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A New Method for Observer Design Using Eigenstructure Assignment and its Application on Fault Detection and Isolation

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Abstract: In this paper the design of an observer for Multi-input Multi-output (MIMO) systems using eigenstructure assignment and the employment of this observer for fault diagnosis are investigated. Moreover, the designed observer will be implemented in a model of unmanned aircraft. Furthermore, the state feedback design with eigenstructure assignment has been accomplished on the aircraft. The design aim is to obtain suitable gains for the observer and the state feedback, as well. Using this method, a complete parametric expression for the observer gains and the state feedback gains are established in terms of a set of parametric vectors and closed loop poles. The existence of a set of parametric vectors and closed loop poles represents some degrees of freedom in the design. The simulation results corroborate the effectiveness and simplicity of the proposed method.

Key word: Eigenstructure assignment . State observer . State feedback . Fault detection and isolation

INTRODUCTION

In recent years, automatic control systems and relevant algorithms are becoming complicated. For the complex systems, there is no absolute guarantee over their safety performances and therefore, much improvement in order to enhance reliability and safety of systems is still desirable. This could be achieved not only by applying much reliable components, but also by designing properly Fault Detection and Isolation (FDI) units. So called "Fault", sounds malfunction in real dynamic system that may lead to fail the overall system. Fault detection unit provides a mechanism to identify and isolate faults. Analytical techniques require a model or mathematical descriptions of the system to detect faults. It means that during simultaneous operations of analytical model and real system, if a different behavior is observed, fault detection unit acts immediately. For this purpose, an observer is used and faults are judged due to residual signal. Therefore, to attain a good estimation of system output, an appropriate observer design is a major problem. Luenberger observer theory generated strong interest among many researchers [1,2]. In recent years, many works are carried out in the filed of FDI [3-8], even though in many cases results are not practically validated on real model.
Two aims of this article are:

- Transition closed loop system poles to intended position by suitable design of state feedback gain, using Eigenstructure Assignment (EA) method. It is obvious that in multivariable systems, for a set of given poles, state feedback gain matrix are nonsingular and various ways may use to determine it [9].
- A new parametric approach based on EA method, to calculate Luenberger observer gain matrix more expediently. The method provides a parametric description of the observer gain and corresponding eigenvectors. Finally, fault diagnosis is carried out using residual signals.

PROBLEM FORMULATION

Consider a linear system given as below:

$$\begin{cases} \dot{X}(t) = AX(t) + BU(t) \\ Y(t) = CX(t) \\ X \in R^n \\ U \in R^r \\ Y \in R^m \end{cases} \tag{1}$$

where, X, U and Y are state, input and output vectors, respectively and A, B and C matrices are coefficient matrices. Let's suppose the system is controllable and observable.
So observer equation is as below:

$$\begin{cases} \dot{\hat{X}}(t) = A\hat{X}(t) + BU(t) + L(Y(t) - \hat{Y}(t)) \\ \hat{Y}(t) = C\hat{X}(t), \hat{X} \in R^n,\ \hat{U} \in R^r,\ \hat{Y} \in R^m,\ L \in R^{n\times m} \end{cases} \tag{2}$$

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If error and residual signals are defined as:

$$e(t) = X(t) - \hat{X}(t)$$

and

$$r(t) = Y(t) - \hat{Y}(t) = Ce(t)$$

respectively, then derivative of error signal becomes:

$$\begin{cases} \dot{e}(t) = \dot{X}(t) - \dot{\hat{X}}(t) = (A - LC)e(t) = A_e e(t) \\ r(t) = Ce(t) \end{cases} \quad (3)$$

If L is determined suitably such that all the eigenvalues of (A-LC) matrix is located in the left half plane, the error comes to zero.

By applying state feedback control rule as U = KX the Eq. 1 becomes:

$$\dot{X}(t) = A_c X(t), \quad \text{where } A_c = A + BK \quad (4)$$

Suppose that eigenvalues of closed loop system and those of the observer are specified as:

$$\sigma(A_C) = \{s_i \in C, i = 1,2,...,n\}$$

And

$$\sigma(A_o) = \{s_{oi} \in C, i = 1,2,...,n\}$$

respectively. Where, s_i and s_{oi} values are complex conjugate. So corresponding eigenvectors of the eigenvalues can be written as:

$$A_c v_i = s_i v_i, \quad i = 1,2,\ldots,n \quad (5)$$

$$A_o v_{oi} = s_{oi} v_{oi}, \quad i = 1,2,\ldots,n \quad (6)$$

State Feedback Design Using EA Method

If $\Lambda = \text{diag}(s_1, s_2,\ldots,s_n)$ and

$$V = [v_1, v_2, \ldots v_n]$$

so Eq. 5 can be written as:

