# Service Facilities Optimality Index in Government Secondary Health Institutions and Its Socio-Economic Implications in Akwa Ibom State, Nigeria 

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#### Abstract

This study examines the service facilities optimality index in government secondary health institutions and its socio-economic implications in Akwa Ibom State, Nigeria. The study was undertaken to address issues of long queues at service stations in the general hospitals in Akwa Ibom State. The data were collected from randomly selected hospitals spread across Akwa Ibom state and were analyzed using descriptive statistics. From this study, it was discovered that long queues actually occur in general hospitals in Akwa Ibom State. The study recommends improvement in the facilities outlay (Availability) in the general hospitals in the state if these hospitals are to serve their intended purposes, where patients have confidence in the system and visit same for treatment which would improve societal health and subsequent improvement in social and economic well-being of the state. The study concluded that the persistent problem of long queues in general hospitals in Akwa Ibom State can be traced to inadequate facilities provision in terms of conducing rooms, medical laboratory equipment and personnel, few numbers of doctors and nurses in relation to number of patients to attend to, as well as availability of pharmacy facilities.


Key words: Service • Facilities • Optimality • Socio-Economic • Akwa Ibom

## INTRODUCTION

Government institutions are set up primarily to provide essential services to the citizens. Among such institutions are the health institutions. Technically, the health institutions are categorized into three broad types, based on the scope and dimension of operations. We have the primary, secondary and tertiary health institutions. While the Primary Health Care (PHC) provides the first line non-intensive health care to patients, the secondary health institutions provides the first line non-intensive health care to patients, the secondary health institutions provide a more elaborate health care to patients. The first primary include Health centers and maternal and child health (MCH). In the secondary health care category are the General Hospitals while the tertiary category are the Teaching Hospitals, as well as the specialist hospitals.

Secondary health institutions have been seen as a pivot of a some-what comprehensive health care package for the people. Hence, the government deliberately
appropriates the ratios of its distribution throughout the state (Akwa Ibom) using some form of political zoning formula. Akwa Ibom State is structured into three geo-political zones called Senatorial district. These are: Akwa Ibom North West (Ikot Ekpene), Akwa Ibom South (Eket) and Akwa Ibom North East (Uyo), Senatorial Districts. Each of these senatorial districts has fifteen (15) general hospitals, under the supervision of Hospital Management Board of Akwa Ibom State. There are altogether 45 general hospitals in the State.

For these general hospitals to serve their functions, the facilities must be optimal in relation to patients in flow ration. This is what [1] identify as logistic problem. According to them, facilities location models involving the location and selection of distribution centers, warehouses, shelters, medical centres and other locations are an important approach in Disaster Management (DM). They have gone ahead to state that recent researches have emergency humanitarian logistics optimization models which are important elements of disaster facility location problems. Then, to overcome this challenges, two
approaches can be used to solve this problem; (i) a heuristic algorithm and (ii) an exact algorithm...most research studies have usually addressed these problems using a heuristic algorithm because it requires less time to employ and can solve complicated problems, but the results of this approach are of poor quality compared to the exact algorithm.

Although the first approach can overcome the second approach, the second approach is necessary because it can be used to check the heuristic algorithm and moreover, in some real cases, an exact algorithm can also be used to solve the problem. Hence, the use of exact algorithm is important and unavoidable.

Chandan [2] found that facility location problems could be defined across the two elements of space and time, in which space was "A planning area where facilities are located" and time was "The time the location is identified" (Developing a new facility or revising and existing facility). Ezirim and Nwokah [3] submits that effective delivery of healthcare services requires availability of adequate infrastructure, diagnostic medical equipment, drugs and well-trained medical personnel. He asserts that poor funding and mismanagement affect health care service delivery in terms of overage and quality of services rendered. Similarly, Hiller and Lieberman [4] identified the facilities to include equipment as well as staff strength and the ratio of these to patients’ response to health care facilities.

