Application of Brinkman Model in a Stratified Flow of Variable Viscosity Between Two Permeable Layers

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Abstract: Stratified flow of variable viscosity between two permeable beds is studied. The lower bed is infinite with low permeability where the fluid is governed by Darcy law. The upper bed is of finite thickness with high permeability and is bounded above by an impermeable wall. In the upper bed the flow is governed by Brinkman’s model. The free flow between the permeable beds is governed by the usual Navier-Stokes equations. Expressions for the velocity and temperature distributions in the free flow and porous regions, skin friction, mass flux and its fractional increase are obtained. These expressions are evaluated numerically for various values of the parameters. Furthermore, some deductions are carried out to compare the present results previously established. It is observed that the effect of permeable beds is to enhance the mass flux in the channel. Comparison of the present work with Channabasappa and Ranganna [24], it is found that the influence of introducing Brinkman porous bed increases 15% (approximately) mass flux in the porous channel bounded by Brinkman and Darcy permeable beds. The increase in the stratification factor enhances the skin friction at the permeable boundaries.

Key words: Stratified flow • Variable viscosity • Permeable beds • Brinkman’s model

INTRODUCTION

Fluid flow through porous media is governed by various models depending on the porous microstructure, presence of macroscopic boundaries, type of flow considered and the presence or absence of microscopic inertial effects, [1]. Popular among the governing models is Brinkman’s equation [2] which incorporates viscous shear effects into Darcy’s law to facilitate the study of flow through porous media possessing macroscopic boundaries. Validity, limitations and uses of Brinkman’s equation have been discussed by various authors (cf. [3, 4, 5, 6, 7, 8]). In addition, shear stress effects are important in the study of Navier-Stokes flow through a channel over a porous layer. Rudraiah [6] concluded that Brinkman’s equation is a more appropriate model when the porous layer is of finite depth, while Parvazinia et al., [5], concluded that when Brinkman’s equation is used, three distinct flow regimes arise, depending on Darcy number $Da$, namely, a free flow regime ($Da>1$), a Brinkman regime ($10^{-4}<Da<1$) and a Darcy regime ($Da<10^{-4}$). Their investigation, [5], emphasized that the Brinkman regime is a transition zone between the free and the Darcy flows. Nield, [4], argued that the use of Brinkman’s viscous shear term requires a redefinition of the porosity near a solid boundary, while some authors, [9, 10], emphasized the need for a variable permeability Brinkman model. Sacheti [11] applied Darcy and Brinkman models simultaneously for a viscous and incompressible flow through a porous channel. He applied Beavers and Joseph [12] slip condition at the upper permeable wall of low permeability (where Darcy law is valid) and the continuity of tangential stresses at the lower bed (where Brinkman model is considered). The effect of porosity parameters on the velocity distribution and mass flux is determined.

Viscous stratified flow in porous media has important applications in petroleum industry, since the flow behavior of fluids in petroleum reservoir rock depends considerably on viscosity variation and porous properties.
of the rock bed. The usual study of porous medium is to apply Darcy law. Based on Darcy law many researchers like Muskat [13], Collins [14], Schiedegger [15] etc. made investigations in flow through porous media. There are certain flows for which Darcy law is not valid. These flows may be studied by using Brinkman’s model [16]. The application of Brinkman’s model to various flows has been done by Rudraiah et al. [17] and many others.

Stratification of fluid arises due to temperature variations, concentration differences, or the presence of different fluids. In practical situations where the heat and mass transfer mechanisms run parallel, it is interesting to analyze the effect of double stratification (stratification of medium with respect to thermal and concentration fields) on the convective transport in micropolar fluid. The analysis of free convection in a doubly stratified medium is a fundamentally interesting and important problem because of its broad range of engineering applications. These applications include heat rejection into the environment such as lakes, rivers and seas; thermal energy storage systems such as solar ponds; and heat transfer from thermal sources such as the condensers of power plants. Although the effect of stratification of the medium on the heat removal process in a fluid is important, very little work has been reported in literature [18–21]. Cheng [22] considered the combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification. Cheng and Lee [23] analyzed the free convection on a vertical plate with uniform and constant heat flux in a thermally stratified micropolar fluid. Channabasappa and Ranganna [24] studied the effect of stratification and slip velocity on the flow of viscous stratified fluid of variable viscosity past a permeable bed. Using BJ slip boundary condition, the velocity distributions in and over the permeable bed are obtained. The thickness of the boundary layer just below the nominal surface of the porous bed is determined.

