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An Evaluation from Students' Perspective on Visualization Approach Used in Linear Algebra Instructions

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Abstract: The purpose of this study is to evaluate the visualization method approach applied in linear algebra instructions from the aspect of students' perceptions. A twelve-hour visual linear algebra course was held for this purpose. There were written and oral interviews made with the students before and after this course. Senior students were given lectures without going into detail on the concepts of linear algebra lessons, which they had been given in their freshmen year, by means of visualization approach that vectoral geometry was applied in. Rather positive feedbacks were ascertained by post-application interviews about either linear algebra, visualization or the visualization in linear algebra.

Key words: Linear algebra · Visualization · Teaching linear algebra · Visual linear algebra

INTRODUCTION

It is well-known that most problems experienced within the process of mathematics teaching and learning do not pertain to the nature of mathematics merely, yet the same and similar problems can be experienced in other lessons as well. On the other hand, students' developing more negative attitudes towards mathematics when compared to other lessons seems remarkable. Likewise, the same thing is pertinent for the branches within mathematics. For example, some students are observed to develop more negative attitudes towards algebra regarding branches of mathematics. In the nature of mentioned realities, those confirmations lead math instruction and learning process to the structure of the math in general, of internal structure of related math branch in peculiar and to appropriate pedagogy.

The structure of a certain concept or knowledge and accordingly oriented psychology are the most important factors used to determine the instructional pedagogy. Beside having positive effects for students from cognitive and emotional aspects, these factors may also have diverse effects. Instruction process needs to embrace emotions as well considering Bloom's suggestion that one-forth of learning succession achieved by emotional attributes and Peterson's argument on emotion's being the most important catalyst of learning, providing motivational and preparing cognitive processes for learning [1-3].

One should approach epistemologically, psychologically and pedagogically for choosing the appropriate instruction method or technique that fits its structure best. Psychological analysis of items to be taught, pedagogy of how to teach and epistemology are needed. Epistemology expresses cognitive variables of learning an item, psychology expresses affective variables of learning an item and pedagogy expresses instructional and educational knowledge that finds its source in epistemology and psychology. This fact demonstrates that epistemology and psychology are two main factors to determine pedagogical approach.

Analyzing the source, nature and characteristic of concepts in linear algebra, which also means analyzing it from epistemological aspect, one should definitely recognize the geometry exists in the source and nature of those concepts of linear algebra. Both pure mathematicians studying linear algebra and mathematics instructors draw attention to the relation between linear algebra and geometry [4-15]. Dorier *et al.* (2000) emphasized geometric origin by establishing a relation between linear algebra and synthetic geometry and geometry [16], whereas Hillel (2000) expressed the geometric language as one of the three fundamental languages in linear algebra [17]. Baer (1952) and Hestenes

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(1991) indicated the identity between projective geometry and linear algebra [4-6], whereas Mirsky and Nef (1963) pointed out the identity between linear algebra and analytic geometry [10]. As a matter of fact, abstractions in linear algebra are fundamental geometric structures which are made abstract [18-20].

There is a requirement for proper pedagogical approaches to perform meaningful, consistent and available instructions of linear algebra concepts. Whatever is being done in linear algebra instruction process will not change linear algebra. However, available pedagogic method can cause changes in the process of instruction which is a research field of psychology. Concretization can be embraced as a variable that promotes the level of learning pedagogically. Dienes recommends the use concrete materials and visualization as an instructional structure for better apprehension of math concepts and learning of math structures [21,22]. Use of concrete or semi-concrete structures in teaching concepts, will facilities learning, make it more meaningful and help students to develop certain study skills [23,27-29].

Psychologically, on the other hand, concrete or semi-concrete structures are more applicable for developing positive attitude towards math and bringing about self-confidence and motivation when compared to abstract structure [30-32]. Kennedy ve Tipps (1991), recommend studying with concrete models in order to prevent developing negative attitude towards math [1].

Definitions: The term visualization is defined as producing and using geometrical representation of abstract concepts and algebraic structures in this study.

The Purpose of The Study: The aim of this study is to determine the effects of visualization approach, which is thought to psychologically positive pedagogic approach, on students complying with epistemology of linear algebra concepts. It is well known that psychological factors such as attitude and self-confidence have direct effect on learning.

