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Analysis of FRP Strengthened Reinforced Concrete Beams Using Energy Variation Method

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Abstract: Reinforced concrete elements such as beams and columns may be strengthened in flexure through the use of Fiber Reinforced Polymer (FRP) composites epoxy-bonded to their tension zones, with the direction of fibers parallel to that of high tensile stresses. In this paper an improved analytical method is used to predict the deflection of rectangular reinforced concrete beams strengthened by FRP composites applied at the bottom of the beams. To achieve the aim, potential energy model is formed and varied. The validity of the proposed model has been verified by comparing with the results of the finite element model. Results obtained from the energy variation method show very good agreement with results obtained from the finite element method.

Key words: Fiber Reinforced Polymer (FRP) . Reinforced concrete beam . Energy variation . Finite element

INTRODUCTION

The traditional material used in the strengthening of concrete structures is steel. Because of its drawbacks of low corrosion resistance and of handling problems involving excessive size and weight, there is a need for the engineering community to look for alternatives. Due to lightweight, high strength and good fatigue and corrosion properties, Fiber Reinforced Plastics (FRP) have been intensively used in the repair and strengthening of aerospace structures [1-4]. Though the study of using FRP to strengthen reinforced concrete structures just started in the 1990s [5-18], the technology is currently widely used.

Several studies have been conducted on the use of Glass or Carbon FRP sheets as flexural strengthening reinforcement of concrete beams (see references). The researchers showed the behavior in terms of load-deflection, load-strain, failure patterns and structural ductility. All beams showed a considerable increase in ultimate load capacity (from 40 to 200%) with a good energy absorption capability.

Numerous proposals have been made in predicting deflection of reinforced concrete beams which takes into account tension stiffening, level of loading and percentage of reinforcement [7, 19, 20].

This paper presents an analytical method to predict the deflection of rectangular reinforced concrete beams strengthened by FRP composites applied at the bottom of the beams.

ANALYSIS OF SINGLE SPAN FRP-STRENGTHENED REINFORCED CONCRETE BEAM

In this study a single span simply supported beam strengthened with FRP composites is considered. Details of a typical beam used for the modeling and analysis in this study are shown in Fig. 1.

The flexural analysis of concrete sections with externally bonded tensile FRP reinforcement is based on the following assumptions:

- Plane sections remain plane at all time and strain distribution of elements in cross section are linearly on height.
- There is no slip between the steel or FRP reinforcement and concrete.
- Concrete only works in the compressed zone and the stress-strain relationship is linear.

Figure 2 shows the stress distribution and internal sectional forces of elements existing in the cross section.

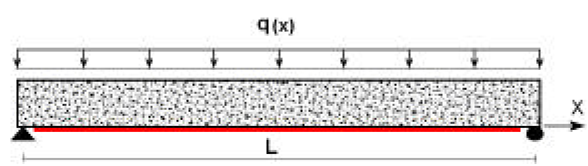

Fig. 1: Simply supported reinforced concrete beam strengthened using FRP sheets

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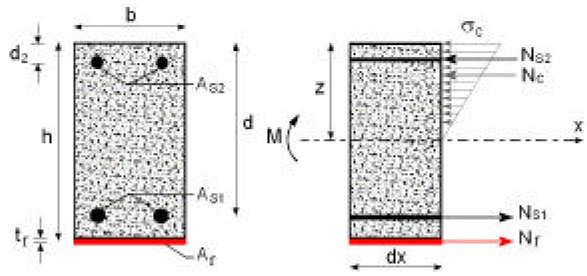


Fig. 2: Stress distribution and internal sectional forces

Variational modeling of the FRP-strengthened reinforced concrete beam: Energy function is written as:

$$Q = U - W \tag{1}$$

$$Q = \frac{1}{2}\int_0^l E_c A_c (u'_c)^2 dx + \frac{1}{2}\int_0^l E_{s1} A_{s1} (u'_{s1})^2 dx + \frac{1}{2}\int_0^l E_{s2} A_{s2} (u'_{s2})^2 dx + \frac{1}{2}\int_0^l E_f A_f (u'_f)^2 dx - \int_0^l q(x).y(x)dx \tag{2}$$

