World Applied Sciences Journal 35 (12): 2643-2648, 2017
ISSN 1818-4952

DOI: 10.5829/idosi.wasj.2017.2643.2648

Dispersion of Pollutants from A point Source in Moderate and Low Wind

[1]Khaled S. M. Essa, [2]Fawzia Mubarak, [1]A.A. Marrouf and [3]Olfat Ebrahim

[1]Mathematics and Theoretical Physics Dept., NRC, AEA, Cairo, Egypt
[2]Rad. Protection Dept., NRC, AEA, Cairo, Egypt
[3]Ben-Suef University, Faculty of Science, Undergraduate, Egypt

Abstract: A generalized simple model was presented to describe the dispersion of a pollutant in downwind distances in stable atmosphericboundary layer (ABL). Advection-Diffusion Equation was solved for a nearly constant emission rate of pollutant from a point source with assumption of well –defined edge plume.The proposed model investigates dispersion and advection of conservative material as it travels downwind. Determination and an explicit approximate expression for the Ground Level Concentration (GLC) were provided. Results were evaluated with the observations obtained from diffusion experiments in moderate and low wind.The present model shows good performance with the observed data and can be used to investigate the short-range dispersion of pollutant. It can be concluded thatthe good parameterizations of input data such as windspeed and plume heightleads to well perfection in the agreement betweencomputed and observed concentrations.

Key words: Plume height · Pollutant concentration · Diffusion equation and stable conditions

INTRODUCTION

The environmental problems caused by dispersion of pollutants into airare complex and strongly influencing many natural processes and hence the ecological balance. For this reason, it is very important to develop our understanding of the pollutants dispersion in the atmosphere and the hazards on the diverse ecosystems involved [1, 2]. The combination of diffusiondue to turbulent eddy motion and advection due to the wind that occurs within the atmospheric boundary layer is called dispersion. Dispersion of pollutants can be described mathematically by atmospheric dispersion modeling. The computational simulation of pollutant dispersion concerning as a very important source of information. Advection–diffusion equation,which is a second-order partial differential equation (PDE) of parabolic type, is a very well description of atmospheric dispersion models [3, 4]. This equation provides a good theoretical dispersion model from a source given appropriate boundary and initial conditions, plus knowledge of the mean wind velocity and concentration of turbulent fluxes [5-8]. For point source releases, advection diffusion equation considered wind speed as a function of vertical height and vertical eddy diffusivity as a function of both vertical height and downwind distance from the source [9, 10]. In this study, we suggest a simple model to predict the downwind concentration from a point source in moderate and low wind conditions. The importance of this study that,in such conditions the pollutants are not able to travel far and thus the near-source areas are affected the most [11].

Theoretical Aspects: A very simple approach, namely the principle of conservation of mass, will be used as a starting point. After steady-state has been reached, the principle of conservation of mass can be written as:

$$Q = \int_0^H (\bar{u})\ C(z)\ dz \qquad (1)$$

where,
$\bar{u}$ (z) is the mean wind speed (m/s),
C(z) is the concentration of pollutants (g/m^3) (Bq/m^3) and
H is the plume height (m) [12].

In subsequent section of this paper the different variables in equation (1), namely, the wind profile, concentration profile and plume height will be discussed in detail. Once these variables are described completely, the integration of equation (1) will lead to the mathematical model.

Corresponding Author: Khaled S.M. Essa, Mathematics and Theoretical Physics Dept., NRC, AEA, Cairo, Egypt.

Table 1: Range of Monin-Obokhov length according to stability classes

Stability class	L range
Very stable	0 < L < 200 m
Stable	200 < L < 1000m
Near neutral	L > 1000m
Unstable	-1000 < L < -200 m
Very unstable	-200 < L < 0 m

Wind Parameterization: Along with increased turbulence, one of the effects of atmospheric convection is to modify the shape of the mean velocity profile. The Obokhov length L is a parameter used to define atmospheric stability. It is defined as that height at which turbulence is generated more by buoyancy than by wind shear. According to Monin and Obukhov [13, 14], the adiabatic wind profile is given by:

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{K_0}\varphi_m(z/L) \quad (2)$$

where,

φ_m Is a function dependent on the dimensionless parameter z/L.

L Is the Monin –Obukhov length scale.

u_* Is the friction velocity (m/s).

k_o Is the Von-Karman constant equals 0.4

Equation (2) was integrated for stable condition to become:

$$u = u_*/k_o(\ln z/z_o + 5z/L) \quad (3)$$

where:

z_o is the roughness height (m) [15, 16].

