Numerical Techniques for Traveling Wave Solutions of Nonlinear Couple KdV Equations

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Abstract: In this paper, we implemented two very reliable techniques which are called He's Homotopy Perturbation (HPM) and Tanh methods for solving evolution equations. The proposed algorithms have been successfully tested on an important evolution equation namely Hirota-Satsuma coupled system. The calculations demonstrate the effectiveness and convenience of Tanh method for nonlinear system of PDEs. Numerical results are very encouraging.

Keywords: Tanh method · homotopy perturbation method · nonlinear system of PDES · exact solutions · hirota-satsuma coupled system

INTRODUCTION

Nonlinear coupled partial differential equations are very important in a variety of scientific fields, especially in fluid mechanics, solid state physics, plasma physics, plasma waves, capillary-gravity waves and chemical physics. The nonlinear wave phenomena observed in the above mentioned scientific fields, are often modeled by the bell-shaped sech solutions and the kink-shaped tanh solutions. The availability of these exact solutions, for those nonlinear equations can greatly facilitate the verification of numerical solvers on the stability analysis of the solution [2, 3]. In this study, we consider two coupled KdV equations. This paper outlines the implementation of two very efficient and reliable techniques which are called He's homotopy perturbation method (HPM) and Tanh method for solving system of coupled equations which are very important in applied sciences. The HPM [8-16, 17-21] and Tanh methods have been successfully tested on Hirota-Satsuma coupled system. It is worth mentioning that homotopy perturbation method (HPM) was developed by He [11-16] by merging the standard homotopy and perturbation. The hyperbolic tangent (tanh) method is a powerful technique to symbolically compute traveling wave solutions of one-dimensional nonlinear wave and evolution equations. In particular, the method is well suited for problems where dispersion, convection and reaction diffusion phenomena play an important role [1].

OUTLINE OF THE TANH METHOD

The tanh method will be introduced as presented by Malfliet [4] and by Wazwaz [5-7]. The tanh method is based on a priori assumption that the traveling wave solutions can be expressed in terms of the tanh function to solve the coupled KdV equations. The tanh method is developed by Malfliet [4]. The method is applied to find out exact solutions of a coupled system of nonlinear differential equations with three unknowns:

$$P_1(u,v,w,u_x,v_x,w_x,u_t,v_t,w_t,u_{xx},v_{xx},w_{xx},..........) = 0$$

$$P_2(u,v,w,u_x,v_x,w_x,u_t,v_t,w_t,u_{xx},v_{xx},w_{xx},..........) = 0$$

$$P_3(u,v,w_x u,y,w,u_t,v_t,w_t,u_{xx},v_{xx},w_{xx},..........) = 0 \quad (2.1)$$

where P_1, P_1, P_1 are polynomials of the variable *u, v, w* and its derivatives. If we consider u (x, t) = u (ξ), v (x, t) = v (ξ), w (x, t) = w (ξ), ξ = k (x-λt), so that u (x, t) = U (ξ), v (x, t) = v (ξ), w (x, t) = W (ξ), we can use the following changes:

$$\frac{\partial}{\partial t} = -k\lambda\frac{d}{d\xi}, \quad \frac{\partial}{\partial x} = k\frac{d}{d\xi},$$

$$\frac{\partial^2}{\partial x^2} = k^2\frac{d^2}{d\xi^2}, \quad \frac{\partial^3}{\partial x^3} = k^3\frac{d^3}{d\xi^3},$$

and so on, then Eq. (2.1) becomes an ordinary differential equation

$$Q_1(U,\dot{U},U',U''',..........) = 0$$

$$Q_2(U,\dot{U},U'',U''',..........) = 0$$

$$Q_3(U,\dot{U},U',U''',..........) = 0 \quad (2.2)$$

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with Q_1, Q_1, Q_l being another polynomials form of there argument, which will be called the reduced ordinary differential equations of Eq. (2.2). Integrating Eq. (2.2) as long as all terms contain derivatives, the integration constants are considered to be zeros in view of the localized solutions. However, the nonzero constants can be used and handled as well [7]. Now finding the traveling wave solutions to Eq. (2.1) is equivalent to obtaining the solution to the reduced ordinary differential equation (2.2). For the tanh method, we introduce the new independent variable

$$Y(x,t) = \tanh(\xi) \qquad (2.3)$$

that leads to the change of variables:

$$\frac{d}{d\xi} = (1 - Y^2)\frac{d}{dY}$$

$$\frac{d^2}{d\xi^2} = -2Y(1 - Y^2)\frac{d}{dY} + (1 - Y^2)^2\frac{d^2}{dY^2}$$

$$\begin{aligned}\frac{d^3}{d\xi^3} = {} & 2(1 - Y^2)(3Y^2 - 1)\frac{d}{dY} \\ & -6Y(1 - Y^2)^2\frac{d^2}{dY^2} + (1 - Y^2)^3\frac{d^3}{dY^3}\end{aligned} \qquad (2.4)$$

