

Reliability Equivalence Factors of a Series System with Multivariate Exponential Distribution

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Abstract: This paper analyses a system consisted of n non-identical components connected in series. The components of the system are assumed to be dependent and their lifetimes follow the multivariate exponential distribution. The concept of the copula is used to generate the reliability function of the original system. Reliability of the system is improved by using reduction method. Other methods of hot duplication and cold, warm duplication with perfect and imperfect switching are established to improve system reliability. A numerical example is introduced to show the results and to compare different improvement methods.

Key words: Copula • Reliability • Series system • Exponential distribution • Reduction method • Cold standby • Warm standby • Hot standby • Equivalence factors

INTRODUCTION

Reliability is the probability that the unit will work successfully for a period of time without failure. In many situations, the reliability function of a system must be improved in order to achieve a desired level. Reliability of a system can be improved by several methods such as reduction and redundancy. In the reduction method, it is assumed that the system design can be improved by reducing the failure rates of a set of its components by a factor ρ such that $(0 < \rho < 1)$ [1]. The redundancy method is divided into some other types such as hot, warm and cold duplication methods.

In reduction method, the failure rates of a set of the system components are reduced by multiplying them by a factor ρ where an integer lies between zero and one. Hot duplication method assumes that some of the system components are duplicated in parallel to achieve high reliability. In cold duplication method, it is assumed that some of the system components are duplicated by a cold redundant standby component via a switch which can be perfect or imperfect. In warm duplication method, it is assumed that some of the system components are duplicated by a warm redundant standby component via a switch which can be perfect or imperfect [2]. Råde [3-5] introduced equivalence factors of reliability and applied this concept for two-component parallel and series systems with independent and identical components with

lifetimes follow exponential distribution. Reliability equivalence factors are defined as the factors by which the failure rates of the components of a system should be reduced in order to reach equality of the reliability of another better system [6]. Sarhan [7] obtained the reliability equivalence factors of n independent and non-identical components of a series system by using the survival function and mean time to failure as characteristics to compare different system designs. Sarhan [8] extended the concept of reliability equivalence from simple series and parallel systems to some complex systems. He considered a radar system in an aircraft, which consists of three independent and non-identical components with constant failure rates. Sarhan [9] introduced the reliability equivalence factors of a bridge network system. Sarhan & Mustafa [10] proposed the reliability equivalence factors of a series system which consists of n independent and non-identical components. Sarhan *et al.* [11] introduced reliability equivalence factors of a parallel-series system assuming that the failure rates of the system components are constant. Sarhan [2] introduced reliability equivalence factors of a general series-parallel system and the system components are assumed to be independent and their lives to have exponential distributions.

Xia & Zhang [12] studied reliability equivalence factors of a parallel system assuming that the failure rates of the system components are functions of time t with a

life distribution of gamma distribution. El-Damcese [13] introduced the reliability equivalence factors of a series-parallel system when the system components are independent and identical with a life distribution of Weibull distribution. Reliability equivalence factors for some systems with mixture Weibull failure rates were introduced by Mustafa [14]. Khan and Jan [15] introduced reliability evaluation of an engineering system using modified Weibull distribution. Mustafa and El-Faheem [16] presented reliability equivalence factors of a system with mixture of n independent and non-identical lifetimes with delay time. Reliability equivalence factor of a parallel system subject to time varying failure rates is studied by El-Damcese and Alltifi [17]. Ezzati and Rasouli [18] improved system reliability using linear-exponential distribution function. El-Damcese and Ayoub [19] obtained the two-dimensional reliability modeling equivalence factors of an independent and identical components parallel system by using bivariate Weibull model. The reliability equivalence factors for the general series-parallel system in the Burr type X distribution are derived by Migdadi and Al-Batah [20]. Yousry *et al.* [21] introduced reliability equivalence factors in exponentiated exponential distribution.

In this paper, a study of reliability equivalence factors of a series system consisting of n dependent and non-identical components is introduced. Reliability function of the original system is derived by using the concepts of copula subject to multivariate exponential distribution. Reliability function of the original system is improved according to reduction, hot, warm and cold duplication methods. Reliability equivalence factors are introduced to compare different system designs. Also, a numerical example is given to interpret how one can utilize the theoretical results obtained in this study and to compare the different reliability factors of the system.

