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A Robust Optimization Approach to R&D Project Selection

[1]Mohammad Modarres and [2]Farhad Hassanzadeh

[1]Department of Industrial Engineering,
Sharif University of Technology, P.O. Box: 11365-9363, Azadi Ave., Tehran, Iran
[2]Department of Industrial Engineering, Sharif University of Technology, Azadi Ave., Tehran, Iran

Abstract: Intense competition in the current business environment leads firms to focus on selecting the most appropriate R&D project portfolio in order to accomplish sustainable growth in the fierce market place. Achieving this goal is tied down by uncertainty which is inherent in all R&D projects. Therefore, investment decisions must be made within an optimization framework, based on the data which is usually unavailable or unreliable. In this paper, a model is developed to hedge against the R&D uncertainty. The proposed model is constructed by applying the concept of "real options". The robust optimization approach is adopted to handle uncertain parameters and determine the optimal project portfolio. The problem is formulated as a robust zero-one integer programming model which is transformed into a standard mixed zero-one linear programming one and solved via an optimization technique. The applicability of the proposed approach is illustrated through an example.

Key words: OR in R&D . portfolio selection . real options valuation . robust optimization

INTRODUCTION

R&D activities are becoming more and more essential to gain competitive advantage, long-term survival and growth for a majority of firms. In fact, R&D enables firms to improve the value of their business through developing new products and services or improving the current ones. Some R&D projects elevate productivity and consequently reduce costs through improving the existing production processes. Beyond them, products and services tend to have shorter and shorter life cycles, which means more and more R&D endeavors are required. R&D projects consume resources, such as money, human resources or laboratories which are limited. Therefore, they must compete for theses common scarce resources.

The purpose of project portfolio selection is to allocate the limited set of resources to various projects in a way that balances risk, reward and alignment with corporate strategy [1]. However, due to long lead times for R&D projects and market and technology dynamics, decision making is a complex task. Furthermore, complexity of these projects and resource interdependencies make portfolio decisions more difficult [2]. Furthermore, R&D projects and in particular, portfolios of R&D projects are well-recognized to possess uncertainty and imprecision properties in that various portfolio estimates might be overly optimistic or pessimistic or even questionable [3]. Therefore, in the R&D project portfolio decision, much of the information required to make decisions is at best uncertain and at worst very unreliable. Moreover, resources or budget availability may be flexible, because additional budget and human resources may be reallocated from other budget categories or projects within the company [4]. However, even with this doubtful information and high complexity, the project portfolio decision still must be made.

Following this introduction, this paper focuses on the problem of selecting a portfolio of R&D projects when uncertainty in data and interactions between candidate projects exist. In particular, we assume no prior data or detail about parameters distributions exists. It must be mentioned that solving this model using robust optimization approach is totally overlooked in the literature. The novelty of this paper is that the application of this technique on portfolio of R&D projects, the employment of real option value of a portfolio of projects as an objective function and the integration of these concepts in a mathematical formulation has, to the best of our knowledge, never been investigated in the literature.

The organization of this paper is as follows. In Section 2, we review the related literature, briefly. Then, we introduce the robust optimization concepts

Corresponding Author: Farhad Hassanzadeh, Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran

and techniques. To handle uncertainty, we adopt robust optimization approach and transform the model into a robust one, in Section 3. The real option valuation approach and the robust portfolio selection model are discussed in Section 4. An example to illustrate the proposed approach is presented in Section 5 and Section 6 concludes the paper.

LITERATURE REVIEW

Studies on R&D project portfolio selection can be divided into three major categories: strategic management tools, benefits measurement methods and mathematical programming approaches [2]. The strategic management tools, such as bubble diagram, portfolio map and strategic bucket method, are used to emphasize the connection of innovation projects to strategy or illuminate issues of risk or strategic balances of the portfolio [1].

Benefits measurement methods determine the preferability figure of each project. A number of approaches, such as analytical hierarchy process [5], net present value [6] and option pricing theory [7] have been utilized in the literature to estimate the benefit of an R&D project. The projects with the highest score may be selected sequentially. The major drawback of most benefits measurement approaches is that neither uncertainty nor resource interactions among projects can be captured. In recent years, some studies used the criterion of conditional stochastic dominance [8] or the mean-Gini analysis [9] to handle R&D uncertainties for risk-averse decision makers.

Mathematical programming models optimize some objective function(s) subject to constraints related to resources, project logics, technology and strategies. The R&D portfolio selection model can be categorized based on mathematical structure as linear, nonlinear, integer, dynamic, goal, multiobjective, or stochastic programming [2]. In recent years, more complicated models were developed to capture the actual situation of R&D project selection. Beaujon *et al.* developed a mixed integer programming model to find an optimal project portfolio and studied the concept of partial funding project and the sensitivity of an estimated project value to the selected portfolio [10]. Dickinson *et al.* proposed the concept of dependency matrix representing complex dependencies between projects and developed an optimal portfolio model over multiple time periods [11].