Based on the transformed cracked section, the neural axis depth z can be solved from:

$$\frac{1}{2} b z^2 + (\alpha_s - 1) A_{s2} (z - d_2) = \alpha_s A_{s1} (d - z) + \alpha_f A_f (h - z) \tag{3}$$

Where:

$$\alpha_s = \frac{E_s}{E_c} \tag{4}$$

$$\alpha_f = \frac{E_f}{E_c} \tag{5}$$

According to the above mentioned assumption:

$$\frac{u_{s1}}{d - z} = \frac{u_c}{\frac{2}{3} z} = \frac{u_{s2}}{z - d_2} = \frac{u_f}{h - z} \tag{6}$$

$$u_c = z_1 u_{s_1} \quad \text{where} \quad z_1 = \frac{\frac{2}{3} z}{d - z} \tag{7}$$

$$u_{s2} = z_2 u_{s_1} \quad \text{where} \quad z_2 = \frac{z - d_2}{d - z} \tag{8}$$

$$u_f = z_3 u_{s_1} \quad \text{where} \quad z_3 = \frac{h - z}{d - z} \tag{9}$$

Based on strain compatibility,

$$tg\varphi = \frac{u_{s1}}{d - z} = \frac{u_{s2}}{z - d_2} = \frac{u_f}{h - z} \cong Y' \tag{10}$$

$$u_{s1} = d_1 Y' \quad \text{where} \quad d_1 = d - z \tag{11}$$

When z is constant,

$$Y'' = \frac{u'_{s1}}{d_1} = \frac{u'_{s2}}{z_2 d_1} = \frac{u'_c}{z_1 d_1} = \frac{u'_f}{z_3 d_1} \tag{12}$$

The energy function can be written as:

$$Q = \frac{1}{2}\int_0^l E_c A_c (z_1 d_1 Y'')^2 dx + \frac{1}{2}\int_0^l E_{s1} A_{s1} (d_1 Y'')^2 dx + \frac{1}{2}\int_0^l E_{s2} A_{s2} (z_2 d_1 Y'')^2 dx + \frac{1}{2}\int_0^l E_f A_f (z_3 d_1 Y'')^2 dx - \int_0^l q(x).y(x)dx \tag{13}$$

Since all characteristics taken are constant, the energy function can be written as:

$$Q = \frac{1}{2} d_1^2 \begin{pmatrix} E_c A_c z_1 + E_{s1} A_{s1} + \\ E_{s2} A_{s2} z_2^2 + E_f A_f z_3^2 \end{pmatrix} \int_0^l Y''^2 dx - \int_0^l q(x).Y(x)dx \tag{14}$$

A polynomial function is used to estimate the deflection of the beam.

$$Y(x) = c_1 x (l - x) + c_2 x^2 (l - x)^2 \tag{15}$$

The values of the constants c_1 and c_2 depend on the boundary conditions used.

For the single span simply supported beam the following boundary conditions are considered.

$$x = 0;\ Y = 0;\ M = 0$$

$$x = l;\ Y = 0;\ M = 0$$

First and second derivatives of the function are:

$$Y'(x) = c_1 (l - 2x) + c_2 (2xl^2 - 6x^2 l + 4x^3) \tag{16}$$

$$Y''(x) = c_1 (-2) + c_2 (2l^2 - 12xl + 12x^2) \tag{17}$$

So, the energy function can be written as:

$$Q=\frac{1}{2}d_1^2(E_cA_cz_1^2+E_{s1}A_{s1}+E_{s2}A_{s2}z_2^2+E_fA_fz_3^2)\times\int_0^l[-2c_1+c_2(2l^2-12xl+12x^2)]^2dx-q\int_0^l[c_1(xl-x^2)+c_2(2l^2-12xl+12x^2)]dx \quad (18)$$