The friction velocity u_* is related to the frictional resistance that the ground exerts on the wind. It is typically about 10% of the wind speed at z= 10 m. The surface roughness length z_o is a measure of the aerodynamic roughness of the ground and it is typically 3-10% of the height of the surface obstacles [17].

The use of stability classes help to understand the different wind profile with respect to atmospheric stability (Table 1). The dimensionless parameter L/z is positive for stable conditions, negative for unstable and goes to zero for near- neutral conditions [18].

Plume Height and Concentration Parameterization: Several equations have been proposed to predict plume rise. Unfortunately, the predictions of the different models are more than a factor of 10 apart. Briggs (1968) developed the well-conceived equations. They are used in many regulatory models [19, 20].

First, the buoyancy flux parameter f_b is defined as:

$$f_b = \left(1 - \frac{\rho_s}{\rho}\right) g r^2 w \quad (4)$$

with ρ_s the density of the stack gas, ρ the density of the surrounding air (1.17 kg/m^3), g the acceleration due to gravity (9.80665 m/s^2), r the stack radius (1m) and w (m/s) the stack gas velocity in the vertical direction.

The transitional plume rise is the local plume rise, before the plume has reached its maximum height. It is given by the following equation [13, 19]:

$$\Delta h = \frac{1.6\, F_b^{1/3} x^{2/3}}{u} \quad (5)$$

where x (m) is the distance downwind from the source and u is the wind speed (m/s). However, plumes do not rise indefinitely but stabilized at a certain height, the final plume rise height, this height is achieved at a distance x_f(m) from the source which can be calculated as follow:

$$x_f = 49\, F_b^{\frac{5}{8}} for F_b < 55\ m^4 s^{-3} \quad (6)$$

$$x_f = 119\, F_b^{\frac{2}{5}} for F_b > 55\ m^4 s^{-3} \quad (7)$$

Equations (6 & 7) are dimensionally not homogeneous and are valid only when metric units are used. At distances greater than x_f, the plume rise is assumed constant and given by:

$$\Delta h = \frac{1.6\, F_b^{1/3} x_f^{2/3}}{u} \quad (8)$$

In our case F_b<55, then equation (6) was applied. Measurements were at downwind distance x=100 m, thenx_f< x, by using equation (8) the plume height was 40 meter.

In Figure (1), the plume heightwas obtained as a function of downwind distance for some value of stratification parameter L. The advantage of this method is that it is possible to obtain height of plume as a function stratification parameter L, surface roughness z_o and downwind distance from the source x [12].

The steady state transport of radioactive contaminants released from a point source is described by the following partial differential equation:

$$u(z)\frac{\partial c}{\partial x} = \frac{\partial}{\partial z}\left(K(z)\frac{\partial c}{\partial z}\right) \quad (9)$$

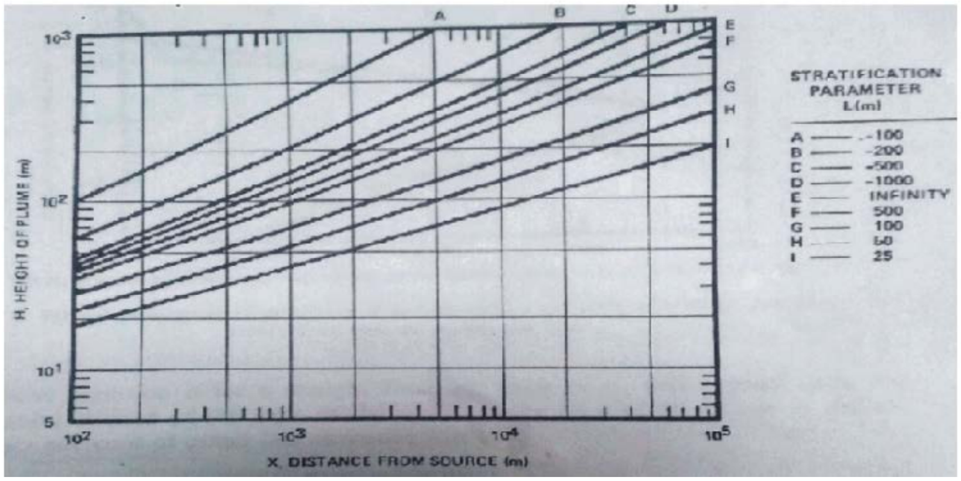

Fig. 1: Height of the plume, H, vs downwind distance, X, for surface roughness, z_o= 0.1 m.