Purpose of the Study: This study was undertaken to address issues of long queues at service stations in the general hospitals in Akwa Ibom State. Persistent long queues (Long waiting time) has the tendency of making a lot of people not to visit these health institutions, there by hindering the general well-being of the society, since "A healthy people makes a healthy society".

The study aimed at addressing issues that would promise timely service delivery. When this would be the culture, more people will access the facilities and the society would be better for it. Therefore, the study would look at the inter-arrival times, mean service time and mean wanting time of patients in the selected hospitals.

Research Hypotheses: The following hypotheses were formulated in order to achieve the purpose of the study:

- The distribution of inter-arrival times does not follow exponential distribution
- The distribution of service time does not follow exponential distribution; and
- The distribution of waiting time does not follow exponential distribution.

Theoretical Underpinning: In service delivery where a service is at a point, the ones to receive service would come in at will and leave when he has been served. The philosophy is on how to render services such that the one serviced would be satisfied and the ones who offer the services would not boundedly stretched. Therefore queuing theory would suggest best practical way to go about this. According to Kurt Lewin, there is nothing so practical as a good theory.

Queuing theory was developed by the French Mathematician, S.D Poisson (1781-1840). The most important application of queuing theory occurred during the late 1800s, when Telephone Company had the problem of knowing the number of operators to place on duty at a given time. Each customers required operator's services. At this point, supervisors were faced with the problem of how many operators were to be on duty to check how many operators would remain idle for minutes at a time. If too few operators then they would be overwhelmed by supply; and perhaps never catching up until additional help was added, 9www. reference@business.com/enclyclopedia [5] submits that A. K. Erlang in 1913, who was a telephone engineer, was responsible for the early theoretical development in the area of queuing.

Okon [6] define a queue as a waiting time of "Customers" (Units) requiring services from one or more severs "(Service facility). A queue forms whenever existing capacity of the service facility, that is, whenever arriving customers cannot receive immediate service due to busy servers. Deriving a model, which is an abstraction of the real situation, Urua [7] assume most queuing models to follow that; customers requiring service are generated over time by an input source, where they enter a queuing system and join a queue. At this point, a member in the queue is selected for service by some rule known as the queue discipline. Usoro [8] posit that a queuing system consists of three parts, viz; (i) the calling population, (ii) the queuing (iii) the service facility. Service is performed on the customer by the service mechanism, which the customer leaves the queuing system.

## Queuing System



Fig. 1: The basic queuing processes
Source: Hillier and Lieberman [4]
Queuing System Elements: The queuing system contains some basic elements [4-8] have identified the elements to include;

- Population - This is the total number of customers that might require service from time to time. This population from which arrivals come is referred to as the calling population. It is either finite or infinite. An infinite population can be seen in banking, petrol stations, among other which the finite population might be observed at work done by computers and in hospital.
- Arrivals - Arrivals are called customers or jobs. The arrival pattern is in terms of; (2) arrival rate distribution which is the number of arrivals per unit time. This is assumed to follows a Poisson distribution, i.e the number of customers generated until any specific time has a Poisson distribution. This is a situation where arrivals to the queuing system occur randomly, but at a certain fixed mean rate, regardless of how many customers already are here (So the size of the input source is infinite); (ii) Inter-arrival time distribution- this is the distribution of unit times between successive arrivals. By assumption, it is the probability distribution of the time between consecutive arrivals which is an exponential distribution. Arrival pattern may be constant or random. For random arrival pattern, arrival rate distribution may be approximated by the Poisson distribution given by;
$P(N t=n)=\frac{e^{-\lambda} \lambda n ;}{n!} n=0,1,2 \ldots$
where:
( $\mathrm{P}(\mathrm{Nt}=\mathrm{n})=$ Probability of arrivals
$\mathrm{N} \quad=$ numbers of arrivals per units length of time
$\lambda \quad=$ Average arrivals rate
e $\quad=2.7183$ (a constant)
Inter-arrival times may be converted into arrival rat and the Poison distribution applied;

Arrival rate $=$ inter- arrival time customers may behave thus; (a) they may join the queue and wait for sometimes and they are served; (b) they may join the queue, wait for some time and then become impatient and leave the queue - reneging customers and (c) customers may refuse to join the queue because it is too long; this is referred to as balking the assumption is that customers neither renege nor balk.

