Development in Mode Shape-Based Structural Fault Identification Technique

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Abstract: In this paper a major group of fault identification methods, namely mode shape-base methods are reviewed. The methods are based on the fact that mode shape is function of the physical properties of the structure. Therefore, changes in the physical properties will cause detectable changes in the mode shape. Main characteristics of these methods are investigated in two parts. In the first part the methods use changes in mode shapes to identify faults are studied and in the second part those utilize mode shape derivatives to detect faults are investigated.

Key words: Fault identification · Mode shape · Structural assessment · Structural integrity · Strain energy

INTRODUCTION

The occurrence of fault during the operation of structures is often inevitable. Natural events or incorrect usage of a structure can be a cause of fault in the structure. Therefore, Structural Health Monitoring (SHM) and also on time detection of faults are important to increase of safety and reliability and decrease of maintenance and repairing cost. Due to this fact, during the last decades many researches were conducted on Non-Destructive Evaluation (NDE) [1, 2].

It has been ages that local test and local inspection has been used in industry. Method such as acoustic or ultrasonic methods, magnetic field methods, dye penetrant, radiography, eddy current methods or thermal field methods [3, 4] are some of these methods. All of these methods include difficulties which made them impractical, infeasible and expensive in large scale structures [5]. The problems of these methods are that the place of fault is not indicated: either the location of fault has to be determined or the tests have to be given periodically in sensitive points in the structure to introduce the fault of the structure. In this case, regardless of being cost-increasing, the fault may occur at unpredicted locations. Another problem is that the location of the fault maybe at invisible and/or unavailable locations for the tests [1, 6].

In contrast there are some global fault identification methods. In these methods the general behavior of a structure is investigated to find out the existence, location and intensity of the fault [7]. Generally, existing global fault identification methods can be classified in two major categories which are the dynamic and static fault identification methods. In the dynamic fault identification methods, changes in dynamic properties of the fault structures are evaluated to detect the location and severity of the faults. In contrast the static fault identification techniques assess the changes of the static properties of the fault structures to identify the location on severity of the faults [1]. The global fault identification method has been expanded during the last years [8] and the main cause is that these methods don’t include the problem mentioned for local and visual inspection methods. Moreover, nondestructive evaluation using dynamic response has been considered a lot [6]. For example FHA (Federal Highway Administration) mandates evaluation of condition of bridge structures. Current federal spending in the United States for replacement of structurally outdated bridges founded on these kinds of methods is approximately $10 billion per year [9].

The basic idea of these methods is that fault will alter the stiffness, mass, or energy dissipation properties of a system, which in turn change the measured dynamic response of the system [10].

In General, the structural fault identification can be divided into five levels as follow [11]:

- detection of fault existence in a structure (existence);
- localization of fault (location);
- identification of the fault type (type);
- quantification of fault severity (extent); and
- prediction of the remaining useful life (prognosis).

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The main problems for structural fault identification are how to ascertain emergence, location and severity of structural fault using the given measured structural dynamic responses. In order to detect structural fault from structural dynamic response signals, the first problem is to select fault feature index to be built [12]. Early researchers have focused on the relationship between fault (i.e., stiffness loss) and resonant frequency changes [13, 14]. However, even though the modal frequency changes have been exhibited to be useful in fault identification problems in mechanical systems with a few Degrees Of Freedom (DOF), to settle the fault location in realistic structures such as bridges and offshore platforms using only frequencies is not applicable [15]. Frequency change depends on the square root of the stiffness alter and the environmental conditions such as moisture content and temperature can easily change the resonant frequencies of a structure. To overcome these disadvantages, some researchers have focused on using mode shape measures to evaluate fault [15, 16].

The mode shape-based methods are sensitive to change in the elements stiffness of structures which has made this methods one of the most practical tool for fault identification. Nevertheless, some of local faults are not able to incitement down modes [17, 18] while down modes are better for measurement [7]; there for the methods in deficient element helping down modes is recognized.