Copula Definitions

Definition (1): Copula (Mangey and Singh [22]): A d -dimensional copula is a distribution function on $[0, 1]^d$ with standard uniform marginal distributions. Let $C(u) = C(u_1, u_2, \dots, u_d)$ be the distribution functions which are copulas. Hence C is a mapping of the form $C: [0, 1]^d \rightarrow [0, 1]$, i.e. a mapping of the unit hypercube into the unit interval. The following three properties must hold:

1. $C(u_1, u_2, \dots, u_d)$ is increasing in each component u_i .
2. $C(1, \dots, 1, u_i, 1, \dots, 1) = u_i$ for all $i \in \{1, \dots, d\}$, $u_i \in [0, 1]$.
3. For all $(a_1, \dots, a_d)(b_1, \dots, b_d) \in [0, 1]$ with $a_i \leq b_i$ we have:

$$\sum_{i_1=1}^2 \dots \sum_{i_d=1}^2 \dots (-1)^{i_1 + \dots + i_d} C(u_{i_1}, \dots, u_{i_d}) \geq 0$$

where $u_{i_1} = u_{i_1}, u_{i_2} = b_{i_2}$ for all $i \in \{1, \dots, d\}$.

Theorem 1: Sklar (Mangey and Singh [22])

Let F be a joint distribution function with margins F_1, \dots, F_d , (not necessarily continuous). Then there exists a copula $C: [0, 1]^d \rightarrow [0, 1]$, such that for all x_1, \dots, x_d in $\mathfrak{R} = [-\infty, \infty]$.

$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)) \quad (1)$$

If the margins are continuous then C is unique; otherwise C is uniquely determined on $\text{Ran } F_1 \times \dots \times \text{Ran } F_d$, where $\text{Ran } F_d$ denotes the range of $F_d: \text{Ran } F_i = F_i(\mathfrak{R})$. Conversely, if C is a copula and F_1, \dots, F_d is distribution functions, then the function F defined in (1) is a joint distribution function with margins F_1, \dots, F_d .

Definition (2): Mangey and Singh [22]

If F is a joint distribution function with marginals F_1, \dots, F_d and theorem (Sklar) holds, we say that C is a copula of F (or a random vector $X \sim F$). If the marginals are continuous then C is the unique copula of F (or X). The copula is the distribution function of the component wise probability transformed random vector. Alternatively, we can evaluate (1) at the arguments $x_i = F_i(u_i)$, $0 \leq u_i \leq 1$, $i = 1, \dots, d$ and use the property of the generalized inverse to obtain

$$C(u_1, \dots, u_d) = F(F_1^-(u_1), \dots, F_d^-(u_d))$$

where F^- is the generalized inverse of F .

Original System: A construction of multivariate distributions that does not suffer from these drawbacks is based on the copula function. To define a copula, begin as you might in a simulation study by considering p uniform random variables u_1, u_2, \dots, u_n , on the unit interval where p is the number of outcomes that you wish to understand. Unlike many applications, it is not assumed that u_1, u_2, \dots, u_n are independent variables. This relationship is described through their joint distribution function as follows [23].

$$C(u_1, u_2, \dots, u_n) = \text{Prob}(U_1 \leq u_1, U_2 \leq u_2, \dots, U_n \leq u_n)$$

where the function C is called a copula and U is a uniform random variable whereas u is the corresponding realization. To complete the construction, an arbitrary marginal distribution functions $F_1(x_1), F_2(x_2), \dots, F_n(x_n)$ are selected and the function:

$$C[F_1(x_1), F_2(x_2), \dots, F_n(x_n)] = F(x_1, x_2, \dots, x_n)$$

defines a multivariate distribution function which evaluated at x_1, x_2, \dots, x_n with marginal distributions F_1, F_2, \dots, F_n .

Now, consider a system consisted of n dependent units connected in series. The joint survival function of the original system is derived as follows

$$S(t) = \text{Prob}(\min(T_1, T_2, \dots, T_n) > t) = \text{Prob}(T_1 > t, T_2 > t, \dots, T_n > t)$$

According to Hougaard's copula family [24], the survival function of the system can be written as

$$S(t) = \exp(-[\{-\ln S_1(t)\}^{1/a} + \{-\ln S_2(t)\}^{1/a} + \dots + \{-\ln S_n(t)\}^{1/a}]^a), \quad a \geq 1$$

Applying this definition of copula, the reliability of the series system of n dependent units with multivariate exponential distribution is deduced and the result is

$$S(t) = \exp\left\{-t\left(\lambda_1^{1/a} + \lambda_2^{1/a} + \dots + \lambda_n^{1/a}\right)^a\right\}, \quad a \geq 1$$

$$R(t) = \exp\left\{-t\left(\sum_{i=1}^n \lambda_i^{1/a}\right)^a\right\}, \quad a \geq 1$$

where λ_i is the failure rate of the ith unit.