METHODS

Data Collection and Analysis: In this study qualitative research method consisting written opinion receiving and oral interviews was performed. First, students' written opinions were received with-in both pre-application and post-application processes and then, oral interviews were performed in order to detail, confirm and approve the knowledge submitted in printed papers. Oral interviews were performed first on one-by-one basis and then with the whole group. There was no restriction of time in those interviews. During the interviews epistemological, pedagogical and psychological variables of linear algebra instruction were taken into account. Considering those variables knowledge structure of linear algebra and its effects on learning, teaching way of linear algebra and its effects on learning and students' positive-negative attitudes (feelings) against linear algebra and their effects on learning were tried to be analyzed with the help of students' opinions. The study was discussed with in trilogy of epistemological introducing factor, pedagogic process and psychological conclusion. Psychological analysis was made in restricted dimension on micro-level and only it was tried to figure out emotional reactions caused within linear algebra instruction process.

Population and Sampling: Course was held within spring term of 2006-2007 educational year. Sample of the study consists of 46 senior students studying (Secondary Schools) Mathematics Education Department of Kazım Karabekir Education Faculty of Atatürk University,14 of which were female and 32 were male out of 46 students mentioned.

Study was performed 4 hours a week to reach a total of 12 hours by one instructor. The content of the course was design as a geometrical summary of concepts in linear algebra and parallel to first year's linear algebra curriculum.

Application: Application focused on the ground line of visualization approach where vector geometry was used and geometrical content was discussed rather than algebraic and abstract content of linear algebra. Doing this algebraic and abstract content of linear algebra wasn't totally left out; they were mentioned at the least. The main purpose of touching on abstract algebraic content leastwise was to make students remember the basic concepts and to exhibit the relation of geometry with abstract concepts and algebraic structures. With this organization it was aimed to make students realize linear algebra concepts' getting into different forms and perceive geometric meanings of them. Lessons got started with a little summary of abstract and algebraic content of the basic concepts and then geometrical content was explained widely.

Pre-application: Before the application got started, students were asked first to write down their opinions on

linear algebra lesson they had studied before and then face-to-face interviews were done both individually and as a whole group. One of the reasons students asked for writing down their opinions is to determine available variables for face-to-face interviews. 46 students participated pre-application interviews. The main subject of face-to-face interviews is to ascertain what students tought about linear algebra and linear algebra instructions.

Application Process: This study began with a discussion of the visualization approach for solving a system of linear equations and concluded it with a discussion of the dot product and projection. The following is a brief description of the main content. The basic concepts and results on systems of linear equations, matrices and determinants, vector subspaces, linear dependence and linear independence, basis and dimension, linear transformations, eigenvalue, dot product and projection were presented by using vector geometry after this introductory section explaining the motivations of the subject from geometrical point of view.

At the beginning of this study, all of the students are given basis knowledges deal with vector concept, such as point setting in R,R^2 and R^3,vector construction, vector addition and scalar multiplication, required for teaching linear algebra because it is usually advisable at the begin to review these concepts. Namely, the introductory section of the lecture is reminded the vector geometry. Then the main tecniques for understanding linear algebra concepts are developed by vector geometry for visualization. That these tecniques are adequate to understand for most of the concepts is considered. All of the techniques in the introductory section are essential for later presentation. In the introductory study, the students have not encountered any geometric concepts or ideas which are essentially new, usually consists of a reexamination of the Cartesian coordinate system, vector and vector operations which should already be familiar for students. This introduction benefits for students who lacked this background. In this way, the abstract concepts and algebraic process can be easily explained on this basis.

Vectors and vector operations provide a powerful tool for visualizing linear algebra concepts. Vectors and some related concepts in R, R^2 and R^3 can be visualized by means of graphs using the coordinate system. Because of this, the concept of vector is very important in the study of visualization. Vector can already be used to geometric interpretation of abstract concepts in linear algebra. Geometric language enabling geometrical vector concept is used to express abstract and algebraic ideas or concepts in linear algebra.

After this introduction, the concepts in linear algebra were presented according to order given above. In the presentation, the concepts were briefly given in abstract and algebraic content and then they were presented in geometrical content.

First, the concept was defined and then a geometric interpretation of it was made. Both the definition of the concept and theorems and examples related to it were presented through drawable spaces. Primarily, the examples were chosen from R, R^2 and R^3. Later, the examples in the non-drawable spaces were carried to the drawable ones by the help of isomorphism and geometrically investigated. Discussing on these drawings, it was tried make the meanings of the algebraic or abstract terms and the existence of these terms in different spaces be perceived.

An Activity Example for Application: An example concerned with the use of visualization through vector geometry in teaching linear algebra are below.

First, the definition of the subspace concept was reminded and then the examples below and some others were solved. During this process, the algebraic solutions, although not always, were briefly mentioned as well.