The majority of mathematical formulations in the literature are developed based on deterministic data. However, as already mentioned, R&D projects comprise a high degree of uncertainty, which generally precludes the availability of obtaining exact data regarding benefit, resource usage and interactions between projects. Fuzzy set theory is one of the mathematical tools to model imprecise information in such environments. It can also be used to represent uncertain project information and may provide an alternative and convenient framework for handling uncertain project parameters (e.g., project value, cost, etc.), while there is lack of certainty in data or lack of available historical data. The literature on fuzzy set application on R&D portfolio selection is fairly abundant. As an early work, Pereira and Junior formulated a simple fuzzy multi-criteria R&D project portfolio selection problem that represented project appraisals for each criterion as fuzzy set and developed an algorithm to find non-dominated solutions [12]. Coffin and Taylor developed a model that includes fuzzy logic in a beam search approach to select and schedule R&D projects under multiple objectives [13]. Kuchta used fuzzy numbers to present the uncertain NPV and resource consumption of each project and considered benefits, outcomes and resource interactions among projects [14]. Wang and Hwang and Carlsson *et al.* used options approach instead of traditional discounted cash flow to evaluate the value of each R&D project and developed a fuzzy zero-one integer programming model to determine the optimal project portfolio [3, 4].

One drawback of fuzzy set theory in R&D portfolio selection shows up when there is a possible range for each project parameter, but the most plausible value(s) within each range cannot be estimated. For example, a technological expert may state that if the project were to start now, then it would be possible to apply for a patent after a few years to protect a new production method. However, the expert cannot give any detail (e.g. probability or possibility distribution) about the likelihood that the patent would be granted or that the technological foresight would eventually come true. In such a case, it would not make much sense if the technological expert presented some distribution to characterize outcomes that are unknown. Thus, an expectation involving this kind of uncertainty can not be stated to represent some known future state. Other approaches must then be employed to address such problems.

Robust optimization is a new approach which incorporates the random character of problem parameters without making any assumptions on their distributions. Robust optimization addresses the problem of data uncertainty by guaranteeing the feasibility and optimality of the solution for the worst instances of the problem. As it is intrinsically a worst case approach, feasibility often comes at the cost of performance and generally leads to over conservative solutions. For instance, Soyster proposed a model to handle column-wise uncertainty in linear programming problems, where every uncertain parameter has to be taken equal to its worst case value in the set [15]. Future research efforts led by Ben-Tal and Nemirovski and El-Ghaoui *et al*. to address over conservativeness, applied robust optimization to linear programming problems with ellipsoidal uncertainty sets, thus obtaining conic quadratic programs [16-18]. Founded on the idea that simultaneous large deviations in all uncertain problem parameters actually occur with negligible probability, Bertsimas and Sim developed the budget of uncertainty approach to control the cumulative conservativeness of all uncertain problem parameters [19]. As this technique which is specifically tailored for polyhedral uncertainty has some interesting characteristics, we will employ it in this paper to develop a practical approach for project portfolio selection in the presence of data uncertainty.

Selecting an appropriate set of R&D projects in an uncertain environment involves effective valuation of all R&D projects. Traditionally, discounted cash flow and expected net present value analyses are the most frequently used methods for valuation of R&D projects. However, these analyses can underestimate R&D project values and lead to high-risk and high-payoff projects that will not be chosen [20]. In recent years, the real options methods have gained growing attention in R&D project valuation [7, 21-23]. The basic idea of the real options approach is to transfer the sophisticated options pricing models used in capital market theory to the valuation of risky R&D projects. The real options approach has received great attention in recent years, because an initial investment of an R&D project is similar to the purchase of an option on a future investment. An R&D project usually involves several phases and the decision makers have the option to stop or defer the project at the end of each phase. Therefore, each phase is an option that is contingent on the earlier exercise of other options. If the project is a technical success, it creates the option to make a significantly larger investment in the continuing project with relatively higher expected net benefit. If the project fails to achieve the technical success, there is no need to commit any further resources and therefore the downside risk is limited to the initial investment cost of the R&D project.

Robust optimization approach is a novel one for R&D project portfolio selection which has not been considered and analyzed so far. The objective of this paper is to develop a robust portfolio selection model to optimize the R&D portfolio for the conservative decision maker in an uncertain R&D environment. Robust optimization approach is used to describe uncertain and flexible project information. Since traditional financial analysis approaches usually underestimate the R&D project value [21], a compound options approach is used to estimate the value of each R&D project. A robust portfolio selection model that can handle both uncertain and flexible parameters is developed to select the optimal R&D portfolio. The resulting model is transformed into a standard mixed 0-1 integer programming model to solve the basic model from a conservative perspective. An optimal R&D portfolio can be obtained by solving the transformed model using an optimization technique. Depending on the level of uncertainty, sensitivity analyses are performed to evaluate the appropriateness of the selected project portfolios.

ROBUST OPTIMIZATION APPROACH FOR MODELLING UNCERTAINTY

We rely extensively on the robust optimization tools developed by Bertsimas and Sim to handle uncertain parameters. Therefore, we review this subject briefly in this section.