Reddappa et al. [25] discussed Convection in a Stratified Flow of a Jeffrey Fluid in an Inclined Channel with Permeable Boundaries. This paper deals with viscous, incompressible and stratified flow of variable viscosity between two permeable beds. The beds have different permeabilities. The lower bed is infinite with low permeability. Here the fluid is governed by Darcy law. The upper bed is of finite thickness with high permeability and is bounded above by an impermeable wall. In the upper bed the flow is governed by Brinkman’s model [16]. The free flow between the permeable beds is governed by the usual Navier-Stokes equations. The velocity fields in the porous and non-porous regions are determined. When the upper bed is replaced by an impermeable one, the results obtained are in agreement with the results of Channabasappa and Ranganna [24].

Nomenclature:

- \( x, y \): Cartesian coordinates
- \( U, V, W \): Velocity components in -direction in the regions 1, 2 and 3 respectively
- \( u, v, w \): Dimensionless velocity components in the regions 1, 2 and 3 respectively
- \( \rho \): Density of the fluid
- \( \mu \): Coefficient of viscosity
- \( k_1 \): Permeability of the lower porous bed
- \( k_2 \): Permeability of the upper porous bed
- \( Q \): Darcy velocity
- \( p \): Pressure
- \( h \): Distance between two porous beds
- \( a-h \): Width of the upper porous bed
- \( V_b \): Slip velocity
- \( v_b \): Dimensionless slip velocity
- \( b, \pi, \eta, \xi, \sigma, \theta \): Dimensionless parameters
- \( U_m \): Average velocity
- \( R = \frac{p U_m h}{\mu} \): Reynolds number
- \( \gamma = \beta h \): Stratification factor
- \( T_0 \): Temperatures of the lower plate
- \( T_1 \): Temperatures of the upper plate
- \( Ec \): Eckert number
- \( Pr \): Prandtl number

Mathematical Formulation: Consider the steady, viscous and incompressible flow of variable viscosity between two permeable beds (Fig.1). The flow in the lower porous bed viz. Region 1 is according to Darcy law. The clear flow between the porous beds viz. Region 2 is governed by the well known Navier-Stokes equations. The flow in the upper porous bed of finite thickness-Region 3 is according to Brinkman model. The upper porous bed is of width \((a-h)\) and is bounded by a rigid wall. The lower porous medium is of low permeability and the upper porous bed is of high permeability. Cartesian coordinate system is used. \(x\)-axis is taken along the normal surface of the lower porous bed and \(y\)-axis perpendicular to it.
We make the following assumptions and derive the basic equations of motion:

- The flow is steady, viscous and fully developed.
- The viscosity coefficient and density decays exponentially with respect to $y$, i.e. $\mu = \mu_0 e^{-\beta y}, \rho = \rho_0 e^{-\beta y}$
- The flow is in $x$-direction. All the physical quantities except the pressure are functions of $y$ only.
- The motion is caused by a constant pressure gradient $\frac{dp}{dx}$.
- The thickness of the lower porous bed is much larger than that of the upper porous bed.

In view of the above assumptions, the continuity and momentum equation/Darcy velocity take the following form:

**Region 1:**

\[ \frac{\partial Q}{\partial x} = 0 \]  
\[ Q = Q_0 e^{\beta y} \]  
where $Q_0 = -\frac{k}{\mu_0} \frac{dp}{dx}$

**Region 2:**

\[ \frac{d^2 V}{dy^2} - \frac{\beta}{ \mu_0} \frac{dV}{dy} = \frac{1}{ \mu_0} \frac{dp}{dx} e^{\beta y} \]
\[ k \left( \frac{\partial^2 T}{\partial y^2} \right) + \mu_0 \left( \frac{\partial V}{\partial y} \right)^2 = 0 \]

**Region 3:**

\[ \frac{\partial W}{\partial x} = 0 \]
\[ \frac{d^2 W}{dy^2} = \frac{\beta}{ \mu_0} \frac{dW}{dy} \]
\[ = \frac{1}{ \mu_0} \frac{dp}{dx} \]

The boundary conditions covering the three regions are:

**BJ slip condition**

\[ y = 0; V = V_0; \frac{dV}{dy} = \frac{\alpha}{\sqrt{k}} (V_0 - Q_0) \]

**Continuity of tangential stress condition**

\[ y = a; W = 0 \] (no slip condition)

\[ y = 0; T = T_0; y = h; T = T'_0 \]

We introduce the following non-dimensional quantities

\[ u = \frac{Q}{U_0}; v = \frac{V}{U_0}; w = \frac{W}{U_0}; y = x; b = a; \]
\[ \bar{T} = \frac{T - T_0}{T' - T_0}; \]
\[ \bar{Q} = \frac{Q}{U_0}; \bar{T}' = \frac{T' - T_0}{T' - T_0} \]

In view of (12), equations (1)-(11) in dimensionless form are as follows:

**Region 1:**

\[ \bar{Q}' = u e^m \]
where $u = \frac{P}{\sigma}; P = -R \frac{\partial \pi}{\partial \xi}; R = \frac{U_0 h \rho_0}{\mu_0}; \pi = \frac{h}{\sqrt{k}}$

**Region 2:**

\[ \frac{d^2 v}{d\eta^2} - \gamma \frac{dv}{d\eta} = -P e^m \]
\[ \frac{\partial^2 T}{\partial \eta^2} + \left( \frac{dv}{d\eta} \right)^2 E c P r = 0 \]

**Region 3:**

\[ \frac{d^2 w}{d\eta^2} - \gamma \frac{dW}{d\eta} - \sigma_0^2 w = -P e^m \]
where \( \sigma_z = \frac{h}{\sqrt{k_z}} \)

Boundary conditions:

\[
\eta = 0; v = v_s; \frac{dv}{d\eta} = \alpha \sigma (v_s - u)
\]

\[
\eta = 1; v = w; \frac{dv}{d\eta} = \frac{dw}{d\eta}
\]

\[
\eta = h; w = 0
\]

\[
\eta = 0; T = 0; \eta = 1; T = 1
\]

**Solution of the Problem:** The velocity distributions in the three regions are obtained on solving the equations (13), (14) and (16) subject to the boundary conditions (17)-(19).

The velocity distribution in region 1 is

\[
Q' = uw^n
\]

where \( u = \frac{P}{\sigma^v} \)

The velocity distribution in region 2 is given by

\[
v = D_i + C_i (e^\eta - e^v) - \frac{P \eta}{\gamma} e^\eta
\]

The slip velocity at the nominal surface of the lower porous bed can be written as

\[
v_s = \frac{1}{D_i} (D_i + D_h + D_t)
\]

The velocity field in the region 3 is given by

\[
w = A_i e^\eta + A_i e^\eta + \frac{D_t}{A_i} e^\eta
\]

where the expressions for the constants \( A_i, C_i, D_i \) are given in Appendix.

**Temperature Distribution:** Using the boundary conditions (20) and solving equation (15), the temperature distribution in region 2 is given by

\[
T = E_i \eta \gamma - E_i - E_i \left( \eta \gamma - \frac{3}{2 \gamma} - 1 \right) \left( e^\eta - e^v \right) + F_i \eta + F_t
\]

**Deductions:**

**Case (i):** Flow through a channel over a permeable bed of low permeability:

When the permeability parameter \( k_1 \) tends to zero in (22) and (23), we get the velocity distribution for the flow of variable viscosity over a permeable bed, viz.,

\[
v = \frac{P}{\gamma} + \frac{\alpha \sigma}{\gamma} \left( v_s - \frac{P}{\sigma^v} \right) \left( e^\eta - e^v \right)
\]

with slip velocity

\[
v_s = \frac{P}{\gamma} \left( \sigma - \alpha \gamma \right) \left( 1 - e^v \right) + \gamma \sigma e^v
\]

This is in agreement with the result of Channabasappa and Ranganna [24].