After solving the problem the energy function is expressed as:

$$Q=(E_cA_cz_1^2+E_{s1}A_{s1}+E_{s2}A_{s2}z_2^2+E_fA_fz_3^2)d_1^2\times\left(2c_1^2l+\frac{2}{5}c_2^2l^5\right)-q\left(c_1\frac{l^3}{6}+c_2\frac{l^5}{30}\right) \quad (19)$$

c_1 and c_2 coefficients are determined from varying energy function.

$$\frac{\partial Q}{\partial c_1}=0 \quad (20)$$

$$\frac{\partial Q}{\partial c_2}=0 \quad (21)$$

$$c_1=\frac{ql^2}{24d_1^2(E_cA_cz_1^2+E_{s1}A_{s1}+E_{s2}A_{s2}z_2^2+E_fA_fz_3^2)} \quad (22)$$

$$c_2=\frac{q}{24d_1^2(E_cA_cz_1^2+E_{s1}A_{s1}+E_{s2}A_{s2}z_2^2+E_fA_fz_3^2)} \quad (23)$$

Finally, after determining c_1 and c_2 coefficients the deflection function takes the following form:

$$Y=\frac{q\times[l^2x(l-x)+x^2(l-x)^2]}{24d_1^2(E_cA_cz_1^2+E_{s1}A_{s1}+E_{s2}A_{s2}z_2^2+E_fA_fz_3^2)} \quad (24)$$

Finite element modeling of reinforced concrete beam strengthened with FRP laminates: Numerical analysis is performed using the ANSYS finite element program to predict the deflection of rectangular reinforced concrete beam strengthened by fiber-reinforced plastics applied at the bottom of the beam. In the numerical analyses, simply supported reinforced concrete beam is considered (Fig. 3).

Three-dimensional finite element model was developed to examine the structural behavior of the strengthened beam. A quarter of the full beam was used for modeling by taking advantage of the symmetry of the beam and loadings.

An eight-node solid element, solid65, was used to model the concrete [22]. This is capable of cracking in tension and crushing in compression. The LINK8, spar element, was used to represent the reinforcing steel bar.

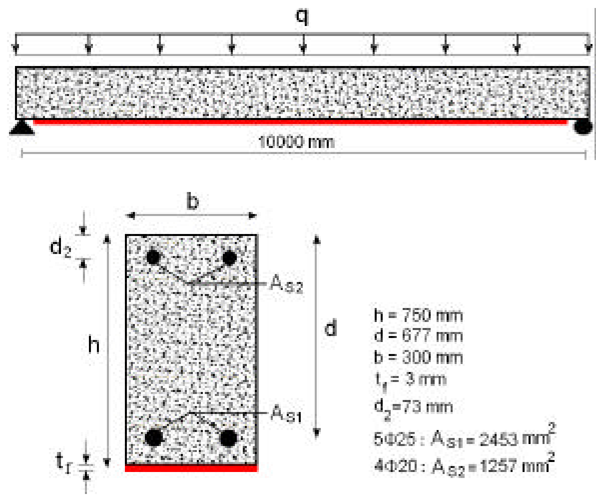


Fig. 3: Details of FRP-strengthened reinforced concrete beam in numerical analysis

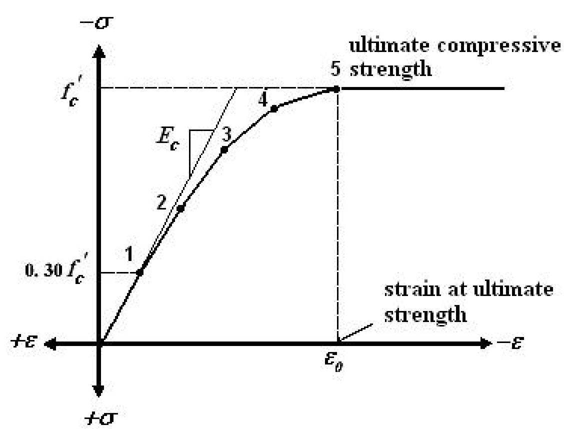


Fig. 4: Simplified compressive uniaxial stress-strain curve for concrete

Table 1: Summary of material properties for CFRP composite used in this study [23-25]