The next crucial step is that the solution we are looking for is expressed in the form

$$u(x,t) = U(\xi) = \sum_{i=1}^{m} a_i Y^i$$

$$v(x,t) = V(\xi) = \sum_{i=1}^{n} b_i Y^i$$

$$w(x,t) = W(\xi) = \sum_{i=1}^{s} c_i Y^i \qquad (2.5)$$

where the parameters *m, n, s* can be found by balancing the highest-order linear term with the nonlinear terms in Eq. (2.2) and $k, \lambda, a_0, a_b, \ldots, a_m, b_0 b_{\flat}, \ldots, b_m, c_0 c_{\flat}, \ldots, c_m$ are to be determined. Substituting (2.5) into (2.2) will yield a set of algebraic equations for $k, \lambda, a_0, a_b, \ldots, a_m, b_0 b_{\flat}, \ldots, b_m, c_0 c_{\flat}, \ldots, c_m$ because all coefficients of Y^i have to vanish. From these relations, $k, \lambda, a_0, a_b, \ldots, a_m, b_0 b_{\flat}, \ldots, b_m, c_0 c_{\flat}, \ldots, c_m$ can be obtained. Having determined these parameters, knowing that m, n, s are positive integers in most cases and using (2.5) we obtain analytic solutions u (x, t), v (x, t), w (x, t) in a closed form [7]. The tanh method seems to be powerful tool in dealing with coupled nonlinear physical models.

HOMOTOPY PERTURBATION METHOD (HPM)

To explain the homotopy perturbation method, we consider a general equation of the type,

$$L(u) = 0 \qquad (3.1)$$

where L is any integral or differential operator. We define a convex homotopy H (u, p) by

$$H(u,p) = (1 - p)F(u) + pL(u) \qquad (3.2)$$

where F (u) is a functional operator with known solutions v0, which can be obtained easily. It is clear that, for

$$H(u, p) = 0 \qquad (3.3)$$

we have

$$H(u, 0) = F(u) \quad H(u, 1) = L(u)$$

This shows that H (u, p) continuously traces an implicitly defined curve from a starting point H (v0, 0) to a solution function H (f, 1). The embedding parameter monotonically increases from zero to unit as the trivial problem F (u) = 0 is continuously deforms the original problem L (u) = 0. The embedding parameter $p \in (0, 1]$ can be considered as an expanding parameter [2-10, 12-16]. The homotopy perturbation method uses the homotopy parameter p as an expanding parameter [5-10] to obtain

$$u = \sum_{i=0}^{\infty} p^i u_i = u_0 + p u_1 + p^2 u_2 + p^3 u_3 + \cdots, \qquad (3.4)$$

if p→1, then (3.4) corresponds to (3.2) and becomes the approximate solution of the form,

$$f = \lim_{p \to 1} u = \sum_{i=0}^{\infty} u_i \qquad (3.5)$$

It is well known that series (3.5) is convergent for most of the cases and also the rate of convergence is dependent on L (u); see [11-16]. We assume that (3.5) has a unique solution. The comparisons of like powers of p give solutions of various orders.

NUMERICAL APPLICATIONS

In this section, we apply the proposed tanh and homotopy perturbation methods for solutions of system of coupled KdV equations.

