Using Newton's Interpolation and Aitken's Method for Solving First Order Differential Equation

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Abstract: In recent years, there has been greater attempt to solving differential equations by analytic methods and numerical methods. Most of researchers treated numerical approach to solve first order ordinary differential equations. These methods such as Runge-Kutta method, Taylor series method and Euler’s method, etc. Faith Chelimo Kosgei studied this problem by combined the newton’s interpolation and Lagrange method. This study will combine of Newton’s interpolation and Aitken's method as hybrid technique by using these two types of interpolation to solve first order differential equation.

Key words: Differential equation • Analytic method • Numerical method • Newton’s interpolation method • Aitken's method

INTRODUCTION

In real life situation many problems can be formulated in the form of ordinary differential equation, specially of first order, hence we need to study and solve the differential equations. A numerical method is used to solve numerical problems. The differential equation problem [1-7], consists of at least one differential equation and at least one additional equation such that the system together have one and only one solution called the analytic or exact solution to distinguish it from the approximate numerical solutions that we shall consider. In this paper, to find the solution of differential equation of first order, Faith C. K [1] studied this problem by using combination of newton’s interpolation and Lagrange method. In this study we will combine of Newton’s interpolation and Aitken's method [2-4, 6, 8]. Finally we verified on a number of examples and numerical results obtained show the efficiency of the method given by present study in comparison with the method of Faith C. K [1]. Let’s consider the following first order differential equation or initial value problem.

\[ y' = f(x, y), \quad y(x_0) = y_0 \quad (1) \]

where \( f(x, y) \) is a known function and the values in the initial conditions are also known numbers.

Method [1]: This study combine both Newton’s interpolation method and Lagrange method. It used newton’s interpolation method to find the second two terms then use the three values for \( y \) to form a quadratic equation using Lagrange interpolation method as follows;

Newton’s Interpolation Method [1, 6]:

\[ f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \ldots + a_n(x - x_0)(x - x_1)...(x - x_{n-1}) \quad (2) \]

where

\[ a_0 = y_0, \quad a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}, \quad a_2 = \frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_2) - f(x_0)}{(x_2 - x_0)} \quad (3) \]

e etc

Lagrange Interpolation Method [1, 6]:

\[ y_n = \frac{(x - x_1) - (x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0) - (x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0) - (x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2 \quad (4) \]
Description of the Method: This method will combine both Newton’s interpolation method and Aitken method. It used Newton’s interpolation method to find the second two terms then use the three values for y to form a linear or quadratic equations using Aitken interpolation method as follows;

\[ f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + ... + a_n(x - x_0)(x - x_1)...(x - x_{n-1}) \]  \hspace{1cm} (5)

where

\[ a_0 = y_0, \quad a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}, \quad a_2 = \frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_2) - f(x_0)}{(x_2 - x_0)} \]  \hspace{1cm} (6)

etc

\[ y_1 = a_0 + a_1(x - x_0) \]  \hspace{1cm} (7)

\[ y_2 = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) \]  \hspace{1cm} (8)

Note: We can use Newton’s Forward Interpolation Formula instead of Newton’s divided Interpolation method in (2.1).

Aitken Interpolation Method [6]:

\[ P_{o,k}(x) = \frac{1}{x_k - x_0} \begin{vmatrix} y_0 & x_k - x_0 \\ x_k & y_k \end{vmatrix} \]  \hspace{1cm} (9)

\[ P_{o,1,2}(x) = \frac{1}{x_2 - x_1} \begin{vmatrix} P_{o,1}(x) & x_1 - x \\ P_{o,2}(x) & x_2 - x \end{vmatrix} \]  \hspace{1cm} (10)

\[ y_n = P_{o,1,2,...,n}(x) = \frac{1}{x_n - x_{n-1}} \begin{vmatrix} P_{o,1,...,(n-1)}(x) & x_{n-1} - x \\ P_{o,1,...,(n-2),n}(x) & x_n - x \end{vmatrix} \]  \hspace{1cm} (11)

Examples: In this section, we will check the effectiveness of the present technique (3). First numerical comparison for the following test examples taken in [1].

Example 1:

Solve \( \frac{dy}{dx} = 1 - y, \quad y(0) = 0 \)

By taking the step \( h = 0.01 \)

First by using Newton's interpolation, we have

\[ \begin{array}{c|c|c}
 x & \text{Combined Newton's Interpolation and Aitken} & y(x) \\
 \hline
 0 & 0 & 0 \\
 0.01 & 0.0100 & 0.009950166251 \\
 0.02 & 0.0199 & 0.019801326 \\
 0.03 & 0.0297 & 0.029654466 \\
 0.04 & 0.0394 & 0.03921056 \\
 0.05 & 0.0490 & 0.048770575 \\
 0.06 & 0.0585 & 0.058235466 \\
 0.07 & 0.0679 & 0.06760618 \\
 0.08 & 0.0772 & 0.076883653 \\
 0.09 & 0.0864 & 0.086068814 \\
 0.1 & 0.0955 & 0.095162581 \\
\end{array} \]

Example 2:

By taking the step \( h = 0.01 \)

First by using Newton's interpolation, we have
Now, forming linear and quadratic using Aitken Method

\[ P_{0,1}(x) = 1 - x \]
\[ P_{0,2}(x) = 1 - 0.99495x \]
\[ P_{0,1,2}(x) = 0.505x^2 - 1.00505x + 1 \]

Hence, we can take the approximation solution of linear and quadratic using Aitken Method, if we take quadratic using Aitken Method, we find the same solution given by Faith C. K [1], Table 2.

Example 3:

Solve \( \frac{dy}{dx} = y - x, \quad y(0) = 0.5 \)

By taking the step \( h = 0.01 \)
First by using Newton's interpolation, we have

\[ a_0 = 0.5 = y_0 \]
\[ a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = \frac{dy}{dx}_{x=0.5} = 0.5 \]
\[ y_1 = 0.5 + 0.5(0.01 - 0) = 0.505 \]
\[ a_2 = \frac{f(x_2) - f(x_1)}{(x_2 - x_1)} = \frac{dy}{dx}_{x=0.01,0.5} - \frac{dy}{dx}_{x=0.5} = -0.25 \]
\[ y_2 = 0.5 + 0.5(0.02 - 0) - 0.25(0.02 - 0)(0.02 - 0.01) = 0.50995 \]

Now, forming linear and quadratic using Aitken Method

\[ P_{0,1}(x) = 0.5 - 0.5x \]
\[ P_{0,2}(x) = 0.5 - 0.4975x \]
\[ P_{0,1,2}(x) = -0.25x^2 + 0.5025x + 0.5 \]

Hence, we can take the approximation solution of linear and quadratic using Aitken Method, if we take quadratic using Aitken Method, we find the same solution given by Faith C. K [1], Table 3.

### REFERENCES