Elastic modulus (MPa)	Poisson's ratio	Tensile strength (MPa)	Shear modulus (MPa)
E_{11} = 62000	ν_{12} = 0.22		G_{12} = 3270
E_{22} = 4800	ν_{13} = 0.22	958	G_{13} = 3270
E_{33} = 4800	ν_{23} = 0.30		G_{23} = 1860

It is uniaxial tension-compression that can also include nonlinear material properties. The SOLID46, 3-D layered structural solid element, was used to represent the FRP materials. Eight nodes having three degrees of freedom at each node, as in the SOLID65 element, define the element. Layer thickness, layer material direction angles and orthotropic material properties also need to be defined. No slippage is assumed between the element layers

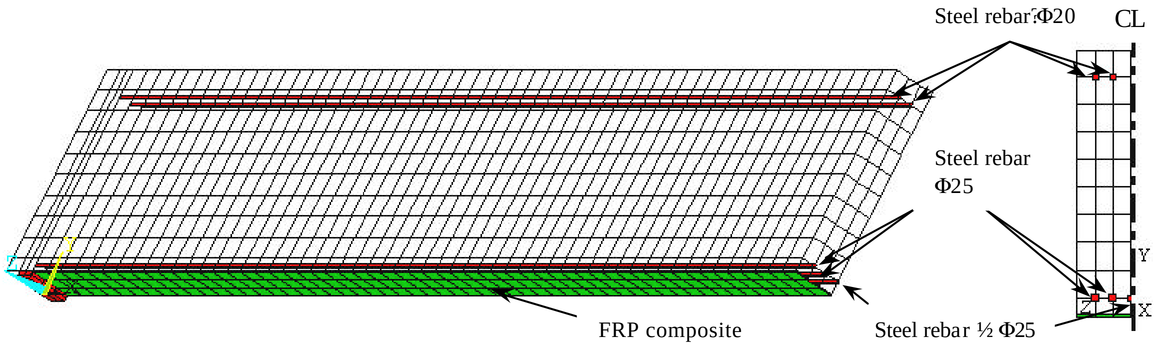


Fig. 5: Finite element model of the reinforced concrete beam strengthened with FRP laminate

(perfect interlaminate bond). The yield strength and young's modulus of reinforcements were taken as 496 MPa and 210 GPa, respectively and the compressive strength of concrete were taken as 25 MPa. The young's modulus and tensile strength of the concrete were calculated as 23668 MPa and 3.15 MPa respectively. Poison's ratio was assumed as 0.2 for concrete and 0.3 for steel rebar. The summary of the properties of FRP composites used in this study are shown in Table 1. The FRP laminate used for the strengthening has 3 layers and each layer has 1mm thickness.

The simplified stress-strain curve for beam model is constructed from six points connected by straight lines. The curve starts at zero stress and strain. Point No. 1, at $0.3f'_c$ is calculated for the stress-strain relationship of the concrete in the linear range. Point Nos. 2, 3 and 4 are obtained from Equation (27), in which ε_0 is calculated from Equation (28). Point No. 5 is at ε_0 and f'_c. In this study, an assumption was made of perfectly plastic behaviour after Point No. 5 [26]. Figure 4 shows the simplified compressive uniaxial stress-strain relationship that was used in this study.

$$f = \frac{E_c \varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2} \quad (27)$$

$$\varepsilon_0 = \frac{2f'_c}{E_c} \quad (28)$$

$$E_c = \frac{f}{\varepsilon} \quad (29)$$

A pure "compression" failure of concrete is unlikely. In a compression test, the specimen is subjected to a uniaxial compressive load. Secondary tensile strains induced by Poisson's effect occur perpendicular to the load. Because concrete is relatively weak in tension, these actually cause cracking and the eventual failure [27, 28]. Therefore, in this study, the crushing capability was turned off and cracking of the