Next problem in this method and generally modal analysis are faced whit is impressible of modal data from external noises [19] which causes some main problems in detection of exact location of fault.

Salawu [20] presented an excellent review on the utilization of resonant frequency changes for fault identification in 1997 and this paper provides an overview of methods to detect, locate and characterize fault in mechanical and structural systems using mode shape. This study is limited to NDE methods that are used to conclude fault from changes in mode shape characteristics of structures. In continuation of this paper the mode shape-base methods are investigated in two parts. In the first part the methods using changes mode shapes are explained and in the second part the methods utilizing mode shape derivatives are studied.

**FAULT IDENTIFICATION VIA MODE SHAPE CHANGES**

A local change of stiffness arises from local change of mode shapes curvature and hence in the mode shapes. For $i^{th}$ mode shape, $\Phi$, the change of the eigenvector is [21]:

$$\Delta \Phi = \Phi_i^d - \Phi_i^0$$

(1)

In the above expression $0$ and $d$ denotes the reference and fault state respectively.

Proper pairing of the eigenvalues/eigenvectors and proper mode shape’s scale are solved by using Modal Assurance Criterion (MAC) and Modal Scale Factor (MSF) [22-24].

The first systematic use of mode shape information for the location of structural fault without the utilization of a prior finite element model (FEM) was presented by West in 1984 [25]. The author utilized the MAC to settle the level of correlation between mode shapes from the test of an intact Space Shuttle Orbiter (SSO) body flap and the mode shapes from the check of the flap after it has been exposed to acoustic loading. The mode shapes are partitioned using various plans and the change in MAC across the different partitioning techniques was utilized to detect the structural fault localization.

Fox [26] showed single-number measures of mode shape changes (such as the MAC) are comparatively insensitive to fault in a beam that has a saw-cut. This work highlights the problem that a lot of data compression can happen in fault identification. “Node line MAC,” (a MAC founded on measurement points nearby a node point for a particular mode shape) was discovered to be a more sensitive indicator of changes in the mode shape caused via fault. Graphical comparisons of relevant changes in mode shapes shown clearly to be the best way of finding the fault location when only mode shapes and resonant frequencies were tested. A simple method of correlating node points with the corresponding peak amplitude points was exhibited to locate the fault. In addition, Fox presented a method of scaling the relative changes in mode shapes to recognize the location of the fault better than before.

Mayes [27] revealed a mode shape-based method for model error localization. The changes known as structural translational and rotational error checking (STRECH) used in this method. By taking ratios of relevant modal parameters displacements, STRECH estimate the preciseness of the structural stiffness between two different structural DOF. STRECH is able to compare the results of an examination with an original FEM or to compare the results of two tests.

Skjaeraek et al. [28] tested the optimal sensor location issue for identifying structural fault based on
changes in mode shapes and modal frequencies by a substructure iteration method.

Ratcliffe [29] presented a technique for locating fault in a beam which utilizes a finite difference approximation of a Laplacian operator on mode data.


Moreover, some other researchers [31-42] provide examples of changes in mode shapes which focused primarily on MAC and coordinate MAC (COMAC) values to identify fault.

Garcia et al. [43] proposed a new non-destructive fault identification method for beam structures. The aim of this study is evaluation of the relevant performance of several Bayesian distance-based pattern recognition models and two non-Bayesian models for non-destructive fault identification. A theory of fault localization, which information on the location of the fault from direct changes in mode shapes, is formulated.

Natke [44] used changes in natural frequencies and mode shapes to identify fault in a FEM of the cable-stayed steel bridge. Fault is simulated by removing the bottom flanges of the longitudinal girders at the last spans. Such a fault case is enormous and the conclusions of such a simulation reflect this situation. In fact, the second vertical bending mode presents a 23% decrease in the corresponding natural frequency and the MAC values of mode shapes differ significantly from unity. Natke justified performing a linear modal analysis of the bridge due to the high pretension of the cables, which generally conclude the bridge to behave in a nonlinear form and essentially ensures that the cables always carry most of the load from the bridge deck, preventing any bilinear stiffness effects of the entire bridge.