Example 4.1.1 Consider the following Hirota-Satsuma coupled system

$$u_t - \frac{1}{2}u_{xxx} + 3uu_x - 3(vw)x = 0$$

$$v_t - v_{xxx} - 3uv_x = 0 \tag{4.1}$$

$$w_t + w_{xxx} - 3uw_x = 0$$

with initial conditions

$$u(x,0) = \frac{1}{3}(\beta - 2k^2) + 2k^2 \tanh^2(kx)$$

$$v(x,0) = \frac{-4 k^2 c_0 (\beta + k^2)}{3c_1^2} + \frac{4k^2 (\beta + k^2)}{3c_1} \tanh(kx) \tag{4.2}$$

$$w(x,0) = c_0 + c_1 \tanh(kx)$$

where c_0, c_1 and β are constants. The exact solution of the problem is given by

$$u(x,t) = \frac{1}{3}(\beta - 2k^2) + 2k^2 \tanh^2(k(x + \beta t))$$

$$v(x,t) = \frac{-4k^2 c_0 (\beta + k^2)}{3c_1^2} + \frac{4k^2 (\beta + k^2)}{3c_1} \tanh(k(x + \beta t)) \tag{4.3}$$

$$w(x,t) = c_0 + c_1 \tanh(k(x + \beta t))$$

Applying homotopy perturbation method (HPM)

$$u_0 + pu_1 + \cdots = u_0(x,t) + p\int_0^t \left(\frac{1}{2}\left(\frac{\partial^3 u_0}{\partial x^3} + p\frac{\partial^3 u_1}{\partial x^3} + \cdots \right) - 3(u_0 + pu_1 + \cdots)\left(\frac{\partial u_0}{\partial x} + p\frac{\partial u_1}{\partial x} + \cdots \right) \right) dx$$

$$+ p\int_0^t \left(3((v_0 + pv_1 + \cdots)(w_0 + pw_1 + \cdots)x) \right) dx$$

$$v_0 + pv_1 + \cdots = v_0(x,t) + p\int_0^t \left(\left(\frac{\partial^3 v_0}{\partial x^3} + p\frac{\partial^3 v_1}{\partial x^3} + \cdots \right) + 3(u_0 + pu_1 + \cdots)\left(\frac{\partial v_0}{\partial x} + p\frac{\partial v_1}{\partial x} + \cdots \right) \right) dx$$

$$w_0 + pw_1 + \cdots = w_0(x,t) + p\int_0^t \left(-\left(\frac{\partial^3 w_0}{\partial x^3} + p\frac{\partial^3 w_1}{\partial x^3} + \cdots \right) + 3(u_0 + pu_1 + \cdots)\left(\frac{\partial w_0}{\partial x} + p\frac{\partial w_1}{\partial x} + \cdots \right) \right) ds$$

Comparing the co-efficient of like powers of p, following approximants are obtained

$$p^{(0)}: \begin{cases} u_0(x,t) = \frac{1}{3}(\beta - 2k^2) + 2k^2 \tanh^2(kx) \\ v_0(x,t) = \frac{-4k^2 c_0 (\beta + k^2)}{3c_1^2} \\ \qquad + \frac{4k^2 (\beta + k^2)}{3c_1} \tanh(kx) \\ w_0(x,y,t) = c_0 + c_1 \tanh(kx) \end{cases} \quad , \quad p^{(1)}: \begin{cases} u_1(x,t) = \frac{1}{3}(\beta - 2k^2) + 2k^2 \tanh^2(kx) \\ \qquad -2\frac{\cosh x + 2t \sinh x}{\cosh^3 x} \\ v_1(x,t) = 2\left(\frac{\cosh^2 x + \cosh x + t \sinh x}{\cosh^2 x} \right) \\ w_1(x,y,t) = 2\left(\frac{-\cosh^2 x + \cosh x + t \sinh x}{\cosh^2 x} \right) \end{cases}$$

$$\vdots$$

The solution is given by

$$u(x,t) = \frac{1}{3}(\beta - 2k^2) + 2k^2 \tanh^2(kx) - 2\frac{\cosh x + 2t \sinh x}{\cosh^3 x} - \frac{2}{\cosh^7 x}\left(\cosh^5 x - 3t^2 \cosh^3 x + 2t\sinh x \cosh^4 x \right)$$

$$- \frac{2}{\cosh^7 x}\left(2t^3 \sinh x \cosh^4 x - 20t^3 \sinh x \cosh^2 x + 2t^2 \cosh^5 x + 24t^3 \sinh x \right) + \cdots$$

$$u(x,t) = \frac{1}{3}(\beta - 2k^2) + 2k^2 \tanh^2(kx) - 2\frac{\cosh x + 2t \sinh x}{\cosh^3 x} - \frac{2}{\cosh^7 x}\left(\cosh^5 x - 3t^2 \cosh^3 x + 2t\sinh x \cosh^4 x \right)$$