Queuing as Poisson process [4] observes that purely random arrivals in queuing mean that the arrival is a Poisson process. A purely random service process means that service time have the exponential distribution. If arrivals follow a Poisson process, it means inter-arrival times have the exponential distribution.

If T is a random variable representing the time between successive arrivals, then; $\mathrm{P}(\mathrm{T}>\mathrm{t})=\exp (-\lambda \mathrm{t})$.

Moreso, the number of arrivals in any time " $t$ " say $\mathrm{A}(\mathrm{t})$, has the Poisson distribution with parameter $\lambda \mathrm{t}^{2}$. That is,
$P(A(t)=n)=\exp (-\lambda)(\lambda t)^{n} / n$
$P\left(A_{t}=n\right)=\frac{e^{-\lambda t}(\lambda t)^{n}}{n!}$
The random arrivals (At) with a discrete random variable N with probability $\lambda=$ mean rate of arrivals and t (The time intervals length), is called a Poisson process when $\lambda$ (Arrival rate) does not change over time. The Poisson process is concerned of some vents in fixed length of time, Usoro [8]. Therefore, $A(t)$ is the number of arrivals given time " t ".

Queuing as Exponential process [4] further says that we picked a point in time at random and found the interval that included that point, rather than picking an interval at random. Suppose you arrive at cab stand according to Poisson process, you arrive at the stand at some random time and wait for the next cab. Your waiting time is exponential, with exactly the same distribution as the time between two successive arrivals (Inter-arrivals) of cab.

Then, suppose a random variable T representing inter-arrival times or service times, T is said to have exponential distribution with parameter $\lambda$ if its probability density function is:
$F y(t)= \begin{cases}0 & \text { for } \mathrm{t}>0 \\ \lambda r^{-\lambda t} & \text { for } \mathrm{t}>0\end{cases}$
Note that there are cases where the service time fails to follow exponential distribution and then other distribution can be used like Erlang distribution.

Relationship Between Poisson Distribution and Exponential Distribution: Inter-arrival times follow exponential distribution in most queuing process, Usoro [8]. The probability distribution of the number of arrivals occurring in any time interval of length " t ", if inter-arrival time are exponential if given by these rules; (i) inter-arrival times are exponential with parameter $\lambda$, if and only if the number of arrivals to occur is an interval of length " $t$ " follows a Poisson distribution with parameter " $\lambda \mathrm{t}$ ". A discrete random variable distribution with parameter $\lambda$ if for $\mathrm{n}=0,1,2 \ldots$.
$P(N=n)=\frac{e^{-\lambda t}(\lambda t) n}{n!}$

Should N be a Poisson random variable, it follows that
$\mathrm{E}(\mathrm{N})=\mathrm{V}(\mathrm{N})=\lambda$

If also, Nt is the number of arrivals occurring during any time interval of length " $t$ ", then it implies that;
$P(N=n)=\frac{e^{-\lambda t}(\lambda t) n}{n!} \quad n=0,1,2 \ldots$
Since, Nt is Poisson with parameter $(\lambda \mathrm{t})$, then it follows that;
$\mathrm{E}(\mathrm{N})=\mathrm{V}(\mathrm{N})=\lambda \mathrm{t}$
We hold these two assumptions for inter-arrival times to e exponential, 9i) Arrivals are defined on non-overlapping time intervals and are independent; (ii) Any small t (any value of " t "), the probability of one arrival occurring between times $t$ and $t+t$ is $\lambda t+0(t)$, where;
$\mathrm{O}(\mathrm{t})$ refers to the quantity satisfying; $\mathrm{Lim} \mathrm{O}(\mathrm{t})=0$
$\mathrm{t} \Delta 1 \mathrm{t} \rightarrow \Delta \Delta \Delta \Delta \Delta$