Uncertainty structure: To begin, consider the following problem subject to data uncertainty:

$$\begin{aligned} &\text{Minimize } \mathbf{c'x} \\ &\text{Subject to } \mathbf{Ax \le b} \\ &\mathbf{1 \le x \le u} \end{aligned} \tag{1}$$

We assume without any loss of generality that the data uncertainty only affects the elements in matrix **A**. Later we show how to rewrite the model, if there is uncertainty on **b** or **c**.

We model data uncertainty in **A** as follows. Each uncertain coefficient a_{ij} is known to belong to an interval centered at its nominal value $\overline{a}_{ij}$ and of half-length $\hat{a}_{ij}$, but its exact value is unknown. As much as it is unlikely that all coefficients are equal to their nominal value, it is also unlikely that they are all equal to their worst-case value. For this reason, the "safest" approach, where all parameters are taken equal to their worst bound, leads to a severe deterioration of the cost without necessarily being justified in practice. Hence, we wish to adjust the uncertainty level of the solution, so that a reasonable trade-off between robustness and performance is achieved.

To quantify this concept in mathematical terms, we define the scaled deviation of parameter a_{ij} from its nominal value as $z_{ij} = (a_{ij} - \overline{a}_{ij})/\hat{a}_{ij}$. The scaled deviation takes a value in interval [-1,1]. Moreover, we impose a budget of uncertainty in the following sense: The total (scaled) variation of the parameters cannot exceed some threshold Γ, not necessarily integer:

$$\sum_{(i,j)\in J} |z_{ij}| \le \Gamma$$

where J is the set of indices of the uncertain parameters. By taking Γ = 0 (Γ = |J|) we obtain the nominal (worst) case. Bertsimas and Sim show that having the threshold Γ vary in (0, |J|) allows greater flexibility to build a robust model without excessively affecting the optimal cost [19]. Intuitively, the budget of uncertainty rules out large deviations in $\Sigma_j a_{ij} x_j$ which play a predominant role in worst-case analysis but actually occur with low probability, since large deviations in the a_{ij} tend to cancel each other out as the number of parameters increases.

The robust approach

Let

$$\Lambda = \left\{ \begin{array}{c} \mathbf{A} \in R^{m\times n} \mid a_{ij} \in [\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]\ \forall i, j, \\ \sum_{(i,j)\in J} \frac{|a_{ij} - \bar{a}_{ij}|}{\hat{a}_{ij}} \le \Gamma \end{array} \right\}$$

The robust problem is then formulated as:

$$\begin{array}{ll} \text{Minimize } \mathbf{c'x} & \\ \text{Subject to } \mathbf{Ax \le b}\ \forall \mathbf{A} \in \Lambda & (2) \\ \mathbf{1 \le x \le u} & \end{array}$$

Theorem 1 [19]: Uncertain linear programming problem (2) has the following robust, linear counterpart:

Minimize $\mathbf{c'x}$

$$\begin{array}{lll} \text{Subject to } \sum_j \bar{a}_{ij} x_j + q_i \Gamma + \sum_{j:(i,j)\in J} r_{ij} \le b_i & \forall i & \\ q_i + r_{ij} \ge \hat{a}_{ij} y_j & \forall (i,j) \in J & (3) \\ \mathbf{-y \le x \le y, 1 \le x \le u} & & \\ \mathbf{q \ge 0, r \ge 0, y \ge 0.} & & \end{array}$$

Proof: [19].

The above robust counterpart is of the same class as the nominal problem, that is, a linear programming problem. This is a highly attractive feature of this approach, since linear programming problems are readily solved by standard optimization packages. Moreover, if in the original problem (2) some of the variables were constrained to be integers, then the robust counterpart (3) would retain the same properties, i.e., the robust counterpart of a mixed integer programming problem is itself another mixed integer programming problem.

Note: If there is uncertainty on **b** or **c**, we can rewrite the linear programming problem as:

$$\begin{array}{ll} \text{Minimize } \tilde{\mathbf{c}}'\tilde{\mathbf{x}} & \\ \text{Subject to } \tilde{\mathbf{A}}\tilde{\mathbf{x}} \ge \mathbf{0} & (4) \\ \tilde{\mathbf{l}} \le \tilde{\mathbf{x}} \le \tilde{\mathbf{u}} & \end{array}$$

with

$$\tilde{\mathbf{x}} = (z, \mathbf{x}, y)',\ \tilde{\mathbf{c}} = (1, \mathbf{0}, 0)',\ \tilde{\mathbf{l}} = (-M, \mathbf{1}, 1)',\ \tilde{\mathbf{u}} = (M, \mathbf{u}, 1)'$$

where M is a large constant and

$$\tilde{\mathbf{A}} = \begin{pmatrix} 1 & -\mathbf{c}' & 0 \\ \mathbf{0} & -\mathbf{A} & \mathbf{b} \end{pmatrix}$$

ROBUST OPTIMIZATION FRAMEWORK FOR R&D PROJECT SELECTION

Selecting among a set of candidate R&D projects is the main problem of this paper, while the input data is uncertain and no prior details of parameters distributions exist. By adopting the approach as well as the results developed by [24] and [7], we first show how each project is valuated on the basis of real options concept. Then the robust optimization model is developed in subsection 4.1 in order to handle the original as well as estimated uncertain parameters.