$$- \frac{2}{\cosh^7 x}\left(2t^3 \sinh x \cosh^4 x - 20t^3 \sinh x \cosh^2 x + 2t^2 \cosh^5 x + 24t^3 \sinh x \right) + \cdots$$

$$w(x,t) = c_0 + c_1 \tanh(kx) + 2\left(\frac{-\cosh^2 x + \cosh x + t\sinh x}{\cosh^2 x}\right) + \frac{1}{\cosh^6 x}\left(2\cosh^5 x + t^2\cosh x - 2t^2 \cosh^3 x\right)$$

$$+ \frac{1}{\cosh^6 x}\left(2t\sinh x\cosh^4 x + 8t^3 \cosh^2 x \sinh x - 2\cosh^6 x - 16t^3 \sinh x\right) + \cdots \qquad (4.4)$$

The closed form solution is given as

$$(u,v,w) = (e^{x+y-t}, e^{x-y+t}, e^{-x+y+t})$$

Example 4.1.2: Re-consider the following Hirota-Satsuma coupled system

$$u_t - \frac{1}{2}u_{xxx} + 3uu_x - 3(vw)x = 0$$

$$v_t - v_{xxx} - 3uv_x = 0 \qquad (4.5)$$

$$w_t + w_{xxx} - 3uw_x = 0$$

Using the traveling wave transformations:

$$u(x,t) = U(\xi) = \sum_{i=1}^{m} a_i Y^i$$

$$v(x,t) = V(\xi) = \sum_{i=1}^{n} b_i Y^i$$

$$w(x,t) = W(\xi) = \sum_{i=1}^{n} c_i Y^i \qquad (4.6)$$

Where

$$Y = \tanh(\xi) \qquad (4.7)$$

$$\xi = k(x + \beta t) \qquad (4.8)$$

The nonlinear system of partial differential equations (4.5) is carried to a system of ordinary differential equations

$$k\beta U' - \frac{1}{2}k^3 U''' + 3kUU' - 3kVW' - 3kWV' = 0$$

$$k\beta V' - k^3 V''' - 3kUV' = 0 \qquad (4.9)$$

$$k\beta W' + k^3 W''' - 3kUW' = 0$$

we postulate the following tanh series in (4.6) and the transformation given in (4.7) and (4.8) the first equation in (4.9) reduces to
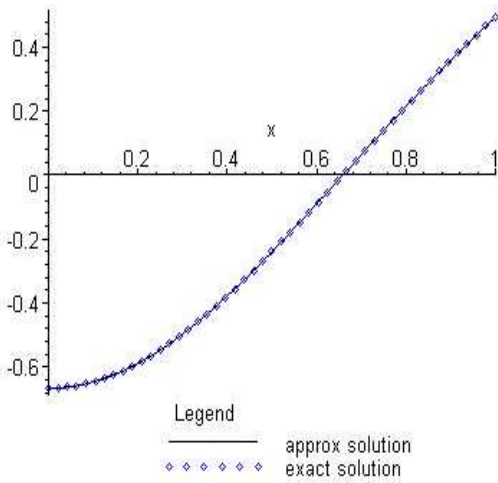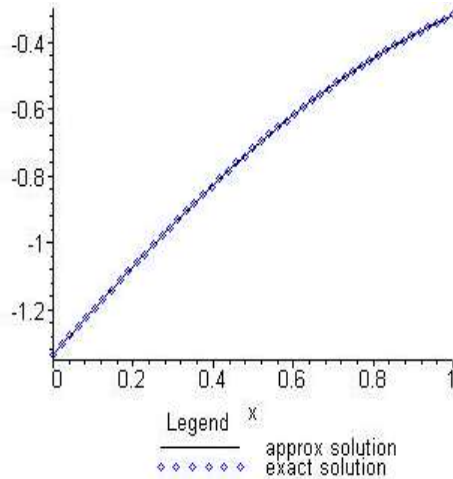

Fig. 1: Solution u (x, t)