## MATERIALS AND METHODS

Service facility Optimality Index (SFOI) data for General Hospitals in Akwa Ibom State was used. The data were collected from randomly selected hospitals spread across the state- Akwa Ibom. The hospitals have been circulated in the same ratio 15 each across the three geo-political zones.

The population of the study comprised of: (i) 45 general hospitals of Akwa Ibom State and, (2) all the patients that visit the hospitals in a week for medical treatment, weekly average of patients turn-out as at January, June and December, 2018. The summary is as shown below;

## Addendum

| 1. | St Luke Hospital, Anua | $=$ | 107 |
| :--- | :--- | :--- | :--- |
| 2. St Theresa Hospital, Use Abat | $=$ | 69 |  |
| 3. | Immanuel General Hospital, Eket | $=88$ |  |
| 4. General Hospital, Iquinta Oron | $=$ | 59 |  |
| 5. General Hospital, Ikot Ekpene | $=104$ |  |  |
| 6. St Mary Hospital, Urua Akpan | $=72$ |  |  |

Total weekly population of the selected hospital $=499$

Therefore, we base our study on the average weekly population of patients seeking medical attention at 499 .

Data were analyzed using descriptive statistics. Average of data obtained in January, June and December 2018 were used.

## Data Analysis

## Hypothesis One:

$\mathbf{H o}_{1}$ : The distribution of inter-arrivals times does not follow exponential distribution.

| Table 1: Mean Inter-Arrival Time |  |  |  |  |
| :--- | :--- | :---: | :---: | :--- |
| S/N | Time (Minutes) | Average Frequency (f) | Mid point | ft |
| 1 | $01-180$ | 138 | 90.5 | 12,489 |
| 2 | $181-360$ | 142 | 270.5 | 38,411 |
| 3 | $361-540$ | 123 | 450.5 | $55,411.5$ |
| 4 | $541-720$ | 96 | 630.5 | 60,528 |
|  | Total | 499 |  | $166,839.5$ |

Source: Field survey 2018

The mean inter-arrival time (I.A.T) $=\mathrm{E}(\mathrm{t})$ is given by:
$E(t)=\frac{\sum f t}{\sum f}$
$\therefore \mathrm{E}(\mathrm{t})=\frac{60,839.5}{499}=334.35=334 \mathrm{~min} /$ patient
$\therefore \lambda=\frac{1}{E(t)}=\frac{1}{334}$
$=0.00299$ patient $/ \mathrm{m}$

Mean inter-arrival rate is 0.0029 patient/minutes
To test if an inter-arrival time follow exponential process, we apply goodness of fit test given by;
$X^{2}=\frac{\sum(f o i j-f e i j)^{2}}{f e i j}$
where $\mathrm{K}-1$ is the d.f and k is the number of independent variables.

The exponential density function (E.d.f), is expressed thus;

$$
\begin{aligned}
\mathrm{Fe}, & =499 \times 0.00299 \mathrm{e}^{-0.00299 \times 90.5} \\
& =134.624 \\
\mathrm{fe}_{4} & =499 \times 0.00299 \mathrm{e}^{-0.00299 \times 630.5} \\
& =937.9=938 \\
\therefore \chi & =938,
\end{aligned}
$$

The significance level $\alpha$ at $5 \%$ and $1 \%$ with K-1 degree of freedom given;

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\(\chi_{\text {tab }} 0.05(3)=7.815, \chi=11.341\)
©๑ Decision Rule: Reject H \(\theta\) if \(\chi \geq \chi\)
©(๔)
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Decision: We therefore reject the null hypothesis $\left(\mathrm{H}_{0}\right)$ which states that "The distribution of inter-arrival times does not follow exponential distribution", since the chi-square goodness-of-fit test result is greater than the table value at both ends. That is $\chi>\chi$ at $5 \%$ and $1 \%$ level of significance, with df of 3 . If is therefore upheld that the distribution of inter-arrival time fellows exponential distribution.