$$AV + BKV = \Lambda V \quad (7)$$

Suppose:

$$W = KV \quad (8)$$

Then we will have

$$AV + BW = \Lambda V \quad (9)$$

Since the system is controllable, so

$$\text{rank}([A - sI_n \quad B]) = n \quad \text{for } \forall s \in C$$

and nonsingular matrices $P(s) \in R^{n\times n}[s]$ and $Q(s) \in R^{(n+r)\times(n+r)}[s]$ can be achieved so that they satisfy conditions given as:

$$P(s)[A - sI_n \quad B]Q(s) = [0 \quad I_n], \forall s \in C \quad (10)$$

Now, if Q(s) matrix is determined as:

$$Q(s) = \begin{bmatrix} Q_{11}(s) & Q_{12}(s) \\ Q_{21}(s) & Q_{22}(s) \end{bmatrix}$$

where $Q_{11}(s) \in R^{n\times r}[s]$, the first theorem can be achieved that is a parametric description of the feedback gain matrix.

Theorem 1: [10]

For Eq. 1, if rank (B) = r and complex conjugate values of s_i are determined for (s_i, i = 1,2,…,n) and Eq. 1 is controllable, it will be possible to represent a parametric description of the feedback gain matrix, K, like below:

$$K = WV^{-1} \quad (11)$$

$$\begin{cases} V = [v_1, v_2, \ldots v_n] \\ v_i = Q_{11}(s_i) f_i, \quad i = 1,2,\ldots,n \end{cases} \quad (12)$$

$$\begin{cases} W = [w_1, w_2, \ldots w_n] \\ w_i = Q_{21}(s_i) f_i, \quad i = 1,2,\ldots,n \end{cases} \quad (13)$$

Where, $f_i \in C^r$, i = 1,2,…,n are a group of free parametric vectors and satisfy the following constraints:

$$s_i = \bar{s}_j \leftrightarrow f_i = \bar{f}_j, \; i,j = 1,2,\ldots,n$$

$$\det(V) \neq 0$$

STATE OBSERVER DESIGN USING EA METHOD

For the system given by Eq. 1 and the observer given by Eq. 2, according to Eq. 3, to become the estimation error to zero, magnitude of L should be determined so that all the eigenvalues of (A-LC) matrix locate in the left half plane. According to EA method, to get an appropriate L and desired roots, the following theorem is represented:

Theorem 2: For Eq. 1 assuming rank(C) = m, complex conjugate values of s_{oi} for (s_{oi}, i = 1,2,…,n) determined and using EA method, a parametric description of gain observer matrix L similar to Eq. 3 will be achieved so that $\sigma(A - LC) = (s_{oi}, i = 1,2,\ldots,n)$ and

$$\begin{cases} L = -(W_o V_o^{-1})^T \\ V_o = [v_{o1}, v_{o2}, \ldots, v_{on}] \\ v_{oi} = Q_{o11}(s_{oi}) f_{oi} \quad , \ i = 1,2,\ldots,n \\ W_o = [w_{o1}, w_{o2}, \ldots, w_{on}] \\ w_{oi} = Q_{o21}(s_{oi}) f_{oi} \ , \ i = 1,2,\ldots,n \end{cases} \tag{14}$$

Where, $f_{oi} \in Cr$, i = 1,2,…,n are a group of free parametric vectors and satisfy the following conditions:

$$s_{oi} = \overline{s}_{oi} \leftrightarrow f_{oi} = \overline{f}_{oi} \quad , \ i,j = 1,2,\ldots,n$$

$$\det(V_o) \neq 0$$

Proof: By use of:

$$\sigma(A - LC) = \sigma\left((A - LC)^T\right) = \{s_{oi} \ , \ i = 1,2,\ldots,n\}$$

And transposed Eq. 3, magnitude of L and eigenvector associated with eigenvalue s_{oi} shown as v_{oi} of $(A - LC)^T$ are evaluated as:

$$\begin{aligned} &(A - LC)^T v_{oi} = s_{oi} v_{oi} \ \Rightarrow \\ &\left(A^T - C^T L^T\right) v_{oi} = s_{oi} v_{oi}, \ i = 1,2,\ldots,n \end{aligned} \tag{15}$$