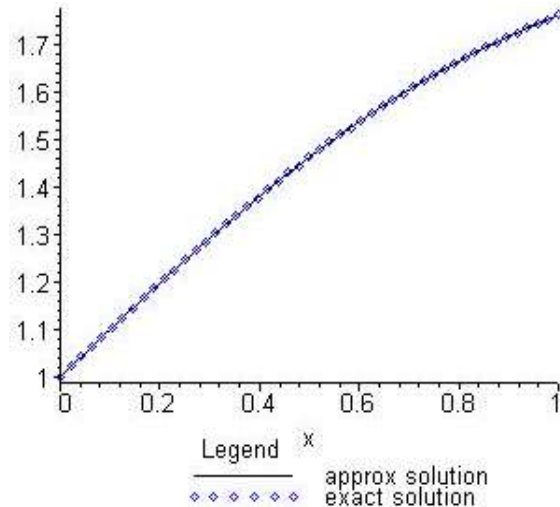

Fig. 2: Solution v (x, t) b = 0, k = t = c_0 = c_1 = 1

Fig. 3: Solution w (x, t)

$$k\beta(1-Y^2)\frac{dU}{dY}-\frac{1}{2}k[2(1-Y^2)(3Y^2-1)\frac{dU}{dY}-6Y(1-Y^2)^2\frac{d^2U}{dY^2}+(1-Y^2)^3\frac{d^3U}{dY^3}]$$

$$+3kU(1-Y^2)\frac{dU}{dY}-3kV(1-Y^2)\frac{dW}{dY}-3kW(1-Y^2)\frac{dV}{dY}=0 \qquad (4.10)$$

the second equation in (4.9) reduces to

$$k\beta(1-Y^2)\frac{dV}{dY}-k[2(1-Y^2)(3Y^2-1)\frac{dV}{dY}-6Y(1-Y^2)^2\frac{d^2V}{dY^2}+(1-Y^2)^3\frac{d^3V}{dY^3}]-3kU(1-Y^2)\frac{dV}{dY}=0 \qquad (4.11)$$

the third equation in (4.9) reduces to

$$k\beta(1-Y^2)\frac{dW}{dY}+k[2(1-Y^2)(3Y^2-1)\frac{dW}{dY}-6Y(1-Y^2)^2\frac{d^2W}{dY^2}+(1-Y^2)^3\frac{d^3W}{dY^3}]-3kU(1-Y^2)\frac{dW}{dY}=0 \qquad (4.12)$$

Now, to determine the parameters m, n and s, we balance the linear term of highest-order with the highest order nonlinear terms. So, in (4.10) we balance $U^{///}$ with $WV^{/}$, to obtain

$$6+m-3=2+n+s-1, \text{ then } m=n+s-2$$

while in Eq. (4.11) we balance $V^{///}$ with $UV^{/}$, to obtain

$$6+n-3=2+m+n-1 \text{ then } n=2, m=s=1$$

The tanh method admits the use of the finite expansion for both:

$$u(x,t)=U(Y)=a_0+a_1Y+a_2Y^2 \quad a_2\neq0 \qquad (4.13)$$

$$v(x,t)=V(Y)=b_0+b_1Y, \quad b_1\neq0 \qquad (4.14)$$

and

$$w(x,t)=W(Y)=c_0+c_1Y, c_1\neq0 \qquad (4.15)$$

Substituting U, $U^{/}$, $U^{//}$, $U^{///}$, V, $V^{/}$, $V^{//}$, $V^{///}$ and W, $W^{/}$, $W^{//}$, $W^{///}$, in (4.10), then equating the coefficient of Y^i, i= 0, 1, 2, 3 leads to the following nonlinear system of algebraic equations

$$Y^0 : \beta a_1 + k^2 a_1 + 3a_0 a_1 - 3b_0 c_1 - 3b_1 c_0 = 0$$

$$Y^1: 2 a_2\beta + 8a_2 k^2 + 3a_1^2 + 6a_0 a_2 - 6b_1 c_1 = 0$$

$$Y^2 : -k^2 a_1 + 3a_1 a_2 = 0$$

$$Y^3 : -2a_2 k^2 + a_2^2 = 0 \qquad (4.16)$$

Substituting U, V, $V^{/}$, $V^{//}$, $V^{///}$ in Eq. (4.11), then equating the coefficient of Y^i, i= 0, 1, 2, 3 leads to the following nonlinear system of algebraic equations

$$Y^0 : [\beta+2k^2-3a_0]b_1=0$$

$$Y^1 : -3 a_1 b_1 = 0$$

$$Y^2 : -2k^2 b_1 - a_2 b_1 = 0 \qquad (4.17)$$

Substituting U, W, $W^{/}$, $W^{//}$, $W^{///}$ in Eq. (4.12), then equating the coefficient of Y^i, i= 0, 1, 2, 3 leads to the following nonlinear system of algebraic equations

$$Y^0 : [\beta-2k^2-3a_0]c_1=0$$

$$Y^1 : -3a_1 c_1 = 0$$

$$Y^2 : 2k^2 b_1 - a_2 b_1 = 0 \qquad (4.18)$$

Solving the nonlinear systems of equations (3.12) and (3.13) with help of Mathematica we can get:

