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Stationary Solitons for Langmuir Waves in Plasmas

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Abstract: This paper talks about the stationary solitons for Langmuir waves in plasmas. The Lie symmetry approach is used to carry out the integration of the governing nonlinear Schrdinger's equation with perturbation terms.

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INTRODUCTION

The nonlinear Schringer's equation (NLSE) in its dimensionless form has important applications in Plasma Physics, Nonlinear Optics, Fluid Dynamics, Mathematical Biology and various other areas in Physical Sciences and Engineering [1-15]. In Plasma Physics, it describes the electron (Langmuir) waves [1, 2]. The NLSE is given by

$$iq_t + aq_{xx} + b|q|^2 q = 0 \quad (1)$$

where a and b are constants. Equation (1) is integrable by the method of Inverse Scattering Transform (IST) [4]. The solutions of (1) are called solitons. Langmuir solitons, in the form of cavitons, were observed in 1974. These cavitons are local regions from which plasma is ousted by the electromagnetic field. In presence of strong magnetic field, cavitons in moving plasma were observed in 1976-77 [1, 4]. Ion acoustic solitons have been detected earlier in 1970-71 [1, 4]. Equation (1) has been extensively studied by Wazwaz in the context of Nonlinear Optics. He integrated this equation in presence of fourth order dispersion [14,15].

The emission of Langmuir waves in the form of small-scale localized electrostatic bursts have been observed directly in waveform data in many space plasma environments, such as solar wind, auroral region and polar cap. The comparison of observations in various space regions has shown that emissions are seen in associations with warm electron fluxes and have common characteristic properties such as burst-like character, an irregular structure, amplitude variations and a low frequency modulation. Langmuir wave bursts occur in association with electron fluxes with energies 100-400 eV propagating from distant regions of the magnetosphere during magnetic disturbances. The results of bicoherence analysis of wave data have shown that usually the parametric decay process does not play an important role in the formation of Langmuir wave bursts. It has been found that a typical power spectrum width of single burst is about 10% of the local plasma frequency, which is larger than the width generated by the thermal effect in Langmuir dispersion. Moreover, power spectra have usually a characteristic form with a dent in the upper part. Power time evolution studies show that these small-scale bursts tend to be correlated with the level of the low frequency wave power. Thus in the framework of the electron beam-plasma interaction, the presence of the low frequency turbulence is expected to play a prominent role in the generation of these plasma oscillations. The theoretical model in the quasi-linear statistical approximation has been developed for beam-plasma

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instability in the magnetized plasma in the presence of low frequency turbulence. It has been shown that the beat-type waveforms of Langmuir emissions can be explained by interference between waves excited by an electron beam and scattering off the density fluctuations. The frequency width of the burst spectrum increases sufficiently due to the resonant wave scattering providing the wave power access to phase space regions with low growth rates [8, 9].

An important property of (1) is that it has infinitely many integrals of motion. The first three of which are given by [1].

$$N = \int_{-\infty}^{\infty} |q|^2 \, dx \tag{2}$$

$$M = ia \int_{-\infty}^{\infty} (q q_x^* - q^* q_x) \, dx \tag{3}$$

$$H = \int_{-\infty}^{\infty} (|q_x|^* - |q|^4) \, dx \tag{3}$$

where N represents the plasmon number, while, M is the linear momentum of the soliton and H gives the Hamiltonian.

Perturbation Terms: The perturbed NLSE that is going to be considered in this paper is [4, 7]:

$$iq_t + aq_{xx} + b|q|^2 q = a|q|^2 q_{xx} + v\frac{q_{xx}^*}{|q|^2} q^2 \tag{5}$$

where a and v are constants. The a -term arises in the study of interaction between Langmuir waves and ion acoustic waves in plasmas, provided the velocity of the Langmuir waves is small as compared to the sound velocity [2]. The coefficient of v accounts for the propagation of solitons in plasmas with sharp boundaries and dissipation.