Consider an R&D project with two subsequent growth opportunity, as depicted in Fig. 1. Once phase 1 of the project is accomplished successfully, the company has the option to invest in the second phase of the project. In pharmaceutical industry for example, phase 1 of the project might serve for identifying promising active pharmaceutical ingredients (APIs) out of numerous possible compounds. Phase 2 is then equivalent to the four sequential tests from pre-clinical testing to clinical testing III. After identifying the API, the company has the option to invest in further testing. Thus, the option to invest in testing (phase 2) serves as the first option, whereas the second option is represented by the investment in production and market introduction (phase 3). These two opportunities together form a compound option.

The first call option with a time to maturity of τ^* is determined on the value of the first investment opportunity, which in turn depends on the second investment. The exercise price of the first option is equal to the negative net present value resulting from

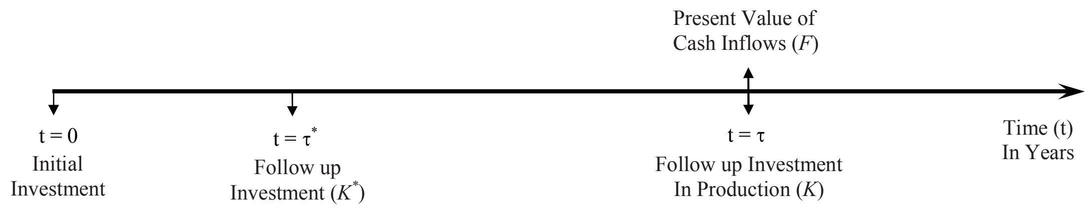

Fig. 1: Simplified illustration of an R&D project

the testing phase, defined as K^*. If the first option is exercised, the company will receive a second call option with a time to maturity of $\tau-\tau^*$. The first option is a compound option since it is the completion of the first investment which provides another option. F is defined as the present value of cash inflows following the second investment (K).

Assuming that the value follows the usual geometric Brownian motion process, these compound options can be valued analytically using Geske's valuation approach, which is based on the Black-Scholes formula and adjusted for real option valuation by Kemna as follows [25]:

$$\begin{aligned} \nu = Fe^{-r\tau}M(k,h;\rho) - Ke^{-r\tau}M(k-\sigma\sqrt{\tau^*},h \\ -\sigma\sqrt{\tau};\rho) - K^*e^{-r\tau^*}N(k-\sigma\sqrt{\tau^*}) \end{aligned} \quad (5)$$

where:
v = real option value

$$h = \frac{\ln(F/K) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$$

$$k = \frac{\ln(F/F_c) + \frac{1}{2}\sigma^2\tau^*}{\sigma\sqrt{\tau^*}}$$

N (.) = univariate cumulative normal distribution function

M (a, b;ρ) = Bivariate cumulative normal distribution function with a and b as upper and lower integral limits and correlation coefficient ρ.

$$\rho = \sqrt{\frac{\tau^*}{\tau}}$$

- τ = Time to maturity of the simple option
- τ^* = Maturity date of the first option (within the compound option)
- F = Present value of cash inflows of the commercial venture as of year τ
- F_C = Critical value of the project above which the first call option will be exercised
- K = Present value of capital expenditures of the commercial venture as of year τ
- K^* = Present value of capital expenditures of the pioneer venture as of year τ^*
- σ = Volatility of the rate of change of the commercial venture
- r = Riskless rate of interest

In the following formulation, we employ the above approach in order to valuate each R&D project.

Model formulation of the R&D portfolio selection: The R&D portfolio selection problem is to select a set of projects from a pool of candidate projects to maximize the expected benefits during the planning horizon. Each candidate project has specific duration and its execution requires the exclusive use of a number of resources (e.g., budget, labor, etc.) while the availability of each resource type is usually limited. To effectively utilize limited resources, it is important to link portfolio selection decisions to the key corporate strategies and to maintain the balance of R&D project portfolio. For example, a balanced portfolio of projects may include investments in breakthrough products, new platforms, derivates and current product support. The percentages of spending on each project category may depend on the goals of the individual companies.

Since uncertainty and flexibility are encountered in making R&D portfolio selection, a robust zero-one integer programming model is proposed here to optimize R&D portfolio decisions in an uncertain R&D environment.