Subject to the boundary conditions

1- $C \rightarrow 0$ at $x \rightarrow \infty$

2- $K\dfrac{\partial c}{\partial z} \rightarrow 0$ at $z \rightarrow 0$

3- $Q = \int_0^x ucdz$ (mass continuity)

Let the concentration profile in the form:

$$C/C_o = 1 + a_1 (z/H) + a_2 (z/H)^2 + \quad (10)$$

where:

C_o Is the concentration at the axis of the plume (Bq/m^3)

C Is the concentration at distance (z) among from the plume axis (Bq/m^3).

H Is the plume height (m).

Q Is the source strength (Bq) and

a1, a2,….are constants.

$$C/C_o = 1 + a_1 (z/H) \quad (11)$$

The above equation is a straight line. The value of a_1 will depend on the concentration at the edge of the plume. If the edge of the plume is defined as having (r) percent of outer line concentration, then

$0.01r = 1 + a_1 \rightarrow a_1 = 0.01r - 1$, if $r = 0 \rightarrow a_1 = -1$

$$\therefore C/C_o = 1 - (z/H) \quad (12)$$

Proposed Model: Substituting from eqn. (3) and (12) in eqn. (1), we get:

$$Q = \int_0^H (\frac{u_*}{K_o}\left(\ln\left(\frac{z}{z_o}\right) + \frac{5z}{L}\right) C_o \left(1 - \frac{z}{H}\right) dz \quad (13)$$

$$Q = \frac{\bar{u}_*}{K_o} C_o \int_0^H \left(\ln\left(\frac{z}{z_o}\right) dz - \frac{z}{H}\ln\frac{z}{z_o} dz + \frac{5z}{L} dz - \frac{5}{LH} z^2 dz\right) \quad (14)$$

$$Q = \frac{\bar{u}_*}{K_o} C_o \int_0^H \ln\left(\frac{z}{z_o}\right) dz - \frac{1}{H}\int_0^H z \ln\frac{z}{z_o} dz + \frac{5H^2}{2L} - \frac{5H^2}{3L} \quad (15)$$

$$Q = \frac{\bar{u}_*}{K_o} C_o \int_0^H \ln\left(\frac{z}{z_o}\right) dz - \frac{1}{H}\int_0^H z \ln\frac{z}{z_o} dz + \frac{5H^2}{6L} \quad (16)$$

- $\int_0^H \ln\left(\frac{z}{z_o}\right) dz$

Let $\ln z/z_o = x \rightarrow e^x = z/z_o \rightarrow e^x dx = dz/z_o$

At $z=0 \rightarrow \ln 0 = x \rightarrow x = -8$

- $\int_0^H \ln\left(\frac{z}{z_o}\right) dz = \int_{-\infty}^{\ln H/z_o} z_o x e^x dx = z_o \int_{-\infty}^{\ln H/z_o} x de^x$

$$= z_o\left[(xe^x)^{\frac{\ln H}{z_o}} - \int_{-\infty}^{\frac{\ln H}{z_o}} e^x dx\right] = z_o\left[\frac{H}{z_o}\ln\frac{H}{z_o} - (e^x)^{\frac{\ln H}{z_o}}\right]$$

$$= H \ln H/z_o - z_o H/z_o = H \ln H/z_o - H \quad (17a)$$

- $-1/H \int_0^H z \ln\frac{z}{z_o} dz$

Let $\ln z/z_o = x \rightarrow z/z_o = e^x \rightarrow z = z_o e^x$, $dz = z_o e^x dx$

At $z = 0 \rightarrow x = \ln 0 = -\infty$

At $z = H \rightarrow x = \ln H/z_o$

- $-1/H \int_0^H z \ln\frac{z}{z_o} dz = -z_o/H \int_{-\infty}^{\ln\frac{H}{z_o}} x\, e^x z_o e^x dx = -z_o^2/H \int_{-\infty}^{\ln\frac{H}{z_o}} x\, e^{2x} dx$

$$= -z_o^2/2 \int_{-\infty}^{\ln\frac{H}{z_o}} x\, de^{2x} = z_o^2/2H\left[(xe^{2x}) - \int_{-\infty}^{\ln\frac{H}{z_o}} e^{2x}\, dx\right]$$

$$= -z_o^2/2H\left[\ln\left(\frac{H}{z_o}\right) e^{2\ln\frac{H}{z_o}} - \int_{-\infty}^{\ln\frac{H}{z_o}} e^{2x}\, dx\right] = -z_o^2/2H\left[\ln\frac{H}{z_o} e^{2\ln\frac{H}{z_o}} - \frac{e^{2\ln\frac{H}{z_o}}}{2}\right]$$

$$= -\frac{H}{z}\ln\left(\frac{H}{z_o}\right) + \frac{z^2}{2H}\left(\frac{H^2}{2z_o^2}\right) = -\frac{H}{2}\ln\left(\frac{H}{z_o}\right) + \frac{H}{4} \quad (17b)$$