This term arises in the Gradov-Stenflo equation [1]. In this paper, the focus is going to be on obtaining the localized stationary solution to (5) of the form [5]

$$q(x,t) = \phi(x) e^{-i\lambda t} \tag{6}$$

where λ is a constant and the function ϕ depends on the variable x alone. Thus, from (5) and (6), $\phi(x)$ satisfies the time independent inhomogeneous nonlinear equation that is given by

$$\lambda\phi + a\phi'' + b\phi^3 = a\phi^2\phi'' + v\phi'' \tag{7}$$

Mathematical Analysis: Equation (7) has a single lie point symmetry, namely $X = \partial/\partial x$. This symmetry will be used to integrate equation (7) once. It can be easily seen that the two invariants are

$$u = \phi \tag{8}$$

and

$$\upsilon = \phi' \tag{9}$$

Treating u as the independent variable and v as the dependent variable, (7) can be rewritten as

$$\frac{d\upsilon}{du} = \frac{\lambda u + b\upsilon^3}{\upsilon(au^2 + \upsilon - a)} \tag{10}$$

Integrating (10) yields

$$\upsilon^2 = \frac{1}{a^2}\left\{bau^2 + (ab - b\upsilon + \lambda a) In(au^2 + \upsilon - a)\right\} + c_1 \tag{11}$$

where c_1 is an arbitrary constant of integration. Now, rewriting (11) in terms of the variable ϕ gives

$$\left(\frac{d\phi}{dx}\right)^2 = \frac{1}{a^2}\left\{ba\phi^2 + (ab - b\upsilon + \lambda a) In(a\phi^2 + \upsilon - a)\right\} + c_1 \tag{12}$$

that leads to the quadrature

$$\frac{x}{a^2} + c_2 = \int \frac{d\phi}{\sqrt{ba\phi^2 + (ab - bv + \lambda a) In(a\phi^2 + v - a) + a^2 c_1}} \tag{13}$$

where c_2 is an arbitrary constant of integration.

CONCLUSIONS

In this paper, the stationary soliton solution is obtained for relativistic plasmas. Here, the Lie symmetry approach is used to carry out the integration of the NLSE with Kerr law nonlinearity. The resulting solution is in quadratures. In future, these results will be generalized to the case of NLSE with power law nonlinearity.

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REFERENCES

1. Biswas, A., 2008. Soliton perturbation theory for Alfven waves in plasmas. Physics of Plasmas. 12(2): 022306.
2. Biswas, A., 2009. Quasi-stationary solitons for Langmuir waves in plasmas". Communications in Nonlinear Science and Numerical Simulations, 14(1): 69-76.
3. E-Emary, M.M. and M.M.A. E-Kareem, 2008. Towards using genetic algorithm for solving nonlinear equation systems. World Applied Sciences Journal, 5(3): 282-289.
4. Hadzievski, L., M.S. Jovanovic, M.M. Skoric and K. Mima, 2002. Stability of one-dimensional electromagnetic solitons in relativistic laser plasmas. Physics of Plasmas, 9(6): 2569-2574.
5. Khalique, M. and A. Biswas, 2009. Optical solitons with parabolic and dual-power law nonlinearity via Lie symmetry analysis. Journal of Electromagnetic Waves and Applications, 23(7): 963-973.
6. Kohl, R., A. Biswas, D. Milovic and E. Zerrad, 2008. Optical soliton perturbation in a non-Kerr law media. Optics and Laser Technology, 40(40: 647-662.
7. Mancic, L. Hadzievski and M.M. Skoric, 2006. Dynamics of electromagnetic solitons in relativistic plasmas. Physics of Plasmas, 13: 052309.
8. Misra, P. and A. Roy Chowdhury, 2002. Modulated ion-acoustic wave near criticalness - A new nonlinear Schrodinger equation in a collisional plasma. Fizika A., 11(4): 163-176. 163-176.
9. Misra, P. and A. Roy Chowdhury, 2006. Modulational instability of dust acoustic waves in a dusty plasma with nonthermal electrons and ions". The European Physical Journal D., 39(1): 49-57.
10. Mohyuddin, S.T. and M.A. Noor, 2008. Solving Schrdinger equations by modified variational iteration method". World Applied Sciences Journal, 5(3): 352-357.
11. Mohyuddin, S.T. and M.A. Noor, 2009. Modified variational iteration method for solving sine-Gordon equations. World Applied Sciences Journal, 6(7): 999-1004.
12. Mohyuddin, S.T. and M.A. Noor, 2009. Modified variation of parameters method for second order integro-differential equations and coupled systems. World Applied Sciences Journal, 6(8): 1139-1146.
13. Tian, F. and F. Austin, 2009. Application of He's max-min approach to a generalized nonlinear oscillator. World Applied Sciences Journal, 6(7): 1005-1007.
14. Wazwaz, A.M., 2009. The variational iteration method for analytic treatment for linear and nonlinear ODEs. Applied Mathematics and Computation, 212(1): 120-134.
15. Wazwaz, A.M., 2006. Reliable analysis for nonlinear Schrdinger equations with a cubic nonlinearity and a power law nonlinearity". Mathematical and Computer Modelling, 43(1-2): 178-184.