**Case (ii):** Flow through a channel bounded by rigid walls:

When the permeability parameters \( k_1 \) and \( k_2 \) tend to zero, the channel will be bounded by two impermeable rigid walls. Then the velocity field (22) reduces to

\[
v = \frac{P}{\gamma} \left( e^\eta - e^v \right) + \frac{P}{\gamma} \left( e^v - \eta e^\eta \right)
\]

**Mass Flux and Skin Friction:** The dimensionless mass flow rate per unit width of the channel bounded by two permeable beds is given as

\[
M_t = \int_0^1 e^{-\eta} v d\eta
\]

\[
= \frac{1}{\gamma} \left[ C_z (1 - e^v) + D_i (1 - e^\eta) \right] + \left( C_z - \frac{P}{2 \gamma} \right)
\]

When the permeability parameter \( k_2 \) in (29) is taken as zero, we obtain the mass flow rate per unit width of the channel bounded below by a permeable bed, namely,

\[
M_t = \frac{P}{\gamma} \left[ 1 + \frac{1}{\gamma} - \frac{e^v}{\gamma} - \frac{e^v}{\sigma^v} \left( \frac{\sigma^v}{\gamma} + \frac{\sigma^v}{\sigma} + \frac{\sigma^v}{\sigma^v} \right) \right]
\]

\[
+ \frac{P}{\gamma} \left[ \frac{1}{\gamma} - \frac{1}{\gamma} - \frac{e^v}{\gamma} \right] + \alpha \sigma \gamma \left[ 1 + \frac{1}{\gamma} - \frac{e^v}{\gamma} \right]
\]

This result agrees with the result of Channabasappa and Ranganna [24].
Further with \( k_i \) taken as zero in (30), we get the mass flow rate per unit width of the channel bounded by two rigid walls, namely,

\[
M_i = \frac{P}{\gamma} \left[ 1 - e^{\gamma} + e^{\gamma} - \frac{\gamma}{2} \right]
\]

(31)

We calculate an expression for the fractional increase in mass flow rate through the channel with permeable boundaries over what it would be if the walls were impermeable. Denoting this by \( \phi_i \), we define that

\[
\phi_i = \frac{M_i - M_{i0}}{M_{i0}}
\]

(32)

If \( \phi_i \) denotes the fractional increase in mass flow rate per unit width of the channel with upper wall rigid and lower wall porous over what it would be if the walls were impermeable, it is easy to see that

\[
\phi_i = \frac{M_i - M_{i0}}{M_{i0}}
\]

(33)

We calculate skin friction at the upper and lower boundary in two cases, i.e. those corresponding to case (i) both walls being porous and case (ii) upper wall being rigid while the lower porous. Denoting the skin friction by \( \tau \), a non-dimensional coefficient of friction can be defined as

\[
\tau = \frac{ch}{2}
\]

where the subscripts \( l \) and \( u \) refer to lower and upper boundary respectively.

The relevant expressions in the case (i) have been found as

\[
(\tau)_l = \frac{\partial v}{\partial \eta} = C_l \gamma - \frac{P}{\gamma}
\]

(34)

\[
(\tau)_u = \frac{\partial v}{\partial \eta} = C_u \gamma e^\gamma - Pe^\gamma \left[ \frac{1}{\gamma} + 1 \right]
\]

(35)

where the subscripts \( l \) and \( u \) refer to lower and upper boundary respectively.

The corresponding expressions for \( \tau \) in the case (ii) may be obtained by letting \( \sigma_i \to \infty \), we obtain

\[
(\tau)_{(ii)} = C_{i2} \gamma - \frac{P}{\gamma}
\]

(36)

\[
(\tau)_{(ii)} = C_{i2} \gamma e^\gamma - Pe^\gamma \left[ \frac{1}{\gamma} + 1 \right]
\]

(37)

RESULTS AND DISCUSSIONS

The velocity field of the flow of stratified fluid of variable viscosity between two permeable beds is studied numerically in order to find the effect of the porous beds. The velocity profiles are drawn for various values of the permeability parameter \( \sigma \) and are depicted in figures 2 to 4. It is seen from figures 1 and 2 that the velocity decreases with the increase in the permeability parameter (both upper and lower cases).

The velocity field attains the maximum value in free fluid region i.e. in the region between two permeable beds. After attaining the maximum value, the velocity decreases with the increase in \( \eta \) and finally becomes zero at the outer rigid wall of the upper porous bed. This justifies the boundary condition (19). From Fig.3 it is observed the velocity increases with increase in the stratification factor \( \gamma \).

The expression for the temperature is given by equation (25). The temperature profiles are plotted in Figures 5 - 7. From Fig.5 we notice that the increase in \( \sigma \) decreases the temperature. Fig.6 shows that the temperature increases with the increase in \( \sigma \). Fig.7 depicts that the temperature increases with increase in the product of Eckert number and Prandtl number is \( Ec Pr \).

