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Comparison of New Aitken Type Method of N. IDE with Several Methods Solving Nonlinear Algebraic Equations

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Abstract: Finding the roots of nonlinear algebraic equations is an important problem in science and engineering. Many mathematical models in physics, engineering and applied science, are applied with nonlinear equations. later many methods developed for solving nonlinear equations. The efficient methods to find the roots of nonlinear equations has been developed by a large number of researchers. In this paper we present the comparison of one important numerical methods given by N. Ide, Starting by King's method, which proposed a modified families of fourth- and eighth-order of convergence iterative methods for nonlinear equations. Finally we verified on a number of examples and numerical results obtained the efficiency of the methodgiven by N. Ide.

Key words: Nonlinear equations · Newton's Method · King's method

INTRODUCTION

Find a solution of the equation f(x)=0, where f(x) is nonlinear function is an important problem in mathematics [1-58]. The well-known method is a Newton's iterative method defined by (1):

$$X_{n+1} = X_n - \frac{f(x_n)}{f(x_n)} \quad (1)$$

New Aitken type Method of N. IDE: This method given in [18] by the scheme,

$$y_0 = x_0 \quad (2)$$

$$X_1 = X_0 - \frac{f(x_0)}{f\left(\sqrt{x_0 \cdot y_0}\right)} = x_0 - \frac{f(x_0)}{f(x_0)} \quad (3)$$

followed by (for n=1)

$$y_n^* = X_n - \frac{f(x_n)}{f'(\sqrt{x_{n-1} \cdot y_{n-1}})} \quad (4)$$

$$y_n = X_n - \frac{f(x_n)}{f\left(\sqrt{x_{n-1} \cdot y_{n-1}^*}\right)} \quad (5)$$

$$z_n^* = y_n - \frac{f(y_n)}{f'\left(\sqrt{y_{n-1} \cdot x_{n-1}}\right)} \quad (6)$$

$$Z_n = y_n - \frac{f(y_n)}{f'\left(\sqrt{y_n \cdot z_n^*}\right)} \quad (7)$$

$$X_{n+1} = Z_n - \frac{f(z_n)}{[y_n, z_n ' f]} \quad (8)$$

where $[y_n, z_n; f]$ denotes the first order divided difference of f on x and y.

Algorithm of the New Aitken Type Method of N. IDE:

- Give x_0 initial value (number real), give the tolerance number □ (for stopping) and take $y_0 = x_0$.
- Calculus of $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
- Calculus (for n ≥ 1):

$$y_n^* = x_n - \frac{f(x_n)}{f'\left(\sqrt{x_{n-1} \cdot y_{n-1}}\right)}$$

$$y_n = x_n - \frac{f(x_n)}{f'\left(\sqrt{x_n \cdot y_{n-1}^*}\right)}$$

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$$z_n^* = y_n - \frac{f(y_n)}{f'\left(\sqrt{y_{n-1}.x_{n-1}}\right)}$$

$$Z_n = y_n - \frac{f(y_n)}{f'\left(\sqrt{y_n.z_n^*}\right)}$$

$$X_{n+1} = Z_n - \frac{f(z_n)}{[y_n, z_n; f]}$$

- Calculus of stopping condition: if $\left|\frac{x_{n+1} - x_n}{x_{n+1}}\right| \le then\ stop,\ else,$
- Take n=n+1 and return to (3).

Several Methods for Comparison: We start by King's method of order four denoted K4 [1], [42] given by:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \frac{f(x_n)}{f'(x_n)} \quad (9)$$

where, $f'(y_n) = f'(x).\frac{f(x_n) + (\beta - 2) f(y_n)}{f(x_n) + \beta f(y_n)}$

The Modified King's Method MK4: We call this family [1], modified King's method MK4. The order of convergence of this family is four, Kung and Traub[1], [42] give the schema of this method by (10),

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \frac{f(x_n)}{f'(x_n)}.\frac{f(x_n) + \beta f(y_n)}{f(x_n) + (\beta - 2) f(y_n)} \quad (10)$$

The MK4 method given by (10) is four-order of convergence.