Case 1 $\quad a_0 = \dfrac{\beta+2k^2}{3}, a_1=0, a_2=+2k^2$

$$b_1 = \frac{4k^2(\beta+3k^2)}{3c_1}, b_0 = \frac{-4k^2c_0(\beta+3k^2)}{3c_1^2}$$

Then:

$$u(x,t)=\frac{\beta+2k^2}{3}+2k^2\tanh^2(k(x+\beta t))$$

$$v(x,t)=\frac{-4k^2c_0(\beta+3k^2)}{3c_1^2}+\frac{4k^2(\beta+3k^2)}{3c_1}\tanh(k(x+\beta t))$$

$$w(x,t)=c_0+c_1\tanh(k(x+\beta t)) \qquad (4.19)$$

Case 2 $\quad a_0 = \dfrac{\beta+2k^2}{3}, a_1=0, a_2=-k^2$

$$b_1 = \frac{-4k^2(\beta + 3k^2)}{3c_1}, \ b_0 = \frac{4\,k^2c_0(\beta + 3k^2)}{3c_1^2}$$

Then:

$$u(x,t) = \frac{\beta + 2k^2}{3} - 2k^2 \tanh^2(\ k\ (x + \beta\ t))$$

$$v(x,t) = \frac{4k^2c_0(\beta + 3k^2)}{3c_1^2} - \frac{4k^2(\beta + 3k^2)}{3c_1}\tanh(\ k\ (x + \beta\ t))$$

$$w(x,t) = c_0 + c_1\tanh(\ k\ (x + \beta\ t)) \tag{4.20}$$

Case 3 $\quad a_0 = \dfrac{\beta + 2k^2}{3}, \ a_1 = 0, \ a_2 = \dfrac{k^2}{3}$

$$b_1 = \frac{2k^2(\beta + 3k^2)}{9c_1}, \ b_0 = \frac{-2\,k^2c_0(\beta + 3k^2)}{9c_1^2}$$

Then:

$$u(x,t) = \frac{\beta + 2k^2}{3} + \frac{k^2}{3} \tanh^2(\ k\ (x + \beta\ t))$$

$$v(x,t) = \frac{-2\,k^2c_0(\beta + 3k^2)}{9c_1^2} + \frac{2k^2(\beta + 3k^2)}{9c_1}\tanh(\ k\ (x + \beta\ t))$$

$$w(x,t) = c_0 + c_1\tanh(\ k\ (x + \beta\ t)) \tag{4.21}$$

Case 4 $\quad a_0 = \dfrac{\beta - 2k^2}{3}, \ a_1 = 0, \ a_2 = +2k^2$

$$b_1 = \frac{4k^2(\beta + k^2)}{3c_1}, \ b_0 = \frac{-4k^2c_0(\beta + k^2)}{3c_1^2}$$

Then:

$$u(x,t) = \frac{\beta - 2k^2}{3} + 2k^2 \tanh^2(\ k\ (x + \beta\ t))$$

$$v(x,t) = \frac{-4\,k^2c_0(\beta + k^2)}{3c_1^2} + \frac{4k^2(\beta + k^2)}{3c_1}\tanh(\ k\ (x + \beta\ t))$$

$$w(x,t) = c_0 + c_1\tanh(\ k\ (x + \beta\ t)) \tag{4.22}$$

Case 5 $\quad a_0 = \dfrac{\beta - 2k^2}{3}, \ a_1 = 0, \ a_2 = -k^2$

$$b_1 = \frac{-4k^2(\beta + k^2)}{3c_1}, \ b_0 = \frac{4k^2c_0(\beta + k^2)}{3c_1^2}$$