System reliability function versus time and failure rate λ_i is plotted in Fig. 1 for the following data: $n = 4$, $\lambda_2 = 0.002$, $\lambda_3 = 0.0015$, $\lambda_4 = 0.0033$, $a = 1.1$

The mean time to failure of the system is given as follows.

$$MTTF_s = \int_0^{\infty} R(t) dt$$

$$MTTF_s = \frac{1}{\left(\sum_{i=1}^n \lambda_i^{1/a}\right)^a}$$

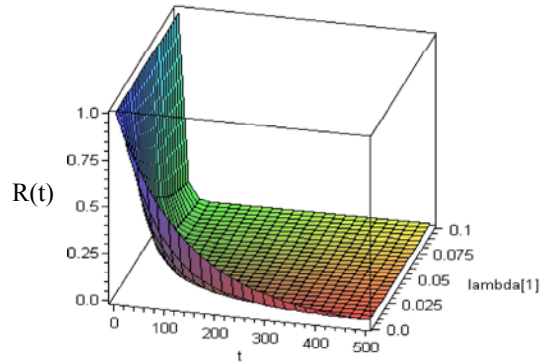


Fig. 1: Reliability functions versus time and failure rate λ_i .

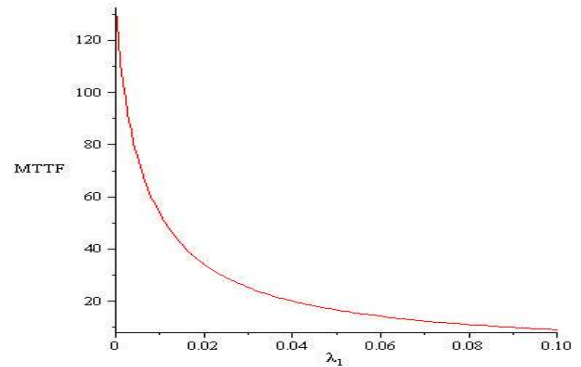


Fig. 2: Mean time to failure versus the failure rate λ_1

Given the same data used in reliability, the mean time to system failure versus the failure rate λ_1 is shown graphically in Fig. 2.

Reduction Method: In this method, it is supposed that the failure rates of k units of the system are decreased by multiplying by a factor ρ , $0 < \rho < 1$ and hence system reliability function is obtained as follows.

$$R_{r,k}(t) = \exp\left\{-t\left(\rho_1 \lambda_1^{1/a} + \rho_2 \lambda_2^{1/a} + \dots + \left(\rho_k \lambda_k^{1/a} + \lambda_{k+1}^{1/a} + \dots + \lambda_n^{1/a}\right)\right)^a\right\}$$

$$R_{r,k}(t) = \exp\left\{-t\left(\sum_{i=1}^k \rho_i \lambda_i^{1/a} + \sum_{i=k+1}^n \lambda_i^{1/a}\right)^a\right\}, \quad \text{for } k = 1, 2, \dots, n$$

In this case the mean time to system failure of improved system will be given as.

$$MTTF_{r,k} = \frac{1}{\left(\sum_{i=1}^k \rho_i \lambda_i^{1/a} + \sum_{i=k+1}^n \lambda_i^{1/a}\right)^a}$$

Cold Standby Duplication Method: In this method, it is supposed that m components of the system are duplicated with a cold standby unit. There are two cases discussed as follows.