Modified King's Method: Modified King's method [1] denoted by MK8a and MK8b, given by the forms (11) and (12).

$$y_n = x_n - \frac{f(x_n)}{f[w_n, x_n]},$$

$$z_n = y_n - \frac{f(y_n)}{g(x_n)}.\frac{f(x_n) + \beta f(y_n)}{f(x_n) + (\beta - 2) f(y_n)} \quad (11)$$

$$x_{n+1} = x_n - \frac{f(x_n) + (m_1 + m_2 + m_3)}{m_1 f[w_n, x_n] + m_2 f[y_n, x_n] + m_3 f[z_n, x_n]}$$

where,

$$m_1 = f(y_n) f(z_n)(z_n - y_n)$$
$$m_2 = f(w_n) f(z_n)(w_n - z_n)$$
$$m_3 = f(w_n) f(y_n)(y_n - w_n)$$

$$y_n = x_n - \frac{f(x_n)}{f[w_n, x_n]},$$

$$z_n = y_n - \frac{f(y_n)}{g(x_n)}.\frac{f(x_n) + \beta f(y_n)}{f(x_n) + (\beta - 2) f(y_n)} \quad (12)$$

$$x_{n+1} = x_n - \frac{f(z)}{c_2 - c_1 c_4}$$

where,

$$c_1 = f(z_n)$$
$$c_2 = f[y_n, z_n] - c_3(y_n - z_n) + c_4 f(y_n)$$
$$c_3 = f[y_n, z_n, w_n] + c_4 f[y_n, w_n]$$

$$C_4 = \frac{f[y_n, z_n, x_n] - f[y_n, z_n, w_n]}{f[y_n, w_n] - f[y_n - x_n]}$$

Numerical Examples: To show the efficiency of the comparison, we compare the New Aitken type Method of N. IDE denoted by NANI [18] with the methods given by schemes K4, MK4, MK8a and MK8b, five examples will be tested. We compare with K4 presented by King [40] and with the derivative-free fourth- and eighthordermethods presented by Yasmin *et al*. [57], Cordero *et al*. [36]; and Zafar *et al*. [58] denoted respectively by YZA4, YZA8, CHMT4, CHMT8 and ZYAJ4, ZYAJ8.

Examples:

Example 1: For the function $f_1 = \cos(x) - x;\ x_0 = 0$

Table 1 give the Comparisons between different methods with the method of New Aitken type Method of N. IDE.

Example 2: For the function $f_2 = sin^2(x) - x^2 + 1;\ x_0 = 1$

Table 2 give the Comparisons between different methods with the method of New Aitken type Method of N. IDE.

Table 1: Comparison of the methos for the function $f_1 = \cos(x) - x$; $x_0 = 0$

Method	Number of Iteration n	x_n	$\|f(x_n)\|$
MK4	4	0.73908513321516064	1.75E-209
K4	4	0.73908513321516064	9.03E-71
YZA4	4	0.73908513321516064	1.60E-128
CHMT4	4	0.73908513321516064	1.49E-207
ZYAJ4	4	0.73908513321516064	0.02E-224
MK8a	3	0.73908513321516064	4.94E-441
MK8b	3	0.73908513321516064	5.03E-466
YZA8	3	0.73908513321516064	9.51E-223
CHMT8	3	0.73908513321516064	5.73E-465
ZYAJ8	3	0.73908513321516064	5.41E-455
NANI	5	0.73908513321516064	1.70E-208

Table 2: Comparison of the methos for the function $f_2 = sin^2(x) - x^2 + 1$; $x_0 = 1$

Method	Number of Iteration n	x_n	$\|f(x_n)\|$
MK4	4	1.4044916482153412	2.69E-176
K4	5	1.4044916482153412	2.19E-68
YZA4	4	1.4044916482153412	1.90E-90
CHMT4	4	1.4044916482153412	4.63E-107
ZYAJ4	4	1.4044916482153412	8.16E-139
MK8a	3	1.4044916482153412	1.44E-333
MK8b	3	1.4044916482153412	2.42E-359
YZA8	3	1.4044916482153412	4.00E-240
CHMT8	3	1.4044916482153412	4.26E-235
ZYAJ8	3	1.4044916482153412	8.38E-307
NANI	5	1.4044916482153412	2.16E-163