Doebling and Farrar [45] tested changes in the natural frequencies and mode shapes of a bridge as a function of fault. This paper focuses on appraising the statistics of the modal parameters using Monte Carlo procedures to check if fault has produced a statistically significant change in the mode shapes. Stanbridge et al. [46] also utilize mode shape changes to identify saw-cut and fatigue crack fault in flat plates. The authors also discuss methods of extracting those mode shapes by laser-based vibrometers.

Another application of SHM using changes in mode shapes and participation agents can be found in Ahmadian, Mottershead and Friswell [47]. They suggested a fault identification procedure that utilizes measured displacements of a structure and an existing analytical model to detect the location of faults. When fault happens in substructures or also a small part of a larger structure, the substructure’s mode shapes will be changed; however the modes of other substructures will be unaffected. In fact, fault in a particular substructure changes the involved factors of the higher mode shapes of the substructure, but not the higher mode shapes of the other substructures.

Ettouney et al. [48] discussed a comparison of three various SHM techniques applied to a complex structure. These three techniques are based on knowing the mode shapes and natural frequencies of the fault and non-fault structure. A FEM was used to extract mode shapes of up to 250 Hz for this work. The first method includes monitoring the change of the stiffness matrix of the structure. The stiffness matrix of a non-fault structure can be calculated from the measured modal parameters and the stiffness matrix for the fault structure and also by the mode shapes and natural frequencies achieved from the fault structure. The change in the stiffness matrix of the fault and non-fault states can then be applied for fault identification. In a similar form, the flexibility matrices can be calculated from the measured modal parameters of the fault and non-fault structures. After that, the flexibility change between the fault and non-fault states can be used for fault identification. The third method is the fault index method described by Stubbs et al. [49]. Unlike the two previous methods, fault in this method is identified at structural element levels rather than at nodal degrees of freedom. Ettouney et al. applied the three methods to a complex steel structure. The overall dimensions of the structure were roughly 3 × 21 × 21 m. In this study fault was introduced as a model by altering the modulus of elasticity for selected structural elements. All these three methods could identify the relative location of the fault elements with acceptable accuracy. However, it is questionable if these methods can work in real-world too because the number of mode shapes and natural frequencies that can be obtained from experimental modal analysis, are often limited.

Kho et al. [50] presented modal analysis techniques to locate fault in a wooden wall structure by evaluating fault-sensitive parameters like resonant pole shifts and mode shapes and fault area is detected by visual comparison of the deformation mode shapes before and after fault.

**FAULT IDENTIFICATION VIA MODE SHAPE DERIVATIVE CHANGES**

An alternative to using mode shapes to gain spatial information about sources of vibration changes is using
mode shape derivatives. It is first noted that for beams, plates and shells there is a straight relationship between curvature and bending strain. Some researchers discuss the feasible issues of direct strain measurement or its computation from displacements or accelerations [51].

The mode shape curvature appears to be more sensitive to loss of stiffness because of member fault than the mode shapes themselves. Loss of a member for instance may cause a sudden change in the mode shape’s first derivative (slope) and second derivative (curvature or strain). Observation of these modal derivatives, specially strain because it is easy to measure, may help locate the fault in the structure [52].

**Mode Shape Curvatures:** A special advantage of using either modal curvature is the modal curvature goes to be more sensitive to local fault than does modal displacement [10]. Defined as mode shape curvature, deformation mode of neutral surface which is associated with cross-section bending rigidity (EI) of members. After fault of some element is occurred, reduce of EI will cause increment of curvature, that is fault locations can be detected by curvature. Basically, curvature is constructive by central difference using displacement mode shapes, as follow [53]:

\[
\Phi_{x,r}^r = \frac{\Phi_{i+1,r}^r - 2\Phi_{i,r}^r \Phi_{i-1,r}^r}{(\Delta h)^2}
\]

Where \(\Delta h = h(s+1) - h(s-1)\) is coordinate difference between measure point and is the \(r^{th}\) term of the \(i^{th}\) mode shape.