Case 1 Imperfect Switching: In this case the reliability of each unit duplicated with an identical cold standby unit via an imperfect switch with probability of success γ is given by

$$R_i(t) = (1 + \gamma\lambda_i t)e^{-\lambda_i t}, \quad 0 < \gamma < 1$$

The reliability function of the system will be given by

$$R_{ci,m}(t) = \exp \left\{ -t \left[\lambda_1 - \ln(1 + \gamma\lambda_1 t) \right]^{1/a} + \dots + (\lambda_m t - \ln(1 + \gamma\lambda_m t))^{1/a} + (\lambda_{m+1} t)^{1/a} + \dots + (\lambda_n t)^{1/a} \right\}$$

$$R_{ci,m}(t) = \exp \left\{ - \left[\sum_{i=1}^m (\lambda_i t - \ln(1 + \gamma\lambda_i t))^{1/a} + \sum_{i=m+1}^n (\lambda_i t)^{1/a} \right] \right\} \text{ for } m = 1, 2, \dots, n$$

Case 2 Perfect Switching: In this case the switch is assumed to be perfect and hence the probability of success γ equals 1 and the reliability function of the system will be given by

$$R_{c,m}(t) = \exp \left\{ - \left[\sum_{i=1}^m (\lambda_i t - \ln(1 + \lambda_i t))^{1/\alpha} + \sum_{i=m+1}^n (\lambda_i t)^{1/\alpha} \right] \right\}, \text{ for } m = 1, 2, \dots, n$$

Warm Standby Duplication Method: In this method, it is supposed that l components of the system are duplicated with a warm standby unit. There are two cases discussed as follows.

Case 1 Imperfect Switching: In this case the reliability of each unit duplicated with a warm standby unit with failure rate λ_s via an imperfect switch with probability of success σ is given by

$$R_i(t) = \frac{\sigma\lambda_i + \lambda_{s_i}}{\lambda_{s_i}} e^{-\lambda_i t} \left(1 - \frac{\sigma\lambda_i}{\sigma\lambda_i + \lambda_{s_i}} e^{-\lambda_{s_i} t} \right), \quad \lambda_{s_i} < \lambda_i, \quad 0 < \sigma < 1$$

The reliability function of the system will be given by

$$R_{wi,l}(t) = \exp \left\{ - \left[\sum_{i=1}^l \left(\lambda_i t - \ln \left(\frac{\sigma\lambda_i + \lambda_{s_i}}{\lambda_{s_i}} - \frac{\sigma\lambda_i}{\lambda_{s_i}} e^{-\lambda_{s_i} t} \right) \right)^{1/\alpha} + \sum_{i=l+1}^n (\lambda_i t)^{1/\alpha} \right] \right\},$$

for $l = 1, 2, \dots, n$

Case 2 Perfect Switching: In this case the switch is assumed to be perfect and hence the probability of success σ equals 1 and the reliability function of the system will be given by

$$R_{w,l}(t) = \exp \left\{ - \left[\sum_{i=1}^l \left(\lambda_i t - \ln \left(\frac{\lambda_i + \lambda_{s_i}}{\lambda_{s_i}} - \frac{\lambda_i}{\lambda_{s_i}} e^{-\lambda_{s_i} t} \right) \right)^{1/\alpha} + \sum_{i=l+1}^n (\lambda_i t)^{1/\alpha} \right] \right\},$$

for $l = 1, 2, \dots, n$

Hot Standby Duplication Method: In this method, it is supposed that q components of the system are duplicated with a hot standby unit. The reliability function of the i th component is given by

$$R_i(t) = e^{-\lambda_i t} (2 - e^{-\lambda_i t})$$

The reliability function of the system is obtained as

$$R_{h,q}(t) = \exp \left\{ - \left[\sum_{i=1}^q (\lambda_i t - \ln(2 - e^{-\lambda_i t}))^{1/\alpha} + \sum_{i=q+1}^n (\lambda_i t)^{1/\alpha} \right]^\alpha \right\}, \text{ for } q = 1, 2, \dots, n$$

Reliability Equivalence Factors: Reliability equivalence factors in case of cold duplication method with imperfect switching are obtained as follows.

$$\begin{aligned} \exp \left\{ -t \left(\sum_{i=1}^k \rho_i \lambda_i^{1/\alpha} + \sum_{i=k+1}^n \lambda_i^{1/\alpha} \right)^\alpha \right\} \\ = \exp \left\{ - \left[\sum_{i=1}^k (\lambda_i t - \ln(1 + \gamma \lambda_i t))^{1/\alpha} + \sum_{i=k+1}^n (\lambda_i t)^{1/\alpha} \right]^\alpha \right\} \end{aligned}$$

$$\sum_{i=1}^k \rho_i \lambda_i^{1/\alpha} = \sum_{i=1}^m \left(\lambda_i - \frac{\ln(1 + \gamma \lambda_i t)}{t} \right)^{1/\alpha} + \sum_{i=m+1}^n \lambda_i^{1/\alpha} - \sum_{i=k+1}^n \lambda_i^{1/\alpha}$$

