Numerical Solution of Modified Reynolds Equation to Study the Roughness Effects Between Two Parallel Plates

Shalini M. Patil, P.A. Dinesh and C.V. Vinay

1, Department of Mathematics, JSS Academy of Technical Education, India
2, Department of Mathematics, MSRIT, India

Abstract: In this paper we investigate the Numerical solution of Modified Reynolds Equation describing the viscous, isothermal, incompressible, electrically conducting couple stress fluid as a lubricant between two rectangular plates of which upper plate has roughness surface and lower plate has porous material in the presence of transverse magnetic field. Using Beaver and Joseph Slip conditions the governing equations are solved. Advance numerical technique Finite difference based Multigrid method is applied to solve the Modified Reynolds Equation to study the effect of roughness parameter, Hartmann number, couple stress and aspect ratio. The investigation revealed that for a couple-stress fluid the pressure distribution and load carrying capacity increases for increasing roughness parameter, compared to the classical Newtonian case.

Key words: Couple Stress Fluid · Roughness · Hartmann Number · Modified Reynolds Equation · Multigrid Method

INTRODUCTION

Various industrial applications adopt the Magnetohydrodynamic lubrication phenomenon in current technology. At very high temperature liquid metal lubricants have great demand. Many analytical and experimental investigations are carried out on this MHD lubrication. The consequence of magnetic field in squeeze film lubrication is encouraging because magnetic field has important applications in the industry with apparent significance to technology-based world. Limited studies of hydromagnetic lubrication are accessible in the literature which consists of MHD slider bearings by Anwar and Das. The externally pressurized thrust bearing in MHD lubrication has been investigated both theoretically and experimentally by Maki. MHD squeeze film bearings by Kamiyam [1] and Hamza [2] has shown the effects of MHD on a fluid film squeezed between two rotating surfaces. In recent times Kudenatti [3], Bujurke [4], Naduvinamani [5] investigated the numerical solution of the MHD Reynolds equation for squeeze film lubrication between porous and rough rectangular plates. There are several theories used to study the roughness structure like Christensen [6], Christensen and Toder [7] used for this work. Some of the references used in this paper are Anwar [8], Maki [9], Stokes [10], Beavers and Joseph [11] and Lin [12].

Physical Configuration of the Problem: The geometry of the problem represented is shown in Fig. 1, where the viscous, isothermal, incompressible, electrically conducting couple stress fluid is considered as a lubricating fluid between two rectangular plates of which upper plate has roughness surface and lower surface has porous material in the presence of transverse magnetic field. The upper plate moves towards lower with constant velocity $dH/dt$. A uniform magnetic field $M_z$ is applied in transverse $z$ direction. The upper and lower plates are separated by the thickness $H$. The thickness is prepared of two fractions. where $h(t)$ represents the actual smooth part of the film geometry and is the altitude accountable for irregularity assessed from the actual level. The irregularity is evaluated by roughness factor $\xi$.

$$H = h(t) + h(x, y, \xi)$$  \hspace{2cm} (1)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$  \hspace{2cm} (2)
\[ \frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma M_0^2 u \] (3)

\[ \frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} - \eta \frac{\partial^4 v}{\partial z^4} - \sigma M_0^2 v \] (4)

\[ \frac{\partial p}{\partial z} = 0 \] (5)

Fig. 1: The Physical configuration of squeeze film between rough and porous rectangular plates

where \( u, v \) and \( w \) denote the velocity components in \( x, y \) and \( z \) directions respectively, \( p \) the pressure, \( \sigma \) the electrical conductivity of the fluid, \( M_0 \) the uniform magnetic field, \( \mu \) the viscosity of the fluid and \( \eta \) the couple stress parameter. The quantity \( \frac{n}{\mu} \) distinguishes the lubricant material length. The fitting limiting conditions are: at the upper rough plate \( z = H \);

\[ u = 0, \quad v = 0, \quad w = \frac{\partial H}{\partial t}, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad \frac{\partial^2 v}{\partial z^2} = 0, \quad \text{and} \quad \frac{\partial w}{\partial z} = 0 \] (6)

at the lower porous plate \( z = 0 \);

\[ \frac{\partial u}{\partial z} = \frac{\alpha}{\sqrt{k}} (u - u^*) \frac{\partial v}{\partial z} = \frac{\alpha}{\sqrt{k}} (v - v^*), \quad w = w^* \]

\[ \frac{\partial^2 u}{\partial z^2} = 0, \quad \frac{\partial^2 v}{\partial z^2} = 0. \] (7)

Here \( \alpha \) represents the non-dimensional slip constant and \( k \) is the permeability of the porous media. Equations (6) and (7) are due to vanishing of couple stresses. The revised form of Darcy’s law controls the stream of couple-stress fluids in porous section which is given by;

\[ u^* = -\frac{k}{\mu (1 - \beta + \phi M_0^2)} \frac{\partial P^*}{\partial x}, \quad v^* = -\frac{k}{\mu (1 - \beta + \phi M_0^2)} \frac{\partial P^*}{\partial y}, \quad w^* = -\frac{k}{\mu (1 - \beta)} \frac{\partial P^*}{\partial z} \] (8)

\[ \nabla \cdot q^* = 0 \] (9)

where \( u^* \) and \( v^* \) are velocity components in \( x, y \) and \( z \) directions respectively. \( P^* \) is the hydrostatic pressure and \( \beta = \frac{n}{\mu} \frac{k}{\mu} \). Assuming Cameron and Morgan [12] approximation and employing interface boundary condition, \( P = P^* \) at \( z = 0 \) equation (9) reduces after simplification to;
where \( D = 1 - \beta + \phi M^2 \).