Notation

- n The total number of candidate projects
- T The number of development phases
- v_i The uncertain real options value of candidate project i
- B_t The budget available for stage t
- c_{it} The uncertain investment cost of candidate project i during stage t
- l_{it} Labor (in working months) required to implement project i at stage t
- L_t Labor (in working months) available to staff projects at stage t

S_j^U The maximum budget percentage that can be spent on projects contributing to achievement of strategy j

S_j^L The minimum budget percentage that can be spent on projects contributing to achievement of strategy j

$$S_{ij} = \begin{cases} 1 & \text{if project } i \text{ contributes to strategy } j \\ 0 & \text{otherwise} \end{cases}$$

$$R_i = \begin{cases} 1 & \text{if project } i \text{ is required to implement} \\ 0 & \text{otherwise} \end{cases}$$

$$P_{ip} = \begin{cases} 1 & \text{if project } i \text{ is required for implementing project } p \\ 0 & \text{otherwise} \end{cases}$$

$$x_i = \begin{cases} 1 & \text{if project } i \text{ is selected for funding} \\ 0 & \text{otherwise} \end{cases}$$

The robust model: In the beginning of section 4, we addressed how each project is evaluated individually based on real options valuation concept. However, since the resources are limited and the parameters are uncertain, we develop a robust optimization model to determine the most appropriate portfolio of projects, as follows:

$$\text{Max} \quad \sum_{i=1}^{n} (v_i - c_{i1}) x_i \qquad \text{(I)} \qquad (6)$$

s.t.

$$\sum_{i=1}^{n} c_{it} x_i \le B_t \quad \forall t \qquad \text{(II)}$$

$$\sum_{i=1}^{n} l_{it} x_i \le L_t \quad \forall t \qquad \text{(III)}$$

$$\sum_{i=1}^{n} \sum_{t=1}^{T} S_{ij} c_{it} x_i \le S_j^U \sum_{t=1}^{T} B_t \quad \forall j \qquad \text{(IV)}$$

$$\sum_{i=1}^{n} \sum_{t=1}^{T} S_{ij} c_{it} x_i \ge S_j^L \sum_{t=1}^{T} B_t \quad \forall j \qquad \text{(V)}$$

$$\sum_{i=1}^{n} R_i x_i \ge \sum_{i=1}^{n} R_i \quad \forall i \qquad \text{(VI)}$$

$$x_i - x_p \ge 0 \quad \forall i, p \text{ such that } p_{ip} = 1 \qquad \text{(VII)}$$

$$x_i \in \{0,1\} \quad \forall i \qquad \text{(VIII)}$$

The objective (I) of this model is to maximize the total benefit of the R&D investment portfolio. The benefit of each project is the project real option value minus its initial investment cost (cost of real options). Constraints (II) ensure that the project spending during the planning horizon does not exceed the predetermined budget for each stage (e.g., project materials, capital equipments, staffing, etc.). Constraints (III) ensure that the required personnel should be within the available man-power capacity for each stage. Constraints (IV) and (V) enforce the desired balance in spending between different R&D strategic goals. The maximum as well as minimum spending required for each strategic goal is specified. Constraints (VI) force to select certain projects. Constraints (VII) ensure that project p can only be selected if all its precedent projects i are selected. Finally, constraints (VIII) specify decision variables as binary variables with the value equal to zero or one.

Note that the investment costs (c_{it}) and option values (v_i) are uncertain and described as intervals. We consider all other parameters, including labor work parameters, to be certain. This is because portfolio decisions are most sensitive to financial metrics and other limitations can typically be overcome by support of top level managers. It's however easy to show that when some of these parameters are uncertain, the structure of the problem remains the same. Therefore, the above approach can be employed without major modifications. It must be mentioned that solving this type of model using robust optimization approach is totally overlooked in the literature.

To refuse over conservative solutions, we add the following constraint to the proposed model (6):

$$\sum_{i=1}^{n} \sum_{t=1}^{T} \frac{|c_{it} - \bar{c}_{it}|}{\hat{c}_{it}} + \frac{|v_i - \bar{v}_i|}{\hat{v}_i} \le \Gamma \qquad (7)$$

$$\Gamma \in [0, n(T+1)]$$

while Γ stands for the common budget of uncertainty for all uncertain parameters of the problem. By sensitivity analysis of Γ, one can simulate various levels of conservativeness to determine appropriate portfolio of projects for different uncertain environments.

ILUSTRATIVE EXAMPLE

In this section, an example of R&D project portfolio selection problem in the pharmaceutical industry is presented to illustrate the developed approach. We adapt the data used by [26] and [4] with some modifications. This example can demonstrate how robust optimization technique assists top managers in forming their optimal portfolio of projects.