For example, consider the set $W=\{(x,y,z)$: $z=4$, x, $y\in R\}$ in R^3. "Is W a subspace of R^3?" The subset W, shown in Figure 1 as a shaded plane, consist of the set of all vectors in R^3. For arbitrary elements of W, such as u and v and for arbitrary real number c, the sum of these elements of W and scalar multiplication $c.u$ are also shown in Fig. 1 and as shown in Fig. 1, these do not belong to W because these do not terminate in the shaded plane. Also, an alternative solution to this example is to realize that every subspace passes through the origin, the zero element (0,0,0), but W does not pass the origin, in other words, W does not contain the zero element.

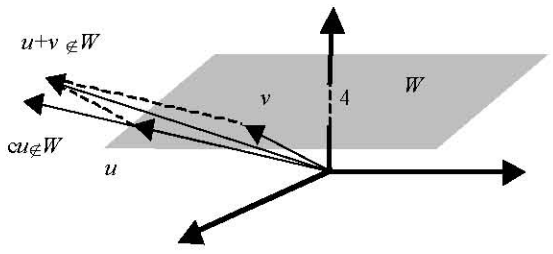

Fig. 1: Example of a subset of R^3 that does not form a subspace of R^3.

Consider the problem "Let W_1 and W_2 be subspaces of a vector space V such that $W_1 \cap W_2 \neq \{\}$, then is $W \cap W_2$ a subspace of V?" Firstly, we examine in this problem in R^2 or R^3 geometrically. Let W_1 and W_2 are the subsets of R^3. In this case, these subsets of R^3 can be shown geometrically as below (Fig. 2).

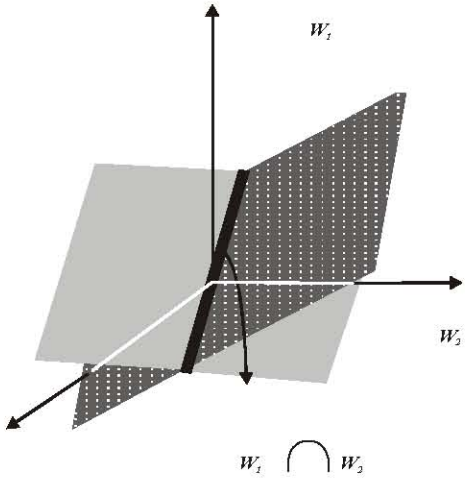

Fig. 2: Example of a subset of R^3 that forms a subspace of R^3.

We know that a line or a plane that passes through the origin represents a subspace of R^2 or R^3, a coordinate system in a space, geometrically. Let V be R^3 and, let W_1 and W_2 be two plane which pass the origin. So, $W_1 \cap W_2$ is a line pass through origin and $W_1 \cap W_2$ is a subspace R^3. This is made to general R^n and other vector space.

Algebraically, for u, $v \in W_1 \cap W_2$ and $c \in F$, if W_1 and W_2 are subspace of R^3, then $cu + v \in W_1$ and $cu + v \in W_2$ and $cu+v \in W_1 \cap W_2$. So, $W_1 \cap W_2$ is a subspace of R^3.

Post-application: 46 senior year students partipated post-application interviews. They were asked first to write down their opinions on visualization approach, linear algebra and use of visualization in linear algebra and then face-to-face interview were done once again. The main subject of face-to-face interviews is to ascertain what students tought about linear algebra and visualization.

FINDINGS

Pre-application Findings: Most students opined negatively as linear algebra's being difficult, abstract, tedious, nonsense, complicated and elusive in pre-application interviews. Regarding those aspects, they also stated that they got cold feet in this lesson and tried to succeed by learnig by heart, without understanding.

The most important reason stated among negative opinions was abstract knowledge structure as those opinions were detailed within face to face interview process. Additionally, lack of foreknowledge, the way and the place the lesson was given to, teacher as well as failing to mention daily life applications of lesson content comprises other reasons.

37 students pointed out that abstractness, lack of foreknowledge, the grade the lesson is given and the way it is performed, students' lack of motivation, tedium, meaninglessness caused a negative prejudice and fear against lesson.