Table 3: Comparison of the methos for the function $\ln(x^2 - x + 1) - 4\sin(x - 1)$; $x_0 = 1.5$

Method	Number of Iteration n	x_n	$\|f(x_n)\|$
MK4	3	1	4.80E-62
K4	4	3.5302670187568383	5.73E-162
YZA4	4	1	2.30E-147
CHMT4	4	1	4.75E-197
ZYAJ4	4	1	3.22E-213
MK8a	3	1	3.75E-430
MK8b	3	1	3.14E-452
YZA8	3	1	1.75E-419
CHMT8	3	1	2.04E-388
ZYAJ8	3	1	2.39E-394
NANI	5	1	1.20E-179

Example 3: For the function $f_3 = \ln(x^2 - x + 1) - 4\sin(x - 1)$; $x_0 = 1.5$

Table 3 give the Comparisons between different methods with the method of New Aitken type Method of N. IDE.

Example 4: For the function $f_4 = e^{-x2} + \cos(x) - x^2$; $x_0 = 1$

Table 4 give the Comparisons between different methods with the method of New Aitken type Method of N. IDE.

Table 4: Comparison of the methos for the function $f_4 = e^{-x2} + \cos(x) - x^2$; $x_0 = 1$

Method	Number of Iteration n	x_n	$\|f(x_n)\|$
MK4	3	0.97416230520054071	8.46E-128
K4	3	0.97416230520054071	1.34E-121
YZA4	3	0.97416230520054071	4.16E-102
CHMT4	3	0.97416230520054071	9.65E-109
ZYAJ4	3	0.97416230520054071	1.11E-111
MK8a	3	0.97416230520054071	1.93E-941
MK8b	2	0.97416230520054071	2.58E-126
YZA8	3	0.97416230520054071	2.18E-851
CHMT8	3	0.97416230520054071	1.06E-892
ZYAJ8	3	0.97416230520054071	2.54E-886
NANI	4	0.97416230520054070	15.98E-207

Table 5: Comparison of the methos for the function $f_5 = \text{aectan}(x) - x^2 + 1$; $x_0 = 1$

Method	Number of Iteration n	x_n	$\|f(x_n)\|$
MK4	3	1.3961536566409308	2.18E-90
K4	3	1.3961536566409308	2.16E-71
YZA4	4	1.3961536566409308	8.75E-232
CHMT4	3	1.3961536566409308	2.25E-68
ZYAJ4	3	1.3961536566409308	2.48E-74
MK8a	3	1.3961536566409308	3.52E-654
MK8b	3	1.3961536566409308	1:65E_707
YZA8	3	1.3961536566409308	3.19E-509
CHMT8	3	1.3961536566409308	1.89E-560
ZYAJ8	3	1.3961536566409308	5.71E-598
NANI	4	1.3961536566409307	51.62E-204

Example 5: For the function $f_5 = \text{aectan}(x) - x^2 + 1$; $x_0 = 1$

Table 5 give the Comparisons between different methods with the method of New Aitken type Method of N. IDE.

CONCLUSION

In this work we compared the method of New Aitken type Method of N. IDE. By the proposed new optimal three derivative-freeroot finding schemes for nonlinear equations [1], these methods given by schemes K4, MK4, MK8a and MK8b, five examples tested. We compared with K4 presented by King [40] and with the derivative-free fourth- and eighth ordermethods presented by Yasmin *et al*. [57], Cordero *et al*. [36]; and Zafar *et al*. [58] denoted respectively as: YZA4, YZA8, CHMT4, CHMT8 and ZYAJ4, ZYAJ8.. We show the efficiency of this method.

REFERENCES

1. Obadah Said Solaiman *et al*., 2019. Optimal fourth- and eight- Order of convergence derivative- free modifications of King's method, journal of King Saud University, Science, 31: 1499-1504, contents lists available at Science Direct.