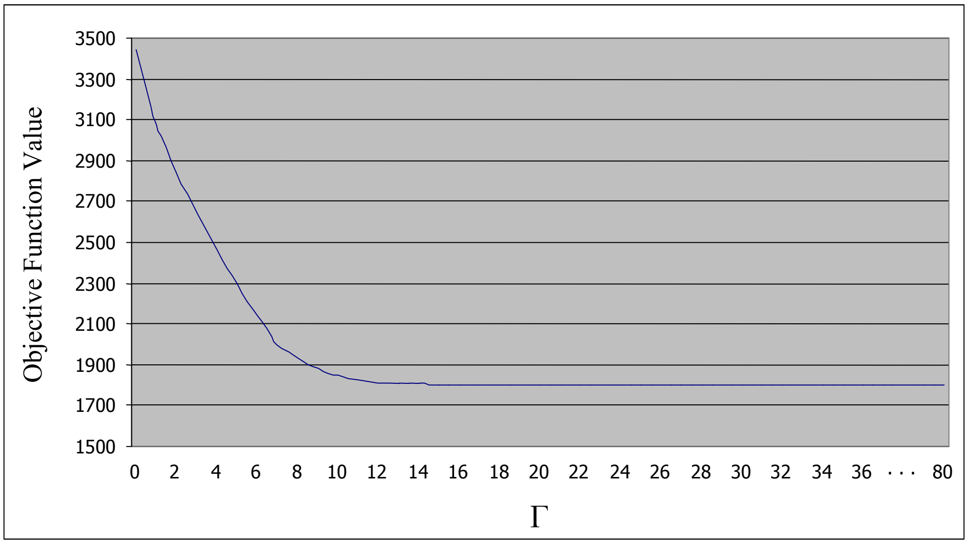

Fig. 2: Objective function value versus Γ

Table 1: Estimated R&D staff required for development phases and projects volatilities

	Required staff (in working months)			
Projects	Phase 1	Phase 2	Phase 3	Volatility (%)
P1	6	72	50	80
P2	12	80	48	70
P3	24	95	70	50
P4	12	100	70	60
P5	32	120	80	50
P6	26	105	75	20
P7	20	85	52	50
P8	12	110	75	70
P9	24	150	90	55
P10	30	155	100	35
P11	14	90	60	45
P12	15	75	70	80
P13	30	180	120	30
P14	45	200	130	40
P15	40	160	110	35
P16	35	190	125	100
P17	36	190	120	60
P18	38	200	130	30
P19	36	220	150	60
P20	48	230	160	20

There are 20 R&D projects that a pharmaceutical company can select among them. Each one has three stages: drug discovery, testing and market introduction. For simplicity, we assume that the times to maturities of the first and second options (τ^* and τ) for all projects are set to 3 and 10 years, respectively. The preferred development budgets for stages 1, 2 and 3 are (in millions) 270, 985 and 1975, respectively. Similarly, the preferred capacities of R&D staff for three stages are (in working months) 375, 1965 and 1320, respectively. We assume no further constraints for the problem.

Table 1 lists estimated volatility (based on historical data) as well as the estimates of required R&D staff for different phases of each project. Table 2 presents the uncertain development costs and estimated present value of cash inflows of each project as interval numbers.

We first employ Geske's valuation approach (section 4) for each of 20 projects to analytically determine their compound option value. We used "R" software to calculate required bivariate cumulative normal distributions. Furthermore, a program in Visual Basic was coded to determine compound option values as intervals. It must be noted that the highest (lowest) option value of a project is realized when highest (lowest) present value of cash inflows and lowest (highest) investment costs in all stages of the project are realized. The last column of Table 2 summarizes the results.

Following option valuation of each project, the problem was formulated, while taking the proposed budget of uncertainty approach. Figure 2 shows the realized value of selected portfolios with respect to different values of Γ in [0, 80]. The non-increasing shape of the objective function verifies the fact that as uncertainty of the environment grows, the uncertain problem parameters gain more volatility. This essentially results in a worse objective function, due to the conservative nature of the robust optimization

Table 2: Costs, present value of cash inflows and calculated option values

Projects	Investment costs Initial (c_{t1})	Phase 2 (c_{t2})	Phase 3 (c_{t3})	NPV of inflows at t = 0	Project option value
P1	(1.8, 2.2)	(27.0, 33.0)	(27.0, 33.0)	(45, 55)	(24.0, 33.9)
P2	(2.7, 3.3)	(45.0, 55.0)	(40.5, 49.5)	(90, 110)	(46.5, 68.2)
P3	(9.0, 11.0)	(67.5, 82.5)	(90.0, 110.0)	(180, 220)	(78.6, 124.9)
P4	(4.5, 5.5)	(58.5, 71.5)	(153.0, 187.0)	(180, 220)	(84.8, 128.3)
P5	(18.0, 22.0)	(76.5, 93.5)	(180.0, 220.0)	(540, 660)	(354.7, 495.0)
P6	(13.5, 16.5)	(36.0, 44.0)	(40.5, 49.5)	(90, 110)	(21.7, 50.7)
P7	(6.3, 7.7)	(31.5, 38.5)	(27.0, 33.0)	(72, 88)	(31.6, 49.7)
P8	(4.5, 5.5)	(49.5, 60.5)	(45.0, 55.0)	(90, 110)	(44.2, 65.5)
P9	(9.0, 11.0)	(67.5, 82.5)	(72.0, 88.0)	(162, 198)	(74.9, 115.4)
P10	(16.2, 19.8)	(76.5, 93.5)	(108.0, 132.0)	(342, 418)	(182.0, 279.9)
P11	(4.5, 5.5)	(31.5, 38.5)	(27.0, 33.0)	(72, 88)	(29.4, 47.9)
P12	(6.3, 7.7)	(36.0, 44.0)	(54.0, 66.0)	(90, 110)	(53.9, 75.3)
P13	(13.5, 16.5)	(85.5, 104.5)	(162.0, 198.0)	(360, 440)	(153.6, 259.2)
P14	(31.5, 38.5)	(108.0, 132.0)	(252.0, 308.0)	(630, 770)	(352.2, 524.2)
P15	(22.5, 27.5)	(63.0, 77.0)	(90.0, 110.0)	(450, 550)	(309.1, 431.1)
P16	(13.5, 16.5)	(85.5, 104.5)	(135.0, 165.0)	(270, 330)	(199.2, 261.5)
P17	(15.3, 18.7)	(72.0, 88.0)	(162.0, 198.0)	(315, 385)	(184.6, 263.6)
P18	(18.0, 22.0)	(81.0, 99.0)	(198.0, 242.0)	(495, 605)	(258.9, 402.2)
P19	(31.5, 38.5)	(108.0, 132.0)	(225.0, 275.0)	(720, 880)	(501.7, 682.8)
P20	(45.0, 55.0)	(117.0, 143.0)	(315.0, 385.0)	(1035, 1265)	(643.7, 940.8)