By defining Λ_o = diag (s_{oi}), i = 1, 2,…, n and $V_o = [v_{o1}, v_{o2}, \ldots, v_{on}]$ we have:

$$A^T V_o - C^T L^T V_o = \Lambda_o V_o \tag{16}$$

If

$$W_o = -L^T V_o \tag{17}$$

Therefore

$$A^T V_o + C^T W_o = \Lambda_o V_o \tag{18}$$

According to dual property, if couple (A, C) is observable, couple (A^T, C^T) will be controllable and rank $(\lfloor A^T - sI_n \ C^T \rfloor) = n$

Therefore, nonsingular matrices $P(s) \in R^{n \times n}[s]$ and $Q_o(s) \in R^{(n+r)\times(n+r)}[s]$ can be obtained so such as they satisfy the conditions given as:

$$P_d(s)\left[A^T - sI_n \ \ C^T\right] Q_o(s) = [0 \ I_n], \forall s \in C \tag{19}$$

If

$$Q_0(s) = \begin{bmatrix} Q_{o11}(s) & Q_{o12}(s) \\ Q_{o21}(s) & Q_{o22}(s) \end{bmatrix}$$

and

$$Q_{o11}(s) \in R^{n \times r}[s]$$

by using first theorem, we have:

$$\begin{cases} V_o = [v_{o1}, v_{o2}, \ldots, v_{on}] \\ v_{oi} = Q_{o11}(s_i) f_{oi} \ , \ i = 1,2,\ldots,n \\ W_o = [w_{o1}, w_{o2}, \ldots, w_{on}] \\ w_{oi} = Q_{o21}(s_i) f_{oi} \ , \ i = 1,2,\ldots,n \end{cases} \tag{20}$$

Where $f_{oi} \in C^r$, i = 1, 2,…, n are a group of free parametric vectors and satisfy the following conditions:

$$s_{oi} = \overline{s}_{oi} \leftrightarrow f_{oi} = \overline{f}_{oi}, i,j = 1,2,\ldots,n$$

$$\det(V_0) \neq 0$$

Now, L can be obtained simply as:

$$W_o = -L^T V_o \ \Rightarrow \ L = -(W_o V_o^{-1})^T \tag{21}$$

Defining residual signal as:

$$r(t) = Y(t) - \hat{Y}(t)$$

fault occurrence can be checked by the rule given by:

$$r(t) = \begin{cases} = 0 & \text{No Fault} \\ \neq 0 & \text{FaultOccure} \end{cases} \tag{22}$$

SIMULATION RESULTS

In this section to show the effectiveness of the EA method, the results are examined through longitudinal motion of unmanned aircraft model described by a linear model as below:

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX \end{cases} \tag{23}$$

$$A = \begin{bmatrix} -0.062 & 0.2859 & 0 & -9.81 & 0 & 0.0125 & 0 \\ -0.562 & -2.3298 & 32.9799 & 0 & 0 & 0 & 5.3170 \\ -0.070 & -.4526 & -0.0499 & 0 & 0 & 0 & -13.5789 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 33 & 0 & 0 & 0 \\ -16.85 & 0 & 0 & 0 & 0 & -1.1968 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10 \end{bmatrix}$$

Table 1: calculate and compare closed loop and observer roots with EA method

Desired roots	-2.8±5j	-1 ± j	-0.1	-3	-20
σ (A + BK) Roots according to the First theorem	-2.8007±5.0072j	-0.9459±0.9968j	-0.1001	-2.9919	-20.9
σ (A - LC) Roots according to the Second theorem	-2.8002±5j	-1±j	-0.1000	-2.9997	-20.0

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 130.0813 \\ 20 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$U = \begin{cases} \eta & \text{ElevatorAngle} \\ T_H & \text{Throttle} \end{cases} \tag{25}$$

$$X = \begin{cases} x_1 & \text{Forwardvelocity} \\ x_2 & \text{Downwardvelocity} \\ x_3 & \text{Pitch rate} \\ x_4 & \text{Pitch angle} \\ x_5 & \text{Height} \\ x_6 & \text{Thrust} \\ x_7 & \text{Elevator actuator mode} \end{cases}$$

So the desired eigenvalues of closed loop system is used as follow.

$$\begin{cases} \lambda_{1,2} = -2.8 \pm 5j \\ \lambda_{3,4} = -1 \pm j \\ \lambda_5 = -0.1 \\ \lambda_6 = -3 \\ \lambda_7 = -20 \end{cases} \tag{26}$$

Using the first and the second theorems and considering desired roots for closed loop system and observer, Table 1 is obtained.