If $\rho_i = \rho$

$$\rho_{r,c}^{(k,m)} = \left(\sum_{i=1}^k \lambda_i^{1/\alpha} \right)^{-1} \left[\sum_{i=1}^m \left(\lambda_i - \frac{\ln(1 + \gamma \lambda_i t)}{t} \right)^{1/\alpha} + \sum_{i=m+1}^n \lambda_i^{1/\alpha} - \sum_{i=k+1}^n \lambda_i^{1/\alpha} \right]$$

If $\rho_i = \rho, m = k$

$$\rho_{r,c}^{(k,k)} = \left(\sum_{i=1}^k \lambda_i^{1/\alpha} \right)^{-1} \left[\sum_{i=1}^k \left(\lambda_i - \frac{\ln(1 + \gamma \lambda_i t)}{t} \right)^{1/\alpha} \right]$$

To obtain reliability equivalence factors in case of cold duplication method with perfect switching, set $\gamma = 1$ in previous relations. Reliability equivalence factors in case of warm duplication method with imperfect switching are obtained as follows.

$$\begin{aligned} \exp \left\{ -t \left(\sum_{i=1}^k \rho_i \lambda_i^{1/\alpha} + \sum_{i=k+1}^n \lambda_i^{1/\alpha} \right)^\alpha \right\} \\ = \exp \left\{ - \left[\sum_{i=1}^l \left(\lambda_i t - \ln \left(\frac{\lambda_i + \lambda_{s_i}}{\lambda_{s_i}} - \frac{\lambda_i}{\lambda_{s_i}} e^{-\lambda_{s_i} t} \right) \right)^{1/\alpha} + \sum_{i=l+1}^n (\lambda_i t)^{1/\alpha} \right]^\alpha \right\} \end{aligned}$$

$$\sum_{i=1}^k \rho_i \lambda_i^{1/\alpha} = \sum_{i=1}^l \left(\lambda_i - \frac{\ln \left(\frac{\lambda_i + \lambda_{s_i}}{\lambda_{s_i}} - \frac{\lambda_i}{\lambda_{s_i}} e^{-\lambda_{s_i} t} \right)}{t} \right)^{1/\alpha} + \sum_{i=l+1}^n \lambda_i^{1/\alpha} - \sum_{i=k+1}^n \lambda_i^{1/\alpha}$$

If $\rho_i = \rho$

$$\rho_{r,w}^{(k,l)} = \left(\sum_{i=1}^k \lambda_i^{1/\alpha} \right)^{-1} \left[\sum_{i=1}^l \left(\lambda_i - \frac{\ln \left(\frac{\lambda_i + \lambda_{s_i}}{\lambda_{s_i}} - \frac{\lambda_i}{\lambda_{s_i}} e^{-\lambda_{s_i} t} \right)}{t} \right)^{1/\alpha} + \sum_{i=l+1}^n \lambda_i^{1/\alpha} - \sum_{i=k+1}^n \lambda_i^{1/\alpha} \right]$$

If $\rho_i = \rho, l = k$

$$\rho_{r,w}^{(k,k)} = \left(\sum_{i=1}^k \lambda_i^{1/\alpha} \right)^{-1} \left[\sum_{i=1}^k \left(\lambda_i - \frac{\ln \left(\frac{\lambda_i + \lambda_{s_i}}{\lambda_{s_i}} - \frac{\lambda_i}{\lambda_{s_i}} e^{-\lambda_{s_i} t} \right)}{t} \right)^{1/\alpha} \right]$$

To obtain reliability equivalence factors in case of warm duplication method with perfect switching, set $\gamma = 1$ in previous relations. Reliability equivalence factors in case of hot duplication method are obtained as follows.

