0.140. As for the vali-dation, the statistic ‘‘% right’’ was computed: it is equal to 43.8%. The choice probabilities for the three vehicle types, as predicted by the Multinomial Logit model, are respectively.



	C	H5	SUV
Choice Rate	14.53%	41.88%	43.59%
MNL	14.48%	41.83%	43.69%
Uncertainty based Crisp Fuzzy (UCF)	14.67%	38.47%	46.86%

Fig. 4: MNL versus UCF approaches



	C	H5	SUV
MNL	14.48%	41.83%	43.69%
Uncertainty based Crisp Fuzzy (UCF)	14.67%	38.47%	46.86%
Center Of Gravity	18.44%	44.38%	37.18%
Minkowski Coefficient	16.72%	43.27%	40.01%
Alpha-weighted valuations	17.67%	43.42%	38.91%

Fig. 5: MNL versus defuzzified outputs by different methods

Uncertainty Effect and Defuzzification: As described in the methodology, one of the main concerns after reaching to the crisp values in fuzzy approach is investigating the uncertainty effects on probability of choosing alternatives.

Figure 4 shows the results of uncertainty based crisp fuzzy approach versus traditional MNL.

This indicates the choice probability of alternatives such as Coupe and SUV are increased (in terms of 1.01 and 1.08) while the choice probability of H5 is decreased (in terms of 0.92) versus relevant choice probabilities in MNL approach.

This result is equal to the relationship between uncertainty of the alternative and the average uncertainty, which obtained over all the alternatives as Table 5.

This experimental study indicates the relationship between changes of choice probability in fuzzy approach versus MNL is equal to the proportion of alternatives uncertainty versus average uncertainty. For instance, the choice probability of SUV type in this fuzzy approach is equal to 1.08 of MNL approach.

In addition to this uncertainty effect, three mentioned standard defuzzification methods applied to this choice context, the results are compared with the MNL and uncertainty based crisp fuzzy as follows:

As result indicates the defuzzified output of choice probabilities in fuzzy approach are different from MNL traditional choice probabilities and from uncertainty based crisp fuzzy probabilities. The results show the choice probability of SUV type is decreased while for two others is increased.

As mentioned before, we can interpret this result so as defuzzification methods are based on typical average of three interval elements (left, central and right) of choice probabilities and this fact results in reordering of choice probabilities.

As summation of three interval choice probabilities of H5 alternative is higher than others, its final choice probability is higher than other alternatives. In addition, the ordering of final choice probabilities is different from MNL and even uncertainty based crisp fuzzy approach as follows:

$$MNL: P_{SUV} > P_{H5} > P_C;$$
$$UCF: P_{SUV} > P_{H5} > P_C;$$
$$SDM: P_{H5} > P_{SUV} > P_C;$$

Whereas MNL as Multinomial Logit, UCF as Uncertainty based Crisp Fuzzy and SDM as Standard Defuzzification Methods are considered.

RESULTS AND DISCUSSION

In this study, a fuzzy-random approach for the predic-tion of consumer's choice behavior has been implemented aiming at deal with the uncertainty and variability involved in vehicle choice contexts considering three alternatives and two attributes.

As the experimental study indicated the MNL model is not able to capture effects of both random and uncertainty variability because of adoption of a unique param-eter to explain these sources of uncertainty while the fuzzy-random model is capable of understanding some additional effects especially uncertainty on family's choice behavior originated from attributes vagueness and preserves consumers' uncer-tainty until the end, expressing choice probabilities in terms of intervals of values and this result justify the accomplishment of the objectives of the study.

Therefore, choice predictions are expressed in two ways: crisp way and fuzzy way in terms of interval of values. This fuzzy value of choice probabilities is directly related to the uncer-tainty commonly associated to the alternative attri-butes: the interval of values represents the effect of vagueness on consumers' choices. Although the max-imum level of possibility is associated to only one value of choice probability, the model allows assuming also different values of choice prob-abilities with lower levels of possibility which results in having a range or interval for the choice probability in stead of a precise value.

However, this is not useful in practical applications and therefore we used a way to calculate a unique probabil-ity, taking into account also the consumers' uncer-tainty.

In this regard, at first, an uncertainty based crisp fuzzy (UCF) approach is introduced and implemented to convert output of fuzzy approach to a crisp value for each alternative. This study indicates the relationship between changes of choice probability in UCF approach versus MNL is equal to the proportion of alternative's uncertainty versus average uncertainty. If the uncertainty of one alternative is higher than average uncertainty, the choice probability in UCF approach is higher than MNL and wise versa.

Specifically, it was found that increasing uncertainty or variability of one alternative rather average uncertainty- which can be calculated over all alternatives-, could make it more attractive in UCF method so as the uncertainty in alternative is higher than average uncertainty, change (positive or negative) of consumer' choice probability to alternative is higher in UCF approach compare to MNL.

In addition, we applied three standard defuzzification methods in order to compare the results of MNL with defuzzified output of fuzzy approach. In this approach, there is a possibility to see the effect of changing definition of attributes into fuzzy interval values on choice probabilities of alternatives which indicate the final choice probability depends on summation of these three choice probabilities and whatever this summation is higher, the final choice probability is higher which finally produces reordering of the choice probabilities. Although the results were different from UCF approach for uniqueness of choice probability in uncertainty conditions, there is a possibility to see the effects of defuzzification on reordering of choice probabilities of alternatives.

The results of the application of the meth-odology have been compared with those arising from the application of a traditional MNL model.

The comparison between the two approaches has allowed defining the possible advantages deriving from the application of the fuzzy-random methodology. The fuzzy-random approach seems to be more able to cap-ture the effects of uncertainty on consumers' percep-tions and consequently on vehicle choice behavior.

This methodology is useful in those cases in which uncertainty, as well random variability, affects the choice behavior and has also a simple structure as the MNL model, which confers it a great easiness of use.

Further research will deal with the application of such models to complex choice contexts as well as the other choice models rather than Multinomial Logit, in order to test the capabilities of these models to predict choices when different levels of variability and uncertainty are involved. And on utilizing defuzzification methods, although we indicated some reasons for different results, which are derived from different defuzzification methods, investigating the difference between these approaches are necessary to investigate in future researches.

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