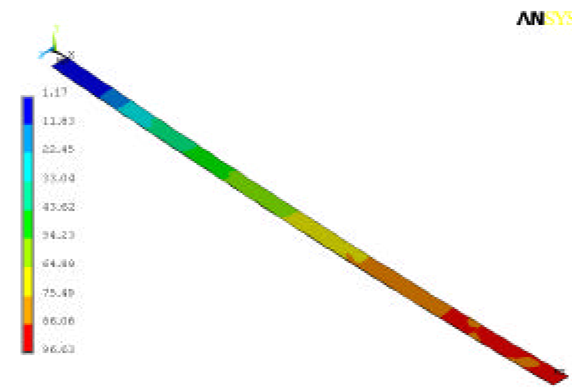


Fig. 6: Von Mises stress contours in the CFRP laminate (MPa) (q = 39.6 KN/m)

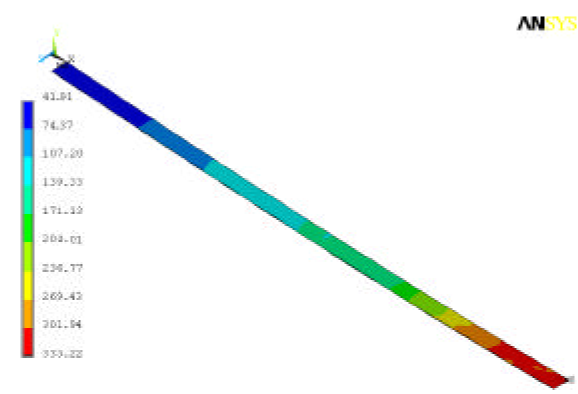
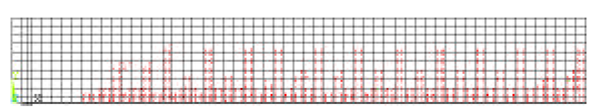
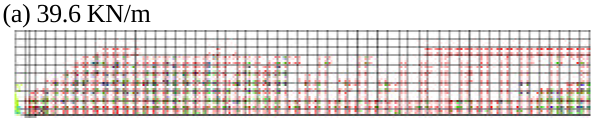


Fig. 7: Von Mises stress contours in the CFRP laminate (MPa) (q = 79.2 KN/m)



(a) 39.6 KN/m



(b) 79.2 KN/m

Fig. 8: Crack patterns of the beam obtained from the finite element analysis

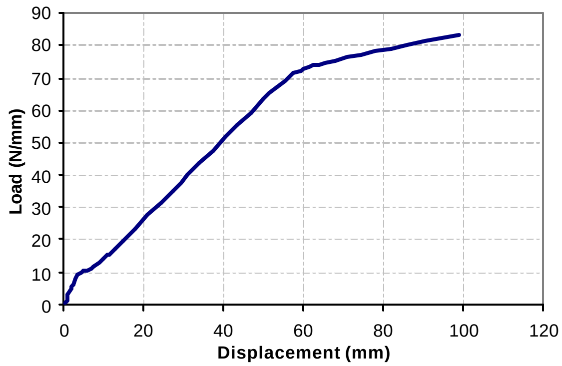


Fig. 9: Load-deflection plot for FRP-strengthened beam resulted from the FE analysis

concrete controlled the failure of the finite element models. Figure 5 shows the FE model of the FRP-strengthened reinforced concrete beam.

Figure 6 and 7 show the Von Mises stress countours in the CFRP laminate at two stages of loading.

Flexural cracks occurred early near the mid span. These cracks were followed by diagonal shear cracks near the support. Crack patterns of the beam at failure obtained from the numerical study are presented in Fig. 8.

Figure 9 shows the Load-deflection curve for the FRP-strengthened beam. The curve has been constituted of three distinct sections, the first section is related to the elastic behavior of beam before cracking. In the continuation of second part, the curve has almost fixed gradient that stems from the process of cracking of the beam by the start of steel yielding. On the third part, the curve gradient reduces and the curve advance to have a horizontal inclination which occurs due to steel yielding.

COMPARISON OF RESULTS OBTAINED FROM THE VARIATION METHOD AND FINITE ELEMENT METHOD

A simply supported single span FRP-strengthened reinforced concrete beam with 60 KN/m uniformly distributed load is used for the purpose of comparison between the variation method and finite element method. Figure 3 shows the details of beam used as a sample in this study.