$$\begin{aligned} \exp \left\{ -t \left(\sum_{i=1}^k \rho_i \lambda_i^{1/\alpha} + \sum_{i=k+1}^n \lambda_i^{1/\alpha} \right)^\alpha \right\} \\ = \exp \left\{ - \left[\sum_{i=1}^q \left(\lambda_i t - \ln(2 - e^{-\lambda_i t}) \right)^{1/\alpha} + \sum_{i=q+1}^n (\lambda_i t)^{1/\alpha} \right]^\alpha \right\} \end{aligned}$$

$$\sum_{i=1}^k \rho_i \lambda_i^{1/\alpha} = \sum_{i=1}^q \left(\lambda_i - \frac{\ln(2 - e^{-\lambda_i t})}{t} \right)^{1/\alpha} + \sum_{i=q+1}^n \lambda_i^{1/\alpha} - \sum_{i=k+1}^n \lambda_i^{1/\alpha}$$

If $\rho_i = \rho$

$$\rho_{r,h}^{(k,q)} = \left(\sum_{i=1}^k \lambda_i^{1/\alpha} \right)^{-1} \left[\sum_{i=1}^q \left(\lambda_i - \frac{\ln(2 - e^{-\lambda_i t})}{t} \right)^{1/\alpha} + \sum_{i=q+1}^n \lambda_i^{1/\alpha} - \sum_{i=k+1}^n \lambda_i^{1/\alpha} \right]$$

If $\rho_i = \rho, q = k$

$$\rho_{r,h}^{(k,k)} = \left(\sum_{i=1}^k \lambda_i^{1/\alpha} \right)^{-1} \left[\sum_{i=1}^k \left(\lambda_i - \frac{\ln(2 - e^{-\lambda_i t})}{t} \right)^{1/\alpha} \right]$$

Numerical Example: As an illustration for the analytical study, the following numerical data is considered:

$$n = 4, \lambda_1 = 0.001, \lambda_2 = 0.002, \lambda_3 = 0.0015, \lambda_4 = 0.0033, \alpha = 1.1, \lambda_{s_2} = 0.0008,$$

$$\lambda_{s_2} = 0.001, \lambda_{s_3} = 0.001, \lambda_{s_4} = 0.002, \gamma = 0.5, \sigma = 0.6$$

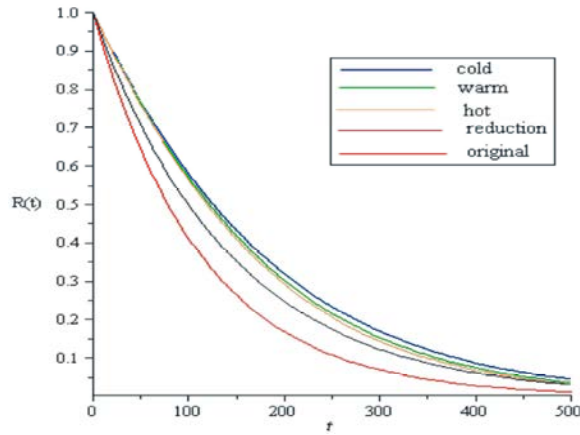


Fig. 3: Reliability function of original system and improved systems when $k, m, l, q = 2, \gamma = 1$

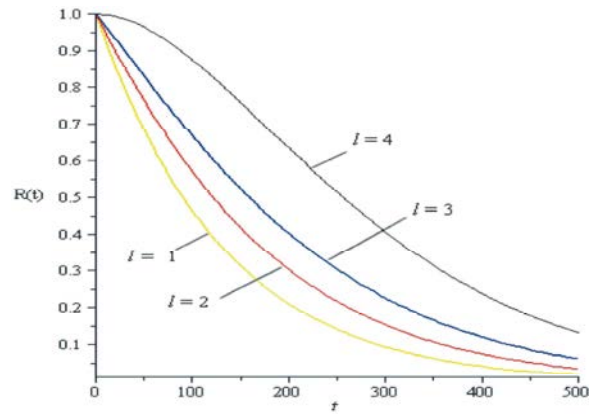


Fig. 6: Reliability functions of the improved system according warm standby method with imperfect switch.

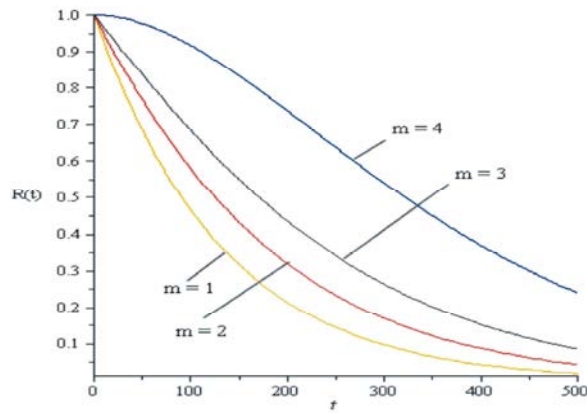


Fig. 4: Reliability functions of the improved system according cold standby method with perfect switch.