On the other hand, 9 students indicated that linear algebra lesson was fun, interesting and useful. Some summary of face-to-face pre-application interviews are stated below:

- Student 1: It was complicated to me. We suddenly confronted with many abstract and unfamiliar concepts. It is quite abstract and elusive. It was a lesson that I was afraid of. I managed to pass this course hardly by memorizing items. I cannot say that I got the logic of it. I wouldn't take it.
- Student 2: I wouldn't understand some subjects such as vector space. But, matrices were easy to me. After all, I only memorized necessary items to pass the course. It was quite abstract and boring. It might be apprehensible in upper-classes.
- Student 3: linear algebra is abstract and nonsense. It becomes really unbearable when the instructor makes the content more abstract. It is hard to learn. I couldn't learn and I couldn't understand it. I just memorized. In my opinion, this course should be taught in upper-classes.
- Student 4: I had hard than with linear algebra. I realized the lack of knowledge in me. It was full of horrifying symbols and operations. I studied it shallowly and sometimes memorized items. I don't think I can solve a linear algebra problem on my own.
- Student 5: It was tough. I couldn't understand what, how and more importantly why we do. It was nonsense and complicated. It feels as if teaching a lesson in English to someone who doesn't know English.
- Student 6: It was quite abstract. That's why I had on hard time. It was very difficult. It is because I couldn't understand its logic that it seems abstract to me. It was the hardest lesson for me. We was afraid of even its name.
- Student 7: It was fun. It seemed interesting to me to study in different spaces. It was one of my favorite lessons.

Post-Application Findings: Most prominent opinions of student coming out of post-application interview are linear algebra's being concrete, comprehensive, easy, logical, meaningful, enjoyable and fluent.

Students mostly think that visualization makes lesson understood easier, simplifies expressions seems complicated otherwise, retrieves lesson from tedium, dispels fears and raise interest against lesson on makes lesson more enjoyable. 36 students expressed their opinions on both linear algebra and visualization. Furthermore, most of the students took a step further to signify the necessity for visualization to be used in other lessons as well.

On the other hand, some students denoted that visualization approach didn't help that much in learning linear algebra other than finding it different, having reviewed it and remembering the learned stuff. To be more precise, there were 7 students sharing this opinion.

One student said that he couldn't learn linear algebra anyway and that it would be beter it it were associated with daily life. Two other students stated that abstract and algebraic presentation of the lesson content was beter than the visualization approach. Some summaries of face-to-face post-application interviews are stated below:

- Student 1: Most of linear algebra terms were not shaped in my mind. I haven't understood the logic of any lesson up to now. Giving the lesson by visualization has engraved linear algebra in my memory. Abstract terms were shaped in my mind. I finally get over learning it by heart only. At least, I have something to say about this lesson. I understood it. Thanks.
- Student 2: I wish we had been taught this lesson this way. I have changed my opinions about this lessons. It was enjoyable and fluent. I know something about linear algebra now, even if it is late.
- Student 3: Visualization of linear algebra lesson has made it easier to teach and learn. Items (subjects) are apprehended easier with visualization. In my opinion, it will be more helpful for students to add visualization into lesson process. I intended it with great interest. It should be used in other lessons as well.
- Student 4: In my opinion, giving linear algebra lesson with this method is important for permanence of knowledge and for learning and understanding lesson contents. It is a usefull lesson. It was pleasure.
- Student 5: For my part, it was great. Considering most questions I could easily understand shape linear dependency, base, vector subspace. I believe in myself from now on. No need for learning by heart. Visualization was fun.
- Student 6: We review our knowledge once again with visualization. Abstract thinking is important. It is better to give this lesson abstractly.
- Student 7: Available drawing in 3D spaces makes everything easier to understand. However, I am not sure if I can draw them by myself (on my own)

CONCLUSION

Prominent student attitudes coming out of the pre-interviews against linear algebra can be classified as epistemological, psychological and pedagogical. The most important reason stated among negative opinions was abstract knowledge structure in linear algebra. Problems derived from epistemology are abstractness, meaninglessness and complexity. Those issues affect psychology either directly or undirectly. Problems stemmed from pedagogy are learning by heart, the grade the lesson is given, the way the lesson is performed, the instructor and last of all; failing to correlate lesson content with daily life experiences. Psychological reflections of those epistemology and pedagogy-originated problems are fear, dislike, tedium and lack of self-confidence.

Visualization approach is tought to be useful; (1) if the knowledge is concretized from *epistemological* point of view; (2) if a process of learning without memorizing, logic and apprehension is established *pedagogically* and (3) if the lesson is retrieved from tedium and liked by students, release the fear and hesitation, increase interest against the lesson and make it more enjoyable *psychologically*.

Furthermore, students' showing great interest to the lesson during application process was encouraging. Considering the short period the application process was managed in, some students' developing positive attitudes towards visualization approach in such a remarkably short time exposes its effectiveness.

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