Then:

$$u(x,t) = \frac{\beta - 2k^2}{3} - 2k^2 \tanh^2(\ k\ (x + \beta\ t))$$

$$v(x,t) = \frac{4k^2c_0(\beta + k^2)}{3c_1^2} - \frac{4k^2(\beta + k^2)}{3c_1}\tanh(\ k\ (x + \beta\ t))$$

$$w(x,t) = c_0 + c_1\tanh(\ k\ (x + \beta\ t)) \tag{4.23}$$

Case 6 $\quad a_0 = \dfrac{\beta - 2k^2}{3}, \ a_1 = 0, \ a_2 = \dfrac{k^2}{3}$

$$b_1 = \frac{2k^2(\beta + k^2)}{9c_1}, \ b_0 = \frac{-2k^2c_0(\beta + k^2)}{9c_1^2}$$

Then:

$$u(x,t) = \frac{\beta - 2k^2}{3} + \frac{k^2}{3} \tanh^2(\ k\ (x + \beta\ t))$$

$$v(x,t) = \frac{-2\,k^2c_0(\beta + k^2)}{9c_1^2} + \frac{2k^2(\beta + k^2)}{9c_1}\tanh(\ k\ (x + \beta\ t))$$

$$w(x,t) = c_0 + c_1\tanh(\ k\ (x + \beta\ t)) \tag{4.24}$$

the solitary wave and behavior of the solutions $u(x, t)$, $v(x, t)$ and $w(x, t)$ are shown in Fig. 4-6 respectively for some fixed values of the parameters ($\beta = 0.5$, $k = 0.5$).

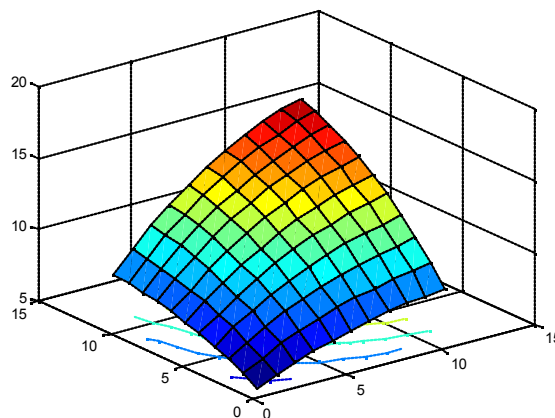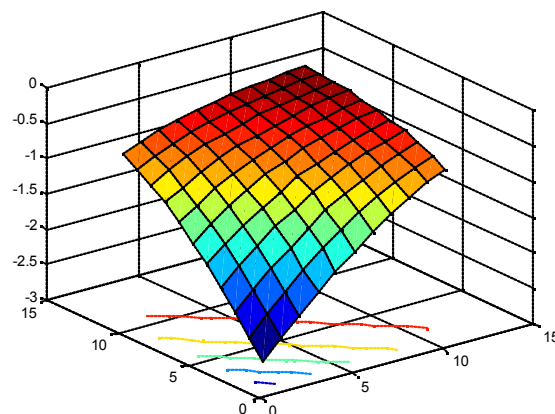Fig. 4: Represents Case 4. Solution $u(x, t)\ k = \beta = 1$

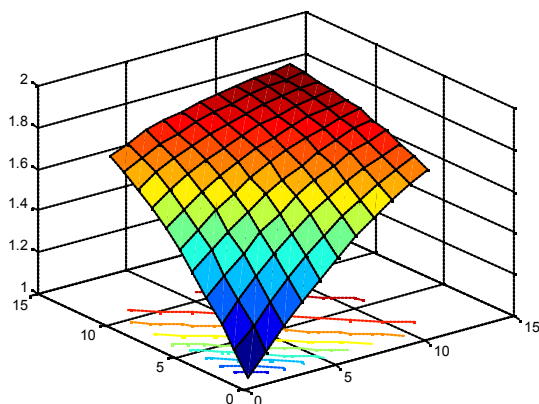Fig. 5: Represents Case 4. Solution $v(x, t)\ k = \beta = c_0 = c_1 = 1$

Fig. 6: Represents Case 4. Solution
w (x, t) $k = \beta = c_0 = c_1 = 1$

RESULTS AND CONCLUSION

In this paper, we applied the powerful Homotopy Perturbation (HPM) and tanh methods for solutions of nonlinear coupled partial differential equations. The tanh method requires transformation formulas. Traveling wave solutions, kinks solutions were derived. It is also observed that the solution of coupled KdV system of PDEs by the HPM is the same as case 4 of the Tanh method.

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