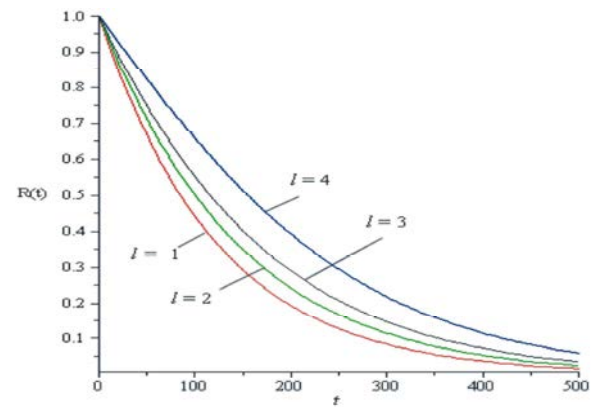


Fig. 7: Reliability functions of the improved system according warm standby method with imperfect switch.

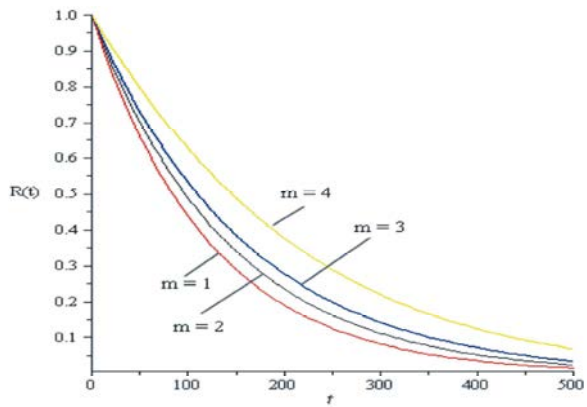


Fig. 5: Reliability functions of the improved system according cold standby method with imperfect switch.

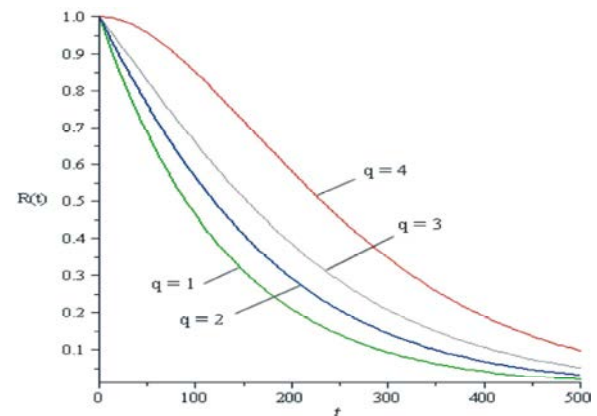


Fig. 8: Reliability functions of the improved system according hot standby method.

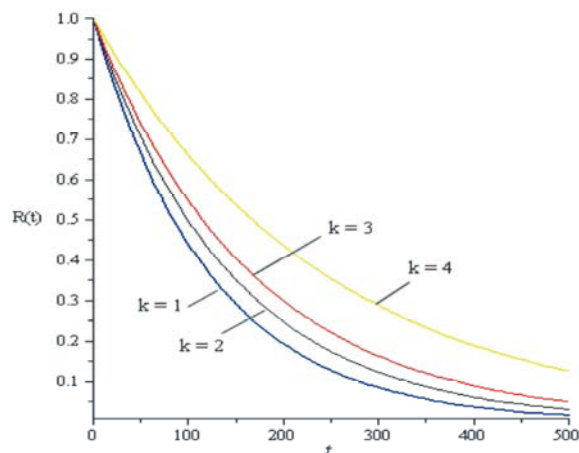


Fig. 9: Reliability function of the improved system according reduction method at $\rho = 0.5$