Solving equations (3) and (4) for \( u \) and \( v \) using the boundary conditions, then substituting in equation (2) for \( u \) and \( v \) and integrating with respect to \( z \) between 0 and \( H \), the Modified Reynolds equation (MRE) expressing the variation of pressure is attained. As there are two types of roughness structures Longitudinal and Transverse roughness structures. However, we are focusing on longitudinal roughness structure running along \( x \) direction.

The roughness effects are represented by equation (11),

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = -\delta \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right)
\]

(10)

In the above equation expectancy operator \( E(\bullet) \) is identified by;

\[
E(\bullet) = \int_{-\infty}^{\infty} f(h_x) dh_x,
\]

(12)

where probability density function is denoted by \( f(h_x) \) and

\[
f(h_x) = \begin{cases} \frac{35}{32c^7} (c^2 - h_x^2)^3, & -c < h_x < c \\ 0 & \text{elsewhere} \end{cases}
\]

where \( c \) is the entire series random thickness, \( \sigma \) being the standard deviation of the probability function which ceases at \( \sigma = \pm 3c \).

The dimensionless factors are:

\[
K_1 = \frac{K_1}{K_2} + \frac{1}{2} \sqrt{K_1^2 - 4K_2}, \quad K_4 = \sqrt{\frac{K_1}{K_2} - \frac{1}{2} \sqrt{K_1^2 - 4K_2}}
\]

\[
K_1 = \tau^2, \quad K_2 = \frac{M^2}{h_0^2} \tau^2, \quad M = \frac{\sigma}{\sqrt{\mu} M_0 h_0},
\]

The non-dimensional parameters are:

\[
\tilde{\tau} = \frac{\tau}{\tilde{h}}, \quad \tilde{\gamma} = \frac{\gamma}{\tilde{h}} \quad \tilde{H} = \frac{H}{\tilde{h}_0}, \quad \tilde{p} = \frac{-h_0^3}{2} E(p), \quad \tilde{\Delta} = \frac{\Delta}{\tilde{h}_0}, \quad \tilde{\psi} = \frac{k\tilde{h}}{\tilde{h}_0}, \quad \tilde{C} = \frac{c}{\tilde{h}_0}
\]

where \( p \) the non-dimensional fluid film pressure, \( C \) is the roughness parameter, \( \tau \) the couple-stress parameter and \( S \) the slip velocity parameter. Longitudinal roughness pattern is applied to equation (11).

The boundary conditions are: \( p = 0 \) at \( x = 0; 1 \) and \( y = 0; 1 \).

**Numerical Solution by Multigrid Method:** The MRE (11) is complex to resolve because of elliptic nature. (FDBMG) Finite difference based Multigrid technique is operated to work out this equation. Using this method variation of pressure can be estimated. The iteration process is carried out till the pressure at two consecutive stages is almost same upto \( 10^{-6} \). The standard second order finite difference scheme applied for equation (11) is given by;

\[
\frac{\partial p}{\partial x} \bigg|_{z=0} = -\delta \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right)
\]
\[ A_0 p_{i+1,j} + A_1 p_{i-1,j} + A_2 p_{i,j+1} + A_3 p_{i,j-1} + A_4 p_{i,j} = -\lambda^2 R_{i,j} \]

where \( R_{i,j} = -M^2, \quad A_4 = -(A_0 + A_1 + A_2 + A_3) \)

RESULTS AND DISCUSSIONS

A mathematical mould has been prepared for analyzing the effects of various physical parameters such as pressure distribution (PD), couple stress parameter, roughness parameter, Hartmann number between two parallel plates in the presence of transverse of magnetic field. The characteristics of squeeze film bearings are obtained as functions of couple-stress parameter \( \eta \), roughness parameter \( C \), aspect ratio \( \lambda \) and Hartmann number \( M \). In Fig. 2, Fig. 3 and Fig. 4 the deviation of pressure distribution with rectangular coordinates \( x \) and \( y \) for various parameters are represented. It is noted that as roughness parameter \( C \) increases from 0.1-0.4, the PD increases. This is due to the fact that, increase in the roughness increases the roughness asperities on bearing surface which further reduces the velocity of the fluid and also reduces the sidewise leakage of the fluid. Thus pressure distribution also enhances.

Fig. 2: Variation of Pressure distribution for \( C=0.1 \)

Fig. 3: Variation of Pressure distribution for \( C=0.3 \)

Fig. 4: Variation of Pressure distribution for \( C=0.4 \)
Load carrying capacity (LCC) is one of the prominent features of hydrodynamic characteristics of the bearing. This can be obtained once the fluid film pressure is calculated. The non-dimensional LCC $W$ of the bearing surface per unit area in a non-dimensional form is

$$\int_{0}^{1} \int_{0}^{1} p(x,y) dx dy$$

The variation of LCC $W$ as a function of aspect ratio $\lambda$ for various roughness parameters keeping other flow parameters constant is shown in Fig. 5. The roughness parameter $C$ increases the LCC. The graph depicts that as $C$ increases the LCC also increases. Furthermore, as aspect ratio increases $\lambda$ from 0.1 to 10 the LCC also increases. As clarified in the earlier section, effects of roughness is to reduce the velocity of the fluid, as a result pressure increases in the fluid film region which yields in increase of the LCC of the bearings.

**CONCLUSIONS**

Using Stokes theory for micro-continuum fluids, the effects of MHD squeeze film lubrication on roughness and couple-stress fluid between two rectangular parallel plates of which the upper plate has rough surface and the lower plate has porous material are studied. BJ slip conditions are used to obtain the Modified Reynolds Equations. Finite difference based Multigrid method is adopted to study the characteristics of bearings. Our investigations revealed that:

- Increase in roughness parameter increases the pressure distribution.
- Increase in roughness parameter increases load carrying capacity for aspect ratio in the range 0.1 to 10.

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