Now in trim conditions and a pitch rate variation of 1 rad/sec simulations are carried out. Closed loop outputs obtained by applying the state feedback are the state observer outputs too and they are shown in Fig. 1 and 2. State variables converge to the steady state values and the estimated state variables follow the real ones by suitable speeds. Then, to confirm efficiency of this method for identifying and detecting the faults, simulation are repeated by increasing the amplitude [Height] of the second output by 0.2 step function at time 15 second. Residual signals are illustrated in Fig. 3 and 4. Figure 3 indicates the difference between real and estimated values of pitch rate and Fig. 4 shows the same difference for height. The fault which is occurred

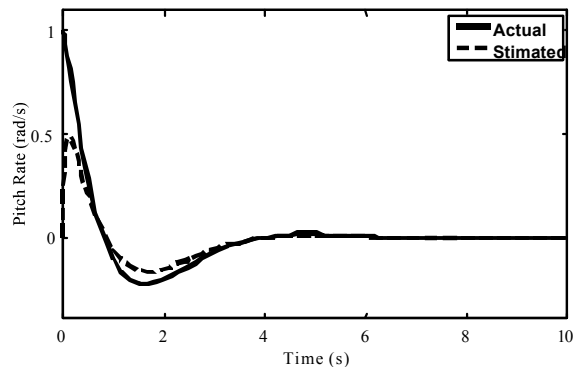

Fig. 1: Real and estimated pitch rate

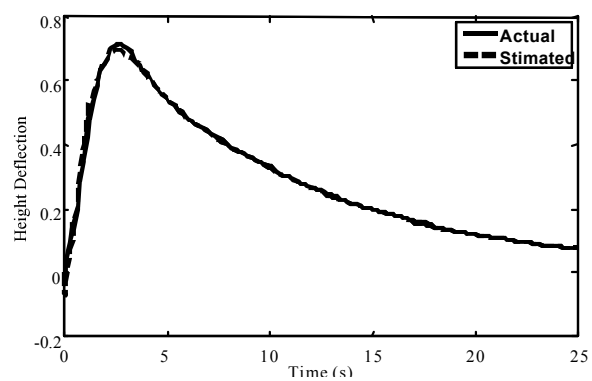

Fig. 2: Real and estimated height

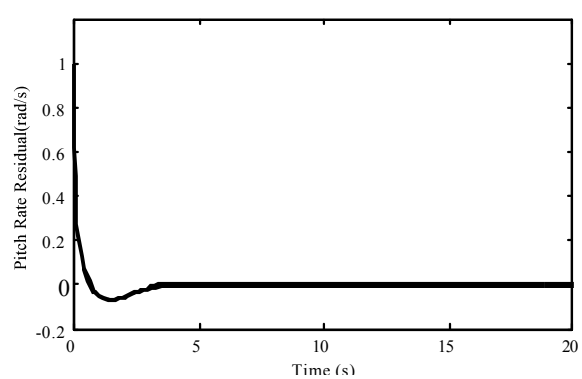

Fig. 3: The difference between real and estimated values of pitch rate, when fault applied to the height sensor at 15th second

at time 15 second can be observed in Fig. 4 obviously. However, Fig. 3 doesn't illustrate the fault occurrence clearly. Moreover, by use of residual signal, the type of

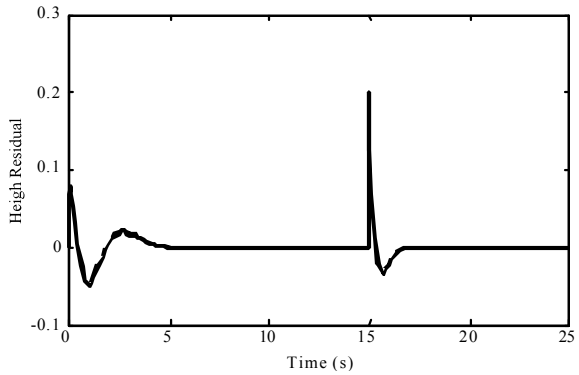

Fig. 4: The difference between real and estimated values of height, when fault applied to the height sensor at 15th second

the fault can be determined and this presents another significant advantage of the proposed method.

CONCLUSION

In this paper, state observer design problem is taken into account by means of EA method. A parametric description for observer gain matrix is presented and eigenvalues of observer and corresponding eigenvectors are obtained. Having a series of parametric vectors and appropriate poles provide possibilities to specify other characteristics such as robustness of system. Longitudinal motion of unmanned aircraft model accompanied with EA based state feedback controller is utilized as an example to illustrate effectiveness of the observer design method. Simulation results confirm capability of the proposed observer design methodology.

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