Pandey et al. [54] for the first time demonstrated that absolute changes in mode shape curvature can be a good fault indicator for the FEM beams structures in 1991.

Chance et al. [55] understood that numerically calculating curvature from mode shapes cause unacceptable errors. They used measured strains in return to measure curvature directly which considerably improved results.

Maeck and De Roeck [56] applied a direct stiffness approach to fault diagnosis, localization and quantification for a bridge structure. The direct stiffness calculation used mode shapes in deriving the dynamic stiffness of a structure and experimental frequencies. This method made utilize of the basic relation that the bending stiffness of a beam is equal to the bending moment divided by the related curvature that is the second derivative of the bending deflection.

Maeck and De Roeck validated their approach by the Z24 prestressed concrete bridge in Switzerland and a reinforced concrete beam, which are both gradually fault. The authors mentioned that when a central difference approximation is utilized for computing of curvatures from mode shapes, this approximation contribute to unwanted oscillations and inaccurate values. Thus, they use a weighted-residual penalty-based optimization technique to account for the inherent errors of the measured mode shapes. This technique, which resembles a finite element technique, estimates the curvatures from the measured mode shapes with minimizing an objective function. An identical study was reported in Maeck, Wahab and De Roeck [57] where the authors approximated the stiffness degradation of a fault reinforced concrete beam by the direct stiffness approach.

Ho and Ewins [58] presented a numerical evaluation of the Fault Index method, which is explained as the quotient squared of a structure’s modal curvature in the non-fault state to the structure’s corresponding modal curvature in its fault state. That quotient depends, of course, on which mode is selected. The first three mode shapes from a numerical model of a cantilever beam are used to compute the Fault Index. The authors made fault by reducing the modulus of elasticity of the beam. They found that the performance of the Fault Index is very sensitive to the presence of noise in the mode shape. The authors showed that noise is amplified in the Fault Index due to curvature is the second derivative of the mode shape and the curvature is squared.

In another study by Ho and Ewins [59] the authors expressed that higher derivatives of mode shapes are more susceptible to fault, but the differentiation process strengthens the experimental variations inherent in these mode shapes. To cope with this difficulty, the authors tried to derive a mode shape-based feature that is sensitive to fault whereas relatively insensitive to experimental variation. They proposed changes in the mode shape slope squared as a feature. Particularity, the first derivative of the mode shape is computed and then squared. Change in this quantity is applied as the fault-sensitive feature. To calculate the derivative of the mode shape, a local polynomial is fit via every four consecutive measurement points and the resulting polynomial is differentiated. The authors note that this way of calculating mode shape derivatives is subject to smaller variations than those with a finite difference approximation, which is typically used to compute the derivatives.
Wang et al. [60] evaluated the fault index method by reviewing the formulation of this method, examining the traditional approaches step by step and then using the method to numerous fault scenarios of a concrete bridge FEM. The evaluation was undertaken because the fault index suffers from some unresolved problems that restrict its effectiveness. First, the sensitivity of this method to fault can change as a function of fault location because of the change in modal strain energy. Therefore, it cannot be supported that the fault index will respond in a uniform way to localized fault at an arbitrary location. Second, it is traditionally supported that the normalized fault measure treats as a standardized normal random variable. However, the authors stated that the fault should be related to an energy criterion because the fault index formulation is expressly a deterministic one. To examine the feasibility of using the fault index to locate fault, tests were conducted by a FEM of a bridge. The authors drew the following results:

Regardless to location or magnitude of fault, the method correctly located fault states approximately 70% of the time. The method is most successful near the center of the structure and endures significantly near the abutment.

Further theoretical numerical studies are essential to determine if the fault index variable is normally distributed as it is supposed to be.