By substituting in (16) we get:

$$Q = \frac{\bar{u}_*}{K_o} C_o\left[H\ln\left(\frac{H}{z_o}\right) - \frac{H}{2}\ln\frac{H}{z_o} - 3H/4 + \frac{5H^2}{6L}\right] \quad (18)$$

$$Q = \frac{\bar{u}_*}{K_o} C_o\left[\frac{12\, H\ln\left(\frac{H}{z_o}\right) - 18LH + 20H^2}{24L}\right] \quad (19)$$

$$\therefore C_o = \frac{24LQK_o}{2Hu_*(6L\ \ln\left(\frac{H}{z_o}\right) - 9L + 10H^2)} \quad (20)$$

$$\therefore C_o = \frac{12LK_oQ}{Hu_*(6L\ \ln\left(\frac{H}{z_o}\right) - 9L + 10H^2)} \quad (21)$$

Substituting from equation (21) in equation (12) we get:

$$C = \frac{4.8\, LQ}{Hu_*(6L\ \ln\left(\frac{H}{z_o}\right) - 9L + 10H^2)}\left(1 - \frac{z}{H}\right) \quad (22)$$

For infinite point source located at a height H from the ground, equation (22), for radionuclides concentration at ground will be:

$$C = \frac{4.8\, LQ}{Hu_*(6L\ \ln\left(\frac{H}{z_o}\right) - 9L + 10H^2)}\left(1 - \frac{z}{H}\right) e^{-\lambda x/u} \quad (23)$$

where, $e^{-\lambda x/u}$ is the radioactive decay term for the specified radionuclide.

RESULTS AND DISCUSSION

Dispersion models generally require steady and horizontally homogeneous hourly surface and upper air meteorological observations. However, 15 minutes and hourly meteorological observations are available for the study area. 15 minutes average wind speed at 10 & 60 meter height was calculated. The representative value of L, the stratification parameter of this study was calculated as 400 m. The average surface roughness for the study area was about 0.1 m. The plume height was calculated to be 40 m. The distance of the observed concentration was 100 m. Table (2) shows Source strength (Bq) and decay constants for the studied fission radionuclides.

Table (3) shows the predicted concentrations due to I-131, I-133 and Cs-138.Results show that the present model is performing well with the observations and can be used to predict the short-range dispersion from a point source release.

For the purposes of verification, the source strength can be adjusted according to the predicted concentrations as shown in equation (24).

Table 2: Source strength (Bq) and decay constants for the studied fission radionuclides.

Exp.	Wind speed (m/s)	I-131	I-133	Cs-138
1	3.1	1347091	1222609	143062
2	2.8	26636	26630	312
3	3.3	21309	2131	249
4	1.9	143836	14383.6	16831
λ		$9.95*10^{-7}$	$9.25*10^{-6}$	$3.8*10^{-4}$

Table 3: Predicted & observed concentrations due to I-131, I-133 and Cs-138

I-131 concentration			I-133 concentration			Cs-138 concentration		
Observed	Predicted	O/P	Observed	Predicted	O/P	Observed	Predicted	O/P
2.4000	1.4708	1.6318	1.4900	1.3345	1.1165	0.1850	0.1543	1.1990
0.0510	0.0291	1.7537	0.0360	0.0291	1.2386	0.0006	0.0003	1.8747
0.0580	0.0233	2.4930	0.0044	0.0023	1.8916	0.0009	0.0003	3.3116
0.2500	0.1571	1.5920	0.0230	0.01570	1.4653	0.1890	0.1800	1.0500

Table 4: Corrected Q for different radionuclides

Experiment	Corrected Q (I-131)	Corrected Q (I-133)	Corrected Q (C_s-138)
1	2198191	1365074	171528
2	46711.72	32982.7	584.891
3	531228.5	4031.02	824.577
4	228982.9	21075.58	17659.9

$$Q_{cal} = \frac{C_{obs}*Q_{obs}}{C_{cal}} \quad (24)$$

Table (4) shows the corrected Q for different radionuclide's.

CONCLUSION

It is concluded that the present model shows good performance with the observed data and can be used to investigate the short-range dispersion of pollutant. In addition, it can be deduced that the suggested model is able to more accurately reproduce the concentrations closer to the source (higher concentrations), where turbulence is based on superficial scales. It can be concluded that the good parameterizations of input data such as wind speed and plume height leads to well perfection in the agreement between computed and observed concentrations.