Table 3: Optimal project portfolios for diverse uncertain environments

Γ	Portfolio of selected projects	Portfolio size	Portfolio value
0	2, 4, 5, 7, 10, 12, 14, 15, 16, 18, 19, 20	12	3439.1
[0.01, 0.14]	1, 5, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20	12	[3390.7, 3430.2]
[0.15, 0.38]	2, 3, 5, 10, 11, 12, 14, 15, 17, 18, 19, 20	12	[3300.3, 3373.4]
[0.39, 0.76]	1, 2, 3, 5, 10, 12, 14, 15, 17, 18, 19, 20	12	[3173.0, 3290.5]
[0.77, 1.16]	1, 2, 5, 7, 10, 11, 12, 14, 15, 17, 18, 19, 20	13	[3045.4, 3151.5]
1.17	1, 3, 5, 10, 11, 12, 14, 15, 17, 18, 19, 20	12	3038.1
[1.18, 1.19]	1, 3, 5, 7, 10, 12, 14, 15, 17, 18, 19, 20	12	[3033.7, 3035.9]
[1.20, 1.32]	1, 3, 5, 10, 11, 12, 14, 15, 17, 18, 19, 20	12	[3004.7, 3031.4]
[1.33, 1.41]	1, 3, 5, 7, 10, 12, 14, 15, 17, 18, 19, 20	12	[2984.6, 3002.4]
[1.42, 1.71]	1, 3, 5, 10, 11, 12, 14, 15, 17, 18, 19, 20	12	[2917.7, 2982.3]
[1.72, 2.00]	2, 3, 5, 10, 12, 14, 15, 16, 18, 19, 20	11	[2843.5, 2905.5]
[2.01, 2.28]	2, 3, 5, 10, 12, 14, 15, 17, 18, 19, 20	11	[2786.1, 2839.2]
[2.29, 2.41]	2, 5, 7, 10, 11, 12, 14, 15, 16, 18, 19, 20	12	[2754.1, 2778.6]
[2.42, 3.88]	2, 5, 7, 10, 11, 12, 14, 15, 17, 18, 19, 20	12	[2498.6, 2751.5]
[3.89, 4.00]	1, 2, 5, 7, 10, 12, 14, 15, 17, 18, 19, 20	12	[2462.9, 2481.2]
[4.01, 4.38]	1, 5, 7, 10, 11, 12, 14, 15, 16, 18, 19, 20	12	[2397.0, 2452.5]
[4.39, 4.75]	1, 5, 7, 10, 11, 12, 14, 15, 17, 18, 19, 20	12	[2336.5, 2389.6]
[4.76, 5.05]	1, 2, 5, 7, 10, 11, 14, 15, 17, 18, 19, 20	12	[2289.4, 2330.6]
[5.06, 6.11]	2, 5, 7, 10, 12, 14, 15, 16, 18, 19, 20	11	[2133.2, 2261.1]
[6.12, 6.70]	1, 2, 5, 10, 12, 14, 15, 16, 18, 19, 20	11	[2058.6, 2116.0]
[6.71, 8.12]	5, 7, 10, 11, 12, 14, 15, 16, 18, 19, 20	11	[1924.0, 2028.2]
[8.13, 9.42]	1, 5, 7, 10, 12, 14, 15, 16, 18, 19, 20	11	[1870.1, 1909.5]
[9.43, 10.42]	1, 2, 5, 7, 11, 12, 14, 15, 16, 18, 19, 20	12	[1841.2, 1863.4]
[10.43, 11.79]	1, 2, 5, 7, 10, 11, 12, 14, 15, 18, 19, 20	12	[1812.2, 1831.9]
[11.80, 20.00]	2, 3, 5, 12, 14, 15, 16, 18, 19, 20	10	[1797.9, 1812.1]
[20.00, 80.00]	2, 3, 5, 12, 14, 15, 16, 18, 19, 20	10	1797.9