According to the sample details, deflection of the FRP-Strengthened beam can be calculated from the variation method as follows:

Based on the transformed cracked section, the neural axis depth z can be solved from the equation (3).

z_1, z_2, z_3 and d_1 can be obtained from the equations (7), (8), (9) and (11), respectively.

Table 2: Calculated parameters for the variational modeling of the beam

α_s	α_f	D_1	z	z_1	z_2	z_3
8.87	2.62	433 mm	244 mm	0.38	0.39	1.17

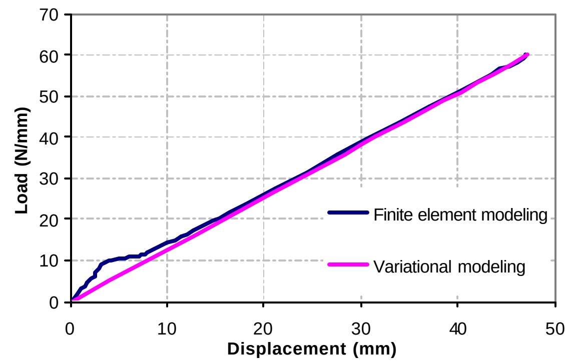


Fig. 10: Load-deflection plot for FRP-strengthened beam resulted from the FE analysis

The calculated above mentioned parameters of the variational modeling of the beam are shown in Table 2.

Finally, after determining the parameters the deflection function (Equation 24) takes the following form:

$$Y = \frac{q}{3.968 \times 10^{15}} \times [l^2 x(l - x) + x^2 (l - x)^2]$$

Mid span deflection of the beam is obtained when

$$x = \frac{l}{2}$$

Then the mid span deflection can be obtained:

$$Y = \frac{ql^4}{1.270 \times 10^{16}} = \frac{60 \times 10000^4}{1.270 \times 10^{16}} = 47.25\text{mm}$$

Figure 10 shows the load versus mid span deflection plots of the beam obtained from the variation method and finite element method.

CONCLUSIONS

The results obtained from the variational modeling show very good agreement with the finite element modeling results. At the first stage there is a difference between the variation method result and finite element method result though not significant. This difference stems from that in the FE modeling the tensile strength of the concrete was considered however in the variational modeling this parameter was neglected. The Energy variation method is an effective solution for predicting deflection at any location along the span of the reinforced concrete beam strengthened by FRP composites applied at the bottom of the beams.