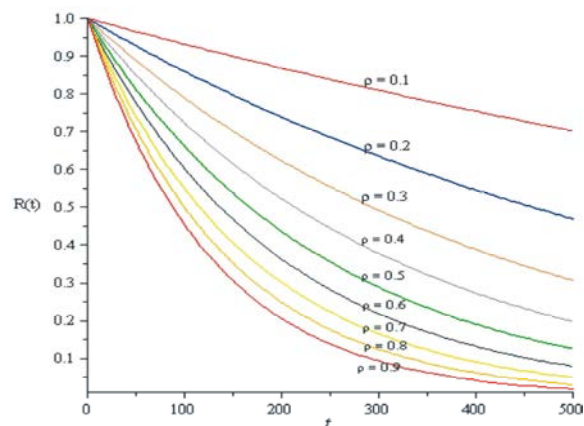


Fig. 10: Comparison of reliability functions of the improved system for different values of ρ at $k = 4$.

Comparison of the reliability function of the original system and the improved systems according to reduction method, hot, cold and warm duplication methods with perfect switching is obtained in Fig. 3. In Fig. 3, it is observed that the reliability function of the original system is improved by using four methods: (I) reduction method, (ii) cold duplication method, (iii) warm duplication method and (iv) hot duplication method. Applying the cold duplication method is the better one. Applying the warm duplication method is better than applying the hot duplication method which is better than applying the reduction method.

In Figs 4 and 5, it is observed that the improved system according cold standby duplication method with perfect switching is better than improved system according cold standby duplication method with imperfect switching. We also can observe that the greater the number of the components which duplicated by a cold standby unit, the more efficient of the reliability function.

In Figs 6 and 7, it is observed that the improved system according warm standby duplication method with perfect switching is better than improved system according warm standby duplication method with imperfect switching. It is also observed that the greater the number of the components which duplicated by a warm standby unit, the more efficient of the reliability function.

In Fig. 8, it is observed that increasing number of the units which duplicated by a hot standby unit increases the system reliability. While in Fig. 9, it is observed that increasing number of the units which improved by using reduction method increases the system reliability.

Table 1a: Reliability equivalence factors for different sets at time $t = 100$.

Reduction r_k	Cold standby c_m				Warm standby w_i				Hot standby h_q			
	c_1	c_2	c_3	c_4	w_1	w_2	w_3	w_4	h_1	h_2	h_3	h_4
r_1	0.0619	-ve	-ve	-ve	0.1033	-ve	-ve	-ve	0.1131	-ve	-ve	-ve
r_2	0.6740	0.0934	-ve	-ve	0.6884	0.1372	-ve	-ve	0.6918	0.1675	-ve	-ve
r_3	0.7830	0.3965	0.0913	-ve	0.7926	0.4257	0.1364	-ve	0.7948	0.4459	0.1640	-ve
r_4	0.8712	0.6418	0.4606	0.1203	0.8769	0.6591	0.4874	0.1781	0.8782	0.6711	0.5038	0.2124

Table 1b: Reliability equivalence factors for different sets at time $t = 100$.

Reduction r_k	Cold standby $(ic)_m$				Warm standby $(iw)_l$			
	$(ic)_1$	$(ic)_2$	$(ic)_3$	$(ic)_4$	$(iw)_1$	$(iw)_2$	$(iw)_3$	$(iw)_4$
r_1	0.5442	-ve	-ve	-ve	0.4734	-ve	-ve	-ve
r_2	0.8416	0.5513	0.3252	-ve	0.8170	0.4862	0.2280	-ve
r_3	0.8945	0.7013	0.5508	0.2553	0.8782	0.6580	0.4861	0.1659
r_4	0.9374	0.8227	0.7333	0.5579	0.9277	0.7970	0.6949	0.5049

Comparison of reliability functions of the improved system according to the reduction method for different values of ρ at $k = 4$ is shown in Fig. 10.

Reliability equivalence factors for different sets at time $t = 100$ are obtained in Table 1.

The mean time to failure is computed for the original system and the improved system according reduction method (for $\rho = 0.1$) and the results are found as follows.

$$MTTF_s = 112.559$$

Reduction	r_1	r_2	r_3	r_4
$MTTF_s$	130.132	182.513	260.835	1417.042

It can be observed that:

$$MTTF_{r,k} > MTTF_s$$

CONCLUSIONS

Copula is a useful tool to construct the reliability function of the systems which consists of dependent units. Multivariate exponential distribution can be used to model the lifetime of set dependent components. Reduction and redundancy methods can be used to improve system reliability. Reliability equivalence factors are used to compare different methods of improving the system.

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