Hypothesis Two:
$\mathbf{H}_{2}$ : Service time does not follow exponential distribution.

| Table 2: Mean Service Time Distribution |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :--- | :---: |
| S/N | Time (Minutes) | Average Frequency (f) | Mid point $(\mathrm{t})$ | ft |  |
| 1 | $01-180$ | 62 | 90.5 | 5,611 |  |
| 2 | $181-360$ | 81 | 270.5 | $21,910.5$ |  |
| 3 | $361-540$ | 72 | 450.5 | 32,436 |  |
| 4 | $541-720$ | 70 | 630.5 | 44,100 |  |
|  | Total | 285 |  | $104,057.5$ |  |

Source: Field survey, 2018

The mean service time (MST)
© $9=\mathrm{E}(\mathrm{t})$ is given by;
©( $\mathrm{E}(\mathrm{t})=$
(9) $\quad \therefore \mathrm{E}(\mathrm{t})=\quad=334.35=334 \mathrm{~min} /$ patient

ఆఅఆ(9 $=265.11 \mathrm{mins} . /$ patient
$\mu=$
$=0.00377$ patient $/ \mathrm{min}$

Mean service time is 0.00377 patients per minute.
To test if service time follows exponential process, we apply the goodness-of-fit test, given as;
$\chi^{2}=\frac{\sum(f o i j-f e i j)^{2}}{f e i j}$
where $\mathrm{K}-1$ is the d.f and k is the number of independent variables. (feij) $=\Sigma \mathrm{f}(\mathrm{t}, \lambda)$ and if $(\mathrm{t}, \lambda)$ is the exponential density function which is expressed as;
$\mathrm{f}(\mathrm{t}, \mu)=\lambda \mathrm{e}^{-\mu \mathrm{t}} ; \mathrm{t}=1,2 \ldots \mathrm{n}$
fe, $=285 \times 0.00377 \mathrm{e}^{-0.00377 \times 630.5}$
$=680$
Significance level $\alpha 5 \%$ and $1 \%$ with k-1 degree of freedom gives;

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© \(\chi=7.815\)
©๑ \(\chi=11.341\)
©(ง)
అ๑Decision Rule: Reject H0 if \(\chi \geq \chi\)
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Decision: we reject the null hypothesis that "The service time does not follow exponential distribution", since the chi-square goodness of fit test result is greater than the table value at both ends. That is $\chi>\chi$ at $5 \%$ and $1 \%$ level of significance, with d.f of 3 . We uphold that service time does follow exponential distribution.

## Hypothesis Three:

$\mathbf{H}_{3}$ : Waiting time does not follow exponential distribution.

| Table 3: Mean Waiting Time |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :--- | :---: |
| $\mathrm{S} / \mathrm{N}$ | Time (Minutes) | Average Frequency (f) | Mid point $(\mathrm{t})$ | ft |  |
| 1 | $01-180$ | 40 | 90.5 | 3,620 |  |
| 2 | $181-360$ | 241 | 270.5 | $65,190.5$ |  |
| 3 | $361-540$ | 133 | 450.5 | $59,916.5$ |  |
| 4 | $541-720$ | 28 | 630.5 | 17,654 |  |
|  | Total | 442 |  | 146,381 |  |

Source: Field survey, 2018

The mean waiting time $\left(w_{t}\right)$ is given as;
©๑Ө $\mathrm{W}_{\mathrm{t}}=$
๑ง( $\mathrm{W}_{\mathrm{t}}==331.18 \mathrm{mins} /$ patient
$\Theta \mu==0.00302$ patient $/ \mathrm{mins}$

Table 3.0 shows that a patient must spend an average of 331.18 minute ( 5.52 hrs ), before he is serviced in the hospital.