2. Nasr Al-Din Ide, 2012. A nonstationary Halley's iteration method by using divided differences formula, Applied Mathematics, Scientific research, USA, 3: 169-171.
3. Nasr Al-Din Ide, 2008. A new Hybrid iteration method for solving algebraic equations, Journal of Applied Mathematics and Computation, Elsevier Editorial, 195: 772-774.
4. Nasr Al-Din Ide, 2008. On modified Newton methods for solving a nonlinear algebraic equations, Journal of Applied Mathematics and Computation, Elsevier Editorial, 198: 138-142.
5. Nasr Al-Din Ide, 2013. Some New Type Iterative Methods for Solving Nonlinear Algebraic Equation", World Applied Sciences Journal, 26(10): 1330-1334, © IDOSI Publications, Doi: 10.5829/idosi.wasj. 2013.26.10.512.(2013).
6. Nasr Al-Din Ide, 2016. A New Algorithm for Solving Nonlinear Equations by Using Least Square Method, Mathematics and Computer Science, SciencePG Publishing, 1(3): 44-47, Published: Sep. 18, (2016).
7. Nasr Al-Din Ide, 2016. Using Lagrange Interpolation for Solving Nonlinear Algebraic Equations, International Journal of Theoretical and Applied Mathematics, Science PG publishing, 2(2): 165-169.
8. Nasr Al-Din Ide, 2008. A new Hybrid iteration method for solving algebraic equations, Applied Mathematics and Computation, 195(2): 772-774.
9. Nasr Al-Din Ide, 2008. On modified Newton methods for solving a non linear algebraic equations, Applied Mathematics and Computation, 198(1): 138-142.
10. Nasr Al-Din Ide, 2013. "Some New Type Iterative Methods for Solving Nonlinear Algebraic Equation", World Applied Sciences Journal, 26(10): 1330-1334, 2013© IDOSI Publications, 2013 DOI: 10.5829/idosi.wasj.2013.26.10.512.
11. Nasr Al-Din Ide, 2015. Application of Iterative Method To Nonlinear Equations Using Homotopy Perturbation Methods, Journal of Basic and Applied Research International Knowledge Press, 5(3).
12. Nasr Al-Din Ide, 2015. Some New Iterative Algorithms by Using Homotopy Perturbation Method for Solving Nonlinear Algebraic Equations, 2015, Asian Journal of Mathematics and Computer Research, (AJOMCOR), International Knowledge Press, 5(3).
13. Nasr Al-Din Ide, 2016. A New Algorithm for Solving Nonlinear Equations by Using Least Square Method, Mathematics and Computer Science, SciencePG Publishing, 1(3): 44-47, Published: Sep. 18, 2016.
14. Nasr Al-Din Ide, 2016. Using Lagrange Interpolation for Solving Nonlinear Algebraic Equations, International Journal of Theoretical and Applied Mathematics. Science PG Publishing, 2(2):165-169, Received: Nov. 14, 2016; Accepted: Dec. 12, 2016; Published: Jan. 22, 2017. pp: 165-169. doi: 10.11648/j.ijtam.20160202.31.
15. Nasr Al-Din Ide, 2018. Improvement of New Eight and Sixteenth Order Iterative Methods for Solving Nonlinear Algebraic Equations by Using Least Square Method , International Journal of Scientific and Innovative Mathematical Research, (IJSIMR), 6(10): 23-27.
16. Nasr Al-Din Ide, 2018. Bisection Method by using Fuzzy Concept, International Journal of Scientific and Innovative Mathematical Research, (IJSIMR), 7(4): 8-11. ISSN No. (Print) 2347-307X & ISSN No. (Online) 2347-3142, DOI: http://dx.doi.org/10.20431/2347-3142.0704002.
17. Nasr Al-Din Ide, 2019. SundusNaji Al Aziz, Using the Least Squares Method with Five Points to Solve Algebraic Equations –Nonlinear-, International Journal of Scientific and Innovative Mathematical Research, (IJSIMR), 7(5): 26-30. ISSN No. (Print) 2347-307X & ISSN No. (Online) 2347-3142 DOI: http://dx.doi.org/10.20431/2347-3142.0705005.
18. Nasr Al-Din Ide, 2018. A New Aitken Type Method by Using Geometric Mean Concept, World Applied Sciences Journal, 37(4): 289-292, ISSN 1818-4952© IDOSI Publications, DOI: 10.5829/idosi. wasj.2019.289.292.
19. Nasr Al-Din Ide, 2019. A New Modified of McDougall-Wotherspoon method for Solving Nonlinear Equations by Using Geometric Mean Concept, Computational and Applied Mathematical Sciences, 4(2): 35-38 © IDOSI Publications. Doi: 10.5829/idosi.cams.2019.
20. Nasr Al-Din Ide, 2019. New Modification Methods for Solving Nonlinear Algebraic Equations by Using Lagrange interpolation approach, Studied in Nonlinear Sciences, 4(2): 23-25, © IDOSI Publications. Doi:10.5829/idosi.sns.2019.
21. Nasr Al-Din Ide, 2019. Iterative Method by Using Tchybcheve integral for Solving Nonlinear Algebraic Equations, World Applied Sciences Journal, 37(8): 661-663, ISSN 1818-4952©IDOSI Publications, DOI: 10.5829/idosi.wasj.2019.661.663.
22. Argyros, I.K. and A.A. Magreñán, 2015. On the convergence of an optimal fourth-order family of methods and its dynamics. Appl. Math. Comput., 252: 336-346.