REFERENCES

1. Baker, A.A., 1987. Fiber composite repair of cracked metallic aircraft components-practical and basic aspects. Composite, 18 (4): 293-308.
2. Ong, C.L., R.C. Chu, T.C. Ko and S.B. Shen, 1990. Composite patch reinforcement of cracked aircraft upper longeron: analysis and specimen simulation. Theor. Appl. Fract. Mech., 14: 13-26.
3. Naboulsi, S. and S. Mall, 1996. Modeling of a cracked metallic structure with bonded composite patch using the three layer technique. Compos Struct., 35: 295-308.
4. Schubbe, J.J., 1999. Mall S. Investigation of a cracked thick aluminum panel repaired with a bonded composite patch. Eng. Fract. Mech., 63: 305-323.
5. Ritchie, P.A., D.A. Thomas, L.-W. Lu and G.W. Connelly, 1991. External reinforcement of concrete beams using fiber reinforced plastics. ACI Struct. J., 88 (4): 490-500.
6. Saadatmanesh, H. and M.R. Ehsani, 1991. RC beams strengthened with GRFP plates. I: Experimental study. J. Struct Engng, 117 (11): 3417-3333.
7. Saadatmanesh, H., M.R. Ehsani and L. Jin, 1997. RC beams strengthened with FRP Plates II: analysis and parametric study. J. Struct. Eng., 117 (11): 3434-3455.
8. Saadatmanesh, H. and M.R. Ehsani, 1991. Repair of earthquake damaged R/C columns with FRP wraps, ACI Struct. J., 94 (2): 206-215.
9. Saadatmanesh, H., M. Ehsani and L. Jin, 1997. Seismic Retrofitting of rectangular bridge columns with composite straps. Model for the Triaxial Behaviour of Concrete. EERI Spectra.
10. Nanni, A., 1993. Flexural behavior and design of RC members using FRP reinforcement. J. Struct. Eng., 119 (11): 3344-3359.
11. Sharif, A., G.J. Al-Sulaimani, I.A. Basunbul, M.H. Baluch and B.N. Ghaleb, 1994. Strengthening of initially loaded reinforced concrete beams using FRP plates. ACI Struct. J., 91 (2): 160-167.
12. Chajes, M.J., T.F. Januszka, D.R. Mertz, T.A. Thomson and W.W. Finch, 1995. Shear strengthening of reinforced concrete beams using externally applied composite fabrics. ACI Struct. J., 92 (3): 295-303.
13. Shahawy, M.A., M. Arockiasamy, T. Beitelman and R. Sowrirajan, 1996. Reinforced concrete rectangular beams strengthened with CFRP laminates. Compos Part B: Eng., 27 (3-4): 225-233.
14. Chambers, R.E., 1997. ASCE design standard for pultruded fiber reinforced-plastic (FRP) structures. J. Compos Construct., 1 (1): 26-38.
15. Malek, A.M. and H. Saadatmanesh, 1998 Analytical study of reinforced concrete beams strengthened with web-bonded fiber reinforced plastic plates or fabrics. ACI Struct. J., 95 (3): 343-352.
16. Mukhopadhyaya, P., N. Swamy and C. Lynsdale, 1998. Optimizing structural of beams strengthened with GFRP plates. J. Compos Construct (ASCE), 2 (2): 87-95.
17. Tedesco, J.W., J.M. Stallings and M. El-Mihilmy, 1999. Finite element method analysis of a concrete bridge repaired with fiber reinforced plastic laminates. Comput. Struct., 72: 379-407.
18. Vecchio, F.J. and F. Bucci, 1999 Analysis of repaired reinforced concrete structures. J Struct. Eng., (ASCE), 125 (6): 644-652.
19. Pesic, N. and K. Pilakoutas, 2005. Flexural analysis and design of reinforced concrete beams with externally bonded FRP reinforcement. Materials and Structures, 38: 183-192.
20. Hu, H.T., F.M. Lin and Y.Y. Jan, 2004. Nonlinear finite element analysis of reinforced concrete beams strengthened by fiber-reinforced plastics. Composite Structures, 63: 271-281.
21. Reddy, J.N., 2001. Energy Principles and Variational Methods in Applied Mechanics.
22. ANSYS User's Manual, Swanson Analysis Systems, Inc.
23. Safari Gorji, M. and K.H. Bargi, 2007. Effects of Column Loading on the Behavior of Concrete Joints Reinforced with Carbon Fiber Reinforced Polymer Laminates, Civil-Comp Press, Stirlingshire, United Kingdom.
24. Safari Gorji, M. and K.H. Bargi, 2007. Study on The Application of FRP Laminates to RC Knee Joints. The First Asia-Pacific Conference on FRP in Structures, APFIS, Hong Kong China.
25. Safari Gorji, M., K.H. Bargi and P. Zarei, 2008. Numerical Investigation on Behavior of FRP-Strengthened Connections in Reinforced Concrete Frames, the Fourth International Conference on FRP Composites in Civil Engineering (CICE2008), Zurich, Switzerland.
26. William, K.J. and E.D. Warnke, 1975. Constitutive model for the triaxial behaviour of concrete. Proceedings of the International Association for Bridge and Structural Engineering.

27. Kachlakev, D., T. Miller and S. Yim, 2001. Finite element modeling of reinforced concrete structures strengthened with FRP laminates.

28. Shah, S.P., S.E. Swartz and C. Ouyang, 1995. Fracture Mechanics of concrete, John Wiely and Sons, Inc. New York.