Table 4: Brief recipe for project selection based on uncertainty level

	Uncertainty level			
Selected projects	Low	Medium	High	Comments
5, 12, 14, 15, 18, 19, 20	█	█	█	Project 12 is not selected only if $4.76 \leq \Gamma \leq 5.05$
2	▓	▓	█	
16	░	▓	█	
10	█	▓	| |	
11	▓	▓	| |	
3	░	| |	█	
1, 7	░	▓	| |	
17	▓	| |	| |	
4, 6, 8, 9, 13	| |	| |	| |	Project 4 is selected only if $\Gamma = 0$

Legend: █ always ▓ often ░ occasionally | | never

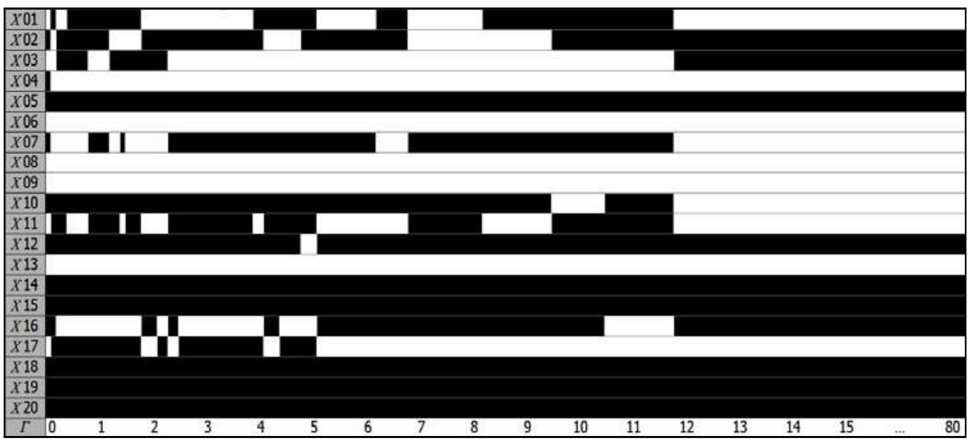

Fig. 3: Schematic of selected projects for various values of Γ

which tends to the worst instances of the problem. It's however obvious that for Γ≥11.80, the objective function fairly remains constant while Γ≥20.00 can impose no further decline on the realized objective function value (1797.9).

Table 3 shows the portfolio of selected projects along with corresponding values and sizes for interval values of Γ with 0.01 approximations. It is observed that when uncertainty is low (Γ≤5), the projects combination of the optimal portfolio changes erratically and the portfolio value drops rapidly, while medium uncertainty (5≤Γ≤10) gives rise to more "robust" portfolios. Furthermore, there is an inclination to form smaller portfolios as the uncertainty grows. This is inevitable, because when parameters uncertainty increases, the look-for-feasible nature of robust optimization confronts tighter budget constraints to satisfy and therefore, fewer projects qualify to enroll the optimal portfolio.

Figure 3 represents the abovementioned results in brief. The black (white) regions in each row indicate the uncertain environments in which the corresponding project is selected (rejected). In addition to the above results, it's immediately observed that inclusion of some projects in the optimal portfolio is less vulnerable to uncertainty (some projects are always or never selected). The remaining projects which are less sensitive to uncertainty show different behaviors under different uncertainty levels, which is a key observation to the project portfolio selection.

Table 4 presents the abovementioned important observation more practically. Following each uncertainty level (column) in the table, it's easy to decide on which projects must be added to the optimal portfolio.

CONLUSIONS

In this paper, a robust optimization approach was developed to select a set of R&D projects from a pool of candidate projects in order to maximize the expected benefits while coping with the uncertain nature of the projects. We adopted real option valuation approach to compensate the deficiencies of classical valuation models. Applying the proposed robust optimization approach on a real-world example from pharmaceutical industry, we showed how detailed projects data can sum up to a simple project selection recipe supported by sophisticated mathematical formulas which account for uncertainty. This in essence provides a very useful decision making instrument for managers who are typically not interested in detailed data and mathematical formulas and rather decide based on descriptive and qualitative tools.

There are many clues hidden in this paper for the observant reader. As an example, we didn't discuss the dynamic nature of the proposed approach. As selected projects evolve from early phases toward completion, more data on projects are collected. Moreover, new project opportunities may arise which must be evaluated and added to the portfolio in case of competency. To update the portfolio, similar approaches can be employed with trivial adjustments to determine projects that must be sacrificed for the promising new ones. As a future direction, we motivate the interested readers to look into this issue in the selection of the R&D portfolio. We also direct them to apply similar approaches on the multiple objective R&D portfolio selection problem.

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