23. Behl, R., I.K. Argyros and S.S. Motsa, 2016. A new highly efficient and optimal family of eighth-order methods for solving nonlinear equations. Appl. Math. Comput., 282: 175-186.
24. Behl, R., A. Cordero, S.S. Motsa and J.R. Torregrosa, 2015. Construction of fourth-order optimal families of iterative methods and their dynamics. Appl. Math. Comput., 271: 89-101.
25. Behl, R., D. González, P. Maroju and S.S. Motsa, 2018. An optimal and efficient general eighth-order derivative free scheme for simple roots. J. Comput. Appl. Math., 330: 666-675.
26. Behl, R., P. Maroju and S.S. Motsa, 2017. A family of second derivative free fourth order continuation method for solving nonlinear equations. J. Comput. Appl. Math., 318: 38-46.
27. Behl, R., S.S. Motsa, M. Kansal and V. Kanwar, 2015. Fourth-order derivative-free optimal families of King's and Ostrowski's methods. In: Agrawal, P., Mohapatra.
28. R., Singh, U., Srivastava, H. (Eds.), Mathematical Analysis and Its Applications. New Delhi: Springer Proceedings in Mathematics & Statistics, 143. Springer, pp: 359-371.
29. Chun, C., 2007. Some variants of King's fourth-order family of methods for nonlinear equations. Appl. Math. Comput., 190: 57-62.
30. Chun, C., 2008. Some fourth-order iterative methods for solving nonlinear equations. Appl. Math. Comput., 195: 454-459.
31. Chun, C., M.Y. Lee, B. Neta, J. Dzˇunic´, 2012. On optimal fourth-order iterative methods free from second derivative and their dynamics. Appl. Math. Comput., 218: 6427-6438.
32. Chun, C. and B. Neta, 2016. Comparison of several families of optimal eighth order methods. Appl. Math. Comput., 274: 762-773.
33. Chun, C. and B. Neta, 2017. Comparative study of eighth-order methods for finding simple roots of nonlinear equations. Numer. Algorithms, 74(4): 1169-1201.
34. Cordero, A., J.L. Hueso, E. Martínez and J.R. Torregrosa, 2010. New modifications of Potra-Pták's method with optimal fourth and eighth orders of convergence. J. Comput. Appl. Math., 234: 2969-2976.
36. Cordero, A., J.L. Hueso, E. Martínez and J.R. Torregrosa, 2013. A new technique to obtain derivative-free optimal iterative methods for solving nonlinearequations. J. Comput. Appl. Math., 252: 95-102.
37. Cordero, A., T. Lotfi, K. Mahdiani and J.R. Torregrosa, 2015. A stable family with high order of convergence for solving nonlinear equations. Appl. Math. Comput., 254: 240-251.
38. Cordero, A., J.G. Maimó, J.R. Torregrosa and M.P. Vassileva, 2016. Stability of a fourth order bi-parametric family of iterative methods. J. Comput. Appl. Math., 312: 94-102.
39. Geum, Y.H., Y.I. Kim and B. Neta, 2018. Constructing a family of optimal eighth-order modified Newton-type multiple-zero finders along with the dynamics behindtheir purely imaginary extraneous fixed points. J. Comput. Appl. Math., 333: 131-156.
40. King, R.F., 1973. A family of fourth order methods for nonlinear equations. SIAM J. Numer. Anal., 10: 876-879.
41. Kogan, T., L. Sapir, A. Sapir and A. Sapir, 2017. To the question of efficiency of iterative methods. Appl. Math. Lett., 66: 40-46.
42. Kung, H.T. and J.F. Traub, Optimal order of one-point and multipoint iteration. J. Assoc. Comput. Mach., 21: 643-651.
43. Lee, M.Y. and Y.I. Kim, 2012. A family of fast derivative-free fourth-order multipoint optimal methods for nonlinear equations. Int. J. Comput. Math., 89(15): 2081-2093.
44. Neta, B. and C. Chun, 2014. Basins of attraction for several optimal fourth order methods for multiple roots. Math. Comput. Simulat., 103: 39-59.
45. Pandey, B. and J.P. Jaiswal, 2017. New seventh and eighth order derivative free methods for solving nonlinear equations. Tbilisi Math. J., 10(4): 103-115.
46. Said Solaiman, O. and I. Hashim, 2018. Two new efficient sixth order iterative methods for solving nonlinear equations. J. King Saud Univ. Sci., 31: 701-705. https://doi.org/10.1016/j.jksus.2018.03.021.
47. Sharifi, S., S. Siegmund and M. Salimi, 2014. Solving nonlinear equations by a derivative-free form of the King's Family with Memory, arXiv 1410.5867v1.
48. Sharma, J.R. and H. Arora 2016. Some novel optimal eighth order derivative-free root solvers and their basins of attraction. Appl. Math. Comput., 284: 149-161.
49. Sharma, J.R. and A. Bahl, 2015. An optimal fourth order iterative method for solving nonlinear equations and its dynamics. J. Comp. Anal., 2015. Article ID 259167.
50. Sharma, J.R. and R.K. Goyal 2007. Fourth-order derivative-free methods for solving non-linear equations. Int. J. Comput. Math., 83(1): 101-106.