Kim et al. [61] extended the fault index method for structures in which no information exists for the non-fault, or baseline, structure. The authors presented a method that can be applied to estimate modal parameters of a non-fault structure using data from the current fault structure and a corresponding FEM. The authors developed a FEM of the structure in question and modal information from the model. Modal information is also gained experimentally from the current fault physical structure. The authors stated an object function that is the norm of the fractional changes in the eigenvalues between the initial FEM and the physical structure. The authors claimed the stiffness parameters for a non-fault baseline model can be achieved by minimizing this object function. The authors evaluated this methodology on a truss bridge with a steel frame. The conclusions from the estimated baseline model were compatible with actual conditions.

Garcia et al. [62] extracted modal parameters by the classical Frequency Response Function (FRF) in the frequency domain and the Auto-Regressive Moving-Average (ARMA) prediction model in the time domain. The authors claimed that modal parameter extraction using the FRF is very user-dependent since the estimation of modal parameters depends highly on the ability and knowledge of the analysis. The ARMA method is a promising technique in that this assumption removes the erroneous user interface related to the FRF technique. The first five mode shapes were extracted from the experiment of the beam. Comparison of the results showed that the ARMA technique was successful in determining the fourth and fifth mode shapes and in locating fault. However, the ARMA method was not successful in reproducing the first mode of the channel.

Garcia and Osegueda [63] in a similar study used parameters of an auto-regressive moving-average (ARMA) model for fault identification to omit the need estimating modal parameters and minimizing user interface in connection with the modal parameter extraction procedure.

Maeck [64] utilized a regularization technique to decrease the errors from the numerical curvature calculation.

**Modal Strain Energy:** Strain mode shapes the same as modal curvature reflex structural instinct character. Based on material mechanics, the below equation is obtained for a straight beam.

\[
\varepsilon_{i,r} = -\Phi_{i,r}^\prime
\]

Where; \(\varepsilon\) = strain of any point in a certain cross section; \(z\) = distance from the point corresponding to neutral axis [53].

Stubbs et al. [65] presented a method based on reduction in modal strain energy between two structural DOF, as described by the curvature of the measured mode shapes. Topole and Stubbs [66, 67] tested the feasibility of using a limited set of modal parameters for structural fault identification. In another publication, Stubbs and Kim [68] examined the feasibility of localizing fault applying this technique without baseline modal parameters.

Zhang et al. [69] proposed a structural fault identification method based on element modal strain energy which utilizes measured mode shapes and modal frequencies from both fault and non-fault structures and also a FEM to locate fault. Simulation studies by beam and truss type structures are accomplished to investigate the feasibility of this method. Modal data from the first five modes are utilized in this method. The authors found that their method is not able to detect fault in a structure when the fault is located in a non-sensitive element to
modal parameter changes. The strain energy method however has demonstrated some ability in locating multiple fault areas. Finally, the authors compared their method with the Element Frequency Sensitivity (EFS), Subspace Rotation Vector (SRV) and the Local Frequency Change Ratio (LFCR) methods of fault identification. These other methods give combined results as to their abilities to detect of fault location in different elements.

Worden et al. [70] presented another strain energy study by a fault index. An aluminum plate stiffened with stringers is examined. Fault is introduced as saw-cuts in the outside stringer.

Carrasco et al. [71] discussed utilizing changes in modal strain energy for locating and quantifying fault within a space truss model. The authors built a scaled-down model of a space truss structure which contained 12 bays with interior cross bracings. The model was instrumented with force transducers and accelerometers; consequently modal information for each structural element and global modal information could be extracted. To achieve baseline modal parameters, five baseline tests were taken on the structure with no fault. The baseline tests were continued by 18 tests with different fault scenarios. This method is according to the fact that fault in a structural member leads changes in the modal strain energy. The total modal strain energy for each structural element can be calculated by experimental mode shapes extracted from the fault and non-fault states. Changes in the modal strain energy between the fault and non-fault states will be localized around the fault structural elements. The authors also pretended that the magnitude of the changes can be used as an indicator for overall magnitude of the fault. Results of the test showed that this methodology performed very well at localizing fault elements within the truss structure.