REFERENCES

1. Davidson Martins Moreira, Antônio Gledson Goulartand Marcelo Romero De Moraes. 2013. A model Employing Integral Transform Method to Simulate Pollutant Dispersion in Atmosphere, RevistaBrasileira De Meteorologia, 28(4): 373-381.
2. Essa, K.S.M. and S.E.M. Elsaid, 2015. Estimation of the Maximum Concentration for Non-Gaussian Under Using Different Schemes of Dispersion Parameters for Isotopes, J. Civil & Environmental Engineering, 5(4): 1-6.
3. John M. Stockie, 2011. The Mathematics of Atmospheric Dispersion Modeling, Society for Industrial and Applied Mathematics, 53(2): 349-372.
4. Buske, D., M.T. Vilhena, T. Tirabassi and B. Bodmann, 2012. Air pollution steady-state Advection diffusion equation: the general three-dimensional solution, JEP, 3: 1124-1134.
5. TizianoTirabassi, Alessandro Tiesi, Marco T. Vilhena, Bardo E.J. Bodmann and Daniela Buske, 2011. An Analytical Simple Formula for the Ground Level Concentration from a Point Source, Atmosphere, 2: 21-35.
6. Camps, J., C. Turcanu, D. Braekers, G. Olyslaegers and J.E.A. Paridaens, 2010. The 'NOODPLAN' early phase nuclear emergency models: an evaluation, Helsinki, Finland, 14-18 June 2010 / IRPA.
7. Pramod Kumar and Maithili Sharan, 2010. An analytical model for dispersion of pollutants from a continuous source in the atmospheric boundary layer, Proc. R. Soc. A, 466: 383-406

8. Pramod Kumar and Maithili Sharan, 2014. A Generalized Analytical Model for Crosswind-Integrated Concentrations with Ground-Level Deposition in the Atmospheric Boundary Layer, Environ Model Assess, 19: 487-501.
9. Anikender Kumar, P. Goyal, 2014. Air Quality Prediction of PM10 through an Analytical Dispersion Model for Delhi, Aerosol and Air Quality Research, 14: 1487-1499, Taiwan Association for Aerosol Research.
10. Glênio A. Gonçalves, Regis S. de Quadros and Daniela Buske, 2013. An Analytical Formulation for Pollutant Dispersion Simulation in the Atmospheric Boundary Layer, Journal of Environmental Protection, 4: 57-64.
11. Davidson M. Moreiraa,_, TizianoTirabassib and Jonas C. Carvalhoa, 2005. Plume dispersion simulation in low wind conditions in stable and convective boundary layers, Atmospheric Environment, 39: 3643-3650.
12. Sharma V. and L.O. Myrup, 1975. Diffusion from A line Source in an Urban Atmosphere, Atmospheric Environment 9: 907-922, Pergamon Press, Britain.
13. Alan J. Cimorelli, Steven G. Perry, Akula Venkatram, Jeffrey C. Weil, Robert J. Paine, Robert B. Wilson, Russell F. Lee, Warren D. Peters, Roger W. Brode and James O. Paumier, 2004. AERMOD Description of model Formulation, U.S. Environmental Protection Agency, North Carolina.
14. Maren Haid, 2013. Experimental analysis of the Monin-Obukhov similarity theory in the lower surface layer, EberhardKarlsUniversitätTübingen, Bachelor thesis, Germany.
15. Garratt, J.R., 1994. The atmospheric boundary Layer, Quarterly Journal of the Royal Meteorological Society, 120(519): 1428-1429.
16. Alfredo Pena, Sven-Erik Gryning and Jakob Mann, 2010. On the length-scale of the wind profile, Quarterly Journal of the Royal Meteorological Society, 136: 2119-2131.
17. Robert Macdonald, 2003. Theory and objectives of Air Dispersion modeling, Modelling Air Emissions for Compliance, MME 474A, Wind Engineering, Canada.
18. Andrea Venora, 2009. Monin-Obukhov Similarity Theory Applied to Offshore Wind Data Validation of Models to Estimate the Offshore Wind Speed Profile in the North Sea, Delft University of Technology, master of science thesis, Holland.
19. Alex De Visscher, 2014. Air Dispersion Modeling, foundation and Application, John Wiley & Sons, USA.
20. Kukkonen, J., J. Nikmo1, M. Sofiev1, K. Riikonen1, T. Petäjä, A. Virkkula1, J. Levula, S. Schobesberger and D.M. Webber, 2014). Applicability of an integrated plume rise model for the dispersion from wild-land fires, Geoscintific. Model Development, 7: 2663-2681.