51. Singh, A. and J.P. Jaiswal, 2016. A class of optimal eighth-order Steffensen-type iterative methods for solving nonlinear equations and their basins of attraction. Appl. Math. Inf. Sci., 10: 251-257.
52. Soleymani, F., S.K. Khattri and S.K. Vanani, 2012. Two new classes of optimal Jarratttype fourth order methods. Appl. Math. Lett., 25(5): 847-853.
53. Steffensen, J.F., 1933. Remarks on iteration. Scand. Actuar. J., 1: 64-72.
54. Traub, J.F., 1964. Iterative Methods for Solution of Equations. Prentice-Hall, Englewood, Cliffs, NJ.
55. Waseem, M., M.A. Noor, A.S. Farooq and K.I. Noor, 2018. An efficient technique to solve nonlinear equations using multiplicative calculus. Turk. J. Math., 42: 679-691.
56. Weerakoon, S. and T.G.I. Fernando, 2000. A variant of Newton's method with accelerated third-order convergence. Appl. Math. Lett., 13: 87-93.
57. Yasmin, N., F. Zafar and S. Akram, 2016. Optimal derivative-free root nding methods based on the Hermite interpolation. J. Nonlinear. Sci. Appl., 9: 4427-4435.
58. Zafar, F., N. Yasmin, S. Akram and M. Junjua, 2015. A general class of derivative free optimal root finding methods based on rational interpolation. The Sci. World J., 2015. Article ID 934260.