The numerical values of the mass flux and magnitude of skin friction at both the walls are computed from equations (32) to (37) for different values of upper and lower permeability parameters \( \sigma_i \) and \( \sigma \) are presented in Table 1 to Table 5. From Table 1 it is observed that for fixed \( \sigma_i \) the mass flux \( (\phi_i) \) decreases with increasing \( \sigma_i \) and the same behavior is observed for fixed \( \sigma \) with increasing \( \sigma \). From Table 2 it is to be noted that skin friction at the lower and upper boundary increases with increasing \( \sigma \). From Table 3 it is found that in the rigid upper boundary case \((\sigma_i \to \infty)\) the mass flux \( (\phi_i) \) decreases and skin friction at the lower and upper boundary are increases with increasing \( \sigma \). From Table 4 we observe that the mass flux \( (\phi_i) \) is increases with increasing stratification factor \( (\gamma) \). From Table 5 we notice that in the rigid upper boundary case \((\sigma_i \to \infty)\) the mass flux \( (\phi_i) \) decreases with increasing stratification factor \( (\gamma) \). Table 6 represents the variation of mass flux \( (M) \) with stratification factor \( (\gamma) \), it is found that for a stratified flow the mass flux increases with the introduction of porous beds in place of rigid plates (i.e., I<II<III). Comparing the present work with Channabasappa and Ranganna [24], we find that the mass flux considerably gets enhanced due to the application of highly permeable bed (obeying Brinkman model) as the upper bounding wall of the channel. Further the effect of increase in the stratification also gives rise to more mass flux in the channel.
Fig. 2: Velocity distribution for various values of $\sigma_i$ (upper) for fixed $\sigma_i = 2.5, \alpha = 0.01, \gamma = 0.2, b = 1.5, P = 5.5$.

Fig. 3: Velocity distribution for various values of $\sigma_i$ (lower) for fixed $\sigma_i = 30, \alpha = 0.01, \gamma = 0.2, b = 1.5, P = 5.5$.

Fig. 4: Velocity distribution for various values of $\gamma$ for fixed $\sigma_i = 30, \sigma_i = 2.5, \alpha = 0.01, \gamma = 0.2, b = 1.5, P = 5.5$.

Fig. 5: Temperature distribution for various values of $\sigma_i$ for fixed $\sigma_i = 2.5, \alpha = 0.01, \gamma = 0.2, b = 1.5, P = 5.5, EcPr = 1.5$.

Fig. 6: Temperature distribution for various values of $\sigma_i$ for fixed $\sigma_i = 30, \alpha = 0.01, \gamma = 0.2, b = 1.5, P = 5.5, EcPr = 1.5$.

Fig. 7: Temperature distribution for various values of $Ec$ for fixed $\sigma_i = 30, \sigma_i = 2.5, \alpha = 0.01, \gamma = 0.2, b = 1.5, P = 5.5$. 

Table 1: Variation of $\phi$ with $\sigma$ and $\alpha$ for $\alpha=0.01$, $\gamma=0.05$, $P=0.5$, $b=1$.

<table>
<thead>
<tr>
<th>$\sigma_i$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
<th>$\phi_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>30</td>
<td>1.5</td>
<td>4.0</td>
<td>1.5</td>
<td>2.5</td>
</tr>
<tr>
<td>0.5</td>
<td>97.55</td>
<td>74.73</td>
<td>47.29</td>
<td>73.66</td>
<td>54.55</td>
</tr>
</tbody>
</table>

Table 2: Variation of $\tau$ with $\sigma_i$ for $\sigma_i=2.5$, $\alpha=0.01$, $\gamma=0.05$, $P=0.5$, $b=1.5$.

<table>
<thead>
<tr>
<th>$\sigma_i$</th>
<th>$\tau_1$</th>
<th>$\tau_2$</th>
<th>$\tau_3$</th>
<th>$\tau_4$</th>
<th>$\tau_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.0424</td>
<td>0.0755</td>
<td>0.1019</td>
<td>0.1237</td>
<td>0.1418</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.4810</td>
<td>-0.4463</td>
<td>-0.4185</td>
<td>-0.3956</td>
<td>-0.3765</td>
</tr>
</tbody>
</table>

Table 3: Variation of $\phi$ and $\tau^*$ with $\sigma_i$ for $\sigma_i=\infty$, $\alpha=0.01$, $\gamma=0.05$, $P=0.5$, $b=1.5$.