To test if service time follows exponential process, we apply the goodness-of-fit test, given as;
$\chi^{2}=\frac{\sum(f o i j-f e i j)}{f e i j}$
where $\mathrm{K}-1$ is the d.f and k is the number of independent variables.
$(\mathrm{Feij})=\Sigma \mathrm{f}(\mathrm{t}, \lambda)$ and if $(\mathrm{t}, \lambda)$ is the exponential density function which is expresses as;
$F(t, \mu)=\lambda \mathrm{e}^{-\mu \mathrm{t}} ; \mathrm{t}=1,2, \ldots \mathrm{n}$
$\mathrm{Fe},=442 \times 0.00302 \mathrm{e}^{-0.00302 \times 630.5}$
$=844$

The significance level $\alpha$ at $5 \%$ and $1 \%$ with k-1 degree of freedom gives;
© $\chi^{2}=7.815$
© $\chi^{2}=11.341$
Decision Rule: Reject H0 if $\chi_{\text {cal }}^{2} \geq \chi_{\text {tab }}^{2}$

Decision: We reject the null hypothesis that "waiting time does not follow exponential distribution" we rather uphold that "waiting time follows exponential distribution".

## DISCUSSION OF FINDINGS AND CONCLUSION

From the study, it was discovered that long queues actually occur in general hospitals in Akwa Ibom State. The mean inter-arrival time was 0.0029 patients/min given 499 arrival captured, while the mean service time was 0.00377 , patients $/ \mathrm{min}$, given 285 serviced patients. However, the weighted inter-arrival time $\left({ }^{499} / 285\right) 1.75 \mathrm{x}$ 0.00299 was 0.00523 patients $/ \mathrm{mins}$ while the weighted service time $\left({ }^{285} / 499\right) 0.571142$ was 0.002153 patient $/ \mathrm{mins}$. The mean waiting time remains at 0.0030 patients $/ \mathrm{min}$ (given 442) patients who some stayed on for services, while others reneged. No balking patient was recorded.

It therefore follows that from the inter-arrival index of 0.00523 patients/minutes compared with service time index of 0.002153 , more patients arrive to the facility to be serviced than what the capacity can carry. This is why the waiting time index was 0.0030 patients/minutes, which actually shows a patient having to wait for 331.18 minutes (5 hours, 51 minutes) in average before he can successfully access medial treatment. The tested hypotheses revealed that inter-arrival time, service time and waiting time have exponential distribution. This finding was in support of the work of Ezirim and Nwokah [3] which found that purely random arrivals in queuing means the arrival process is a poison process. A random service process means that service times have the exponential distribution... if arrivals follow a poison process, then inter-arrival times have the exponential distribution.

This study points to a direction of improvement in the facilities outlay (Availability) in the general hospitals in the state if these hospitals are to serve their intended purposes, where patients have confidence in the system and visit same for treatment which would improve societal health and subsequent improvement in social and economic well-being of the state. The reasoning is that a healthy people make a healthy society. Therefore, government should step up the facilities in the areas of
size and number of service units as well as personnel to cater for the increasing number of medical care receivers.

In conclusion, we have seen that the persistent problem of long queues in general hospitals in Akwa Ibom State can be traced to inadequate facilities provision in terms of conducing rooms, medical laboratory equipment and personnel, few numbers of doctors and nurses in relation to number of patients to attend to, as well as availability of pharmacy facilities. This is evidenced by more patients arriving for treatment than they can receive treatment within a time they would be willing to wait, not run out of their patience. It was recommended that government should provide for more facilities in these hospitals to cater for the increasing number of patients seeking medical care from their hospitals.

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