Choi and Stubbs [72] developed two methods for detecting fault in 2D plate structures. Changes in local modal strain energy is one of these methods. For each method, the 2D plate is separated into several elements for analysis and classical plate theory is used to develop a fault index for each element within the plate.

A method of fault identification using the invariance property of element modal strain energy is presented by Yang et al. [73]. This method is to assign element modal strain energy in two parts as well as defines two fault identification indicators. One is Compression Modal Strain Energy Change Ratio (CMSECR) and the other Flexural Modal Strain Energy Change Ratio (FMSECR). The present modal strain energy is gained using incomplete mode shape and structural stiffness matrix. SHM is therefore accomplished through monitoring the elemental CMSECR and FMSECR.

A more general approach was showed by Ladeveze and Reynier [74] with the MECE (Minimization of the Error in the Constitutive Equations) concept. In conclusion they also get indicators from strain energy expression pointing out the most erroneous locations of the reference model. These locations indicate the changes because of fault.

Other Methods: A new approach was proposed to locate faults using Shannon's sampling theorem [75]. Shannon's sampling theorem is utilized there for the first time to rebuild the mode shapes, resulting from equidistant sampling points gained in this field, in connection with building structures. The feasibility of rebuilding mode shapes of a structural system using a minimum number of sensors is followed. To evaluate the feasibility of this method, the first three mode shapes of a simply supported beam and a mode shape represented by a parabolic equation are rebuilt.

Choi and Stubbs [72] suggested a method of fault identification using the Lagrange Equilibrium Equation presented in the classical plate theory. In this study, the authors utilized the forth order of deformation derivative in faulted and non-faulted plate structures to identify the faults. After that, Rahaei et al. [76] extended this method to available mode shapes.

A new sensitivity-based approach in connection with modal kinetic energies (MKE) was recommended in [77, 78].

Choi et al. [5] proposed a methodology to fault localization in a plate structure. The methodology introduced an index based on the changes in the distribution of the modal compliance of the plate structure due to fault. The changes in the modal compliance distribution are obtained using the mode shapes of the pre-fault and post-fault states of the structure. The moment-curvature relationships and the invariant expression for the sum of bending moments are used to formulate the fault index. The validity of the proposed method is demonstrated using numerical and experimental data.

Gandomi [79] presents a methodology to detect and locate faults in orthotropic plate structures. Specific fault indices have been introduced based on dynamic mode shapes of the fault and non-fault structures. The governing differential equation on vertical deformation, the vertical shear force equations and the invariant expression for the sum of transverse loading of an
orthotropic plate are employed to obtain the aforementioned fault indices. To validate the approaches, the author developed a FEM of a 70 × 50 × 0.3 cm plate that is reinforced by 10 equidistance ribs in a main direction. In this study, two different fault states which are isotropic and orthotropic fault are examined to evaluate the ability of the methodology of identify faults.

CONCLUSIONS AND SUGGESTIONS

The review of methods of fault identification using mode shapes has shown that the approach is potentially useful for routine integrity assessment of structures. Mode shapes obtained from periodic vibration testing can be used to SHM and also assess structural condition. Two major group of the dynamic fault identification methods are those using mode shape changes and the methods utilizing mode shape derivatives. As it has been expressed, the dynamic methods based on mode shape derivatives are more versatile and sensitive to identify faults in the structures.

Many of the proposed methods require either a theoretical model of fault. These methods are thus limited in application to specific structural geometries and the type of fault model assumed thus we should attempt to extend the more effective and applicable methods.

This study shows that the dynamic fault identification methods needs:

- to be evaluated to use down modes to identify faults.
- to be improved to resist the environmental noises.
- to be more practical and easy to accuse measurments.

REFERENCES