<table>
<thead>
<tr>
<th>$\sigma_i$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
<th>$\phi_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.1088</td>
<td>0.4610</td>
<td>0.2154</td>
<td>0.0752</td>
<td>-0.0201</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0230</td>
<td>0.0427</td>
<td>0.0592</td>
<td>0.0732</td>
<td>0.0854</td>
</tr>
<tr>
<td>0.0424</td>
<td>0.0755</td>
<td>0.1019</td>
<td>0.1237</td>
<td>0.1418</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Variation of $\phi$ with $\gamma$ for $\sigma_i=2.5$, $\alpha=0.01$, $\sigma_i=30$, $P=0.5$, $b=1.5$.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
<th>$\phi_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.7336</td>
<td>0.7425</td>
<td>0.7453</td>
<td>0.7466</td>
<td>0.7473</td>
</tr>
</tbody>
</table>

Table 5: Variation of $\phi$ with $\gamma$ for $\sigma_i=\infty$, $\alpha=0.01$, $\sigma_i=30$, $P=0.5$, $b=1.5$.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
<th>$\phi_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>9.7890</td>
<td>2.3068</td>
<td>0.9233</td>
<td>0.4394</td>
<td>0.2154</td>
</tr>
</tbody>
</table>

Table 6: Variation of $M$ with $\gamma$ for $\sigma_i=2.5$, $\alpha=0.01$, $\sigma_i=30$, $P=0.5$, $b=1.5$.

<table>
<thead>
<tr>
<th>$\sigma_i$</th>
<th>$\gamma$</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0417</td>
<td>0.2919</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0427</td>
<td>0.1397</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0438</td>
<td>0.1416</td>
</tr>
<tr>
<td>0.15</td>
<td>0.0449</td>
<td>0.1435</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0460</td>
<td>0.1455</td>
</tr>
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</table>

CONCLUSION

In this paper, we assess the stratified flow of variable viscosity between two permeable beds. It is observed that the effect of permeable beds is to enhance the mass flux in the channel. Comparison of the present work with Channabasappa and Ranganna [24], it is found that the influence of introducing Brinkman porous bed increases 15% (approximately) mass flux in the porous channel bounded by Brinkman and Darcy permeable beds. The increase in the stratification factor enhances the skin friction at the permeable boundaries.

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Appendix

$$a_i = \gamma + \frac{\sqrt{\gamma + 4\sigma_i}}{2}, b_i = \frac{\gamma - \sqrt{\gamma + 4\sigma_i}}{2},$$

$$D_i = A_i e^{a_i} + A_i e^{a_i} + \frac{P}{\sigma_i} e^P + P e^P, C_i = \frac{P + \alpha P}{\gamma} (v_i - \frac{P}{\sigma_i}),$$

$$A_i = \frac{a_i D_i e^{a_i} + r_i + P}{\sigma_i} - a_i D_i e^{a_i}, A_i = \frac{D_i e^{a_i} - D_i e^{a_i}}{\sigma_i},$$

$$D_i = -\alpha C_i (v_i - \frac{P}{\sigma_i}) + \frac{P}{\sigma_i} D_i, D_i = \frac{P}{\sigma_i} D_i,$$

$$D_i = a_i e^{a_i} - a_i e^{a_i}, D_i = a_i e^{a_i} - a_i e^{a_i},$$

$$D_i = 1 - \alpha C_i (1 - e^P) - \frac{a_i e^{a_i} + \alpha P}{\sigma_i} e^{a_i} + \alpha e^{a_i} e^{a_i},$$

$$D_i = 1 \left[ \left( \frac{P}{\sigma_i} + \frac{P}{\sigma_i} \right) e^{a_i} \frac{P}{\sigma_i} \right],$$

$$D_i = \frac{P}{\gamma} e^{\frac{P}{\gamma} \sigma_i} - \frac{P}{\sigma_i} e^{\frac{P}{\gamma} \sigma_i} e^{\frac{P}{\gamma} \sigma_i},$$

where

I: Flow of variable viscosity between parallel plates.

II: Flow of variable viscosity between parallel plates over a permeable bed and bounded by a rigid wall.

III: Flow of variable viscosity between two permeable layers.
REFERENCES


