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Transforming Mathematics Teaching in Pakistan: A Case of Renewed Teacher Education Programme

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Abstract: An initial study of the effect of teacher cognitive performance as a predictor of student achievement in mathematics at Grade VIII level in Pakistan suggested that teaching methodology is located at Douady's second *didactical contract level* where both teachers and students accept the content of mathematics without challenge. In this follow-up study involving quantitative data and qualitative responses of 32 teachers from an urban area, an attempt has been made to explore what teaching methodologies teachers employ during mathematics teaching in Pakistan. An analysis of questionnaire responses to mathematics content items has defined the upper quartile of respondents as *High Content Masters*. An atheoretical factor of *Homework-led teaching* encompasses all the teachers sampled and gives evidence for the significance of low-level transmissive teaching. The *High Content Masters* alone conceptualise additionally a more student-centred methodology operating at a 'Bloom' application level. It is hypothesised that teachers can be placed along a *Mastery continuum* according to their initial training, experience and in-service professional development. Practical suggestions are made to move teaching and learning on to Douady's *fourth level*, where knowledge is of central importance to both teacher and students and need to be negotiated and explored. To achieve this transformation, there is a need to revisit and redesign the teacher education programme, both pre-service and in-service.

Key words: Achievement in Mathematics · Teacher's Influence · Transmissive Teaching · Student Centred Teaching

INTRODUCTION

Importance of mathematics and its teaching at school level hardly needs an argument. Men of all ages have underscored its significance. Galileo Galilee is reported to highlight the importance of mathematics in the following words. "The great book of nature can be read only by those who know the language in which it was written. And that language is mathematics" [1]. According to Smith [2], mathematics is of central importance to modern society. It is not only essential in the physical sciences, technology, business, financial services and many areas of ICT but also of growing importance in biology, medicine and many of the social sciences. It is in realisation that mathematics has been made as essential component of school curricula in every system of education across the globe.

Mathematics is one of the three major subject areas surveyed regularly by the Organisation for Economic Co-operation and Development's (OECD) Programme for International Student Assessment (PISA). The report of a recent survey of mathematics teaching to 15 year olds in 41 countries refers to the need to develop mathematical skills among youth and adolescents in the following words.

"All adults,....now require adequate mathematics proficiency for personal fulfilment, employment and full participation in society....Students about to leave

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compulsory education should thus have a solid understanding of these concepts and be able to apply them to solve problems that they encounter in their daily lives” [3].

The Teaching of Mathematics: Research evidence shows that teachers’ beliefs about mathematics affect the way they teach [4, p:186]. Pakistan is predominantly a Muslim society. The socio-religious culture of Islam can give unquestioning authority to the written word in the book [5, 6]. This environment would appear to condone the transmissive actions of teachers. Yet, the teaching of Islam did urge the individual to be curious and reason about the knowledge to be found in the natural world [7, p:87], and the development of mathematics has a unique place in the history and culture of Islam [8, 9]. The Pakistani mathematics textbook, like those elsewhere, nevertheless tends to treat the subject content as having no history and having been handed down as a permanent body of knowledge [10, p:29]. Thus Pakistani teachers naturally adopt the traditional transmissive approach to mathematics teaching. Douady [11, p:377] sees the learning in mathematics classrooms subject to one of four constraining procedures or *didactical contracts*. In the second of these contracts neither the teachers nor the students debate or explore the meaning and value of mathematics, but proceed through prescribed, repetitive exercises to course and examination completion, both parties having 'survived the experience'. There is evidence that this *second contract* approach is widespread and not limited to Pakistan [12, 13, 14].

The Learning of Mathematics: Reflecting on the state of mathematics education research Niss [15, p: 1303] refers to the complexity of mathematics learning: 'there is no such thing as a well established, unifying theory, supported by a majority of researchers'. Such a state of affairs allows certain flexibility in approach to addressing the focus of this paper on the impact of the teacher on teaching mathematics in Pakistan.

Hiebert and Grouws [13] distinguish between teaching for skill efficiency, which makes use of lower level cognitive skills and learning 'drill' and the teaching for conceptual understanding, where the student has to 'struggle' with concepts in order to assimilate them internally. Lower level learning in mathematics can be assisted by the concrete presentation of concepts and their linking in some hierarchical manner [16]. In an extensive review of the research with concrete learning stimuli, Johnson [17] refers to the increase in achievement, improved understanding, increased flexibility in thinking and improved attitudes following the long term use of concrete learning aids. However, models do not always succeed in all classroom situations in revealing mathematical 'meaning' [18, 13].

The real problem in mathematics learning appears to be how to make the transition from concrete to abstract knowledge. Gravemeijer [18] expands the work of Freudenthal [19] to advocate *realistic mathematics*, where the everyday, informal and hence concrete experiences of the students are described; generalised under teacher guidance; formalised using the learning already assimilated at the lower cognitive levels; expressed as algorithms, theorems and higher order concepts. Once this learning has been mastered, it is not necessary to repeat the’re-invention' of the higher order knowledge but simply apply it in problem-solving. As Gravemeijer points out this is guided discovery learning of an active mathematics, which is 'bottom-up' and allows students to see how mathematics came to be what it is today, rather than a 'revealed truth'. *Realistic mathematics* learning can be seen as operationalizing Hiebert and Grouws 'struggle' process in acquiring conceptual understanding at a higher level. Writing from a United States perspective in 2007, the two authors report that low-level 'recitation' teaching is still commonly employed (pp.392). There is no conceptual 'struggle'. The reasons given are particularly appropriate to teaching and learning in Pakistan; a static culture; teachers lacking sufficient subject and pedagogical knowledge to make any change; limited teacher professional development and 'high-stake' school testing systems [13, p:393].

The Present Research: In the first reported phase of this research into the significance of the mathematics teacher in student achievement [20], five research questions were posed. Three of these were answered in the earlier report. The fourth and fifth questions are:

- Do teacher attitudes support the hypothesis that Pakistani mathematics teaching is essentially a transmission of low-level knowledge?
- If the hypothesis of question 4 is supported, how can mathematics teaching and learning be moved to a higher level?

These questions are the remit of the present paper and their formulation is to some degree a consequence of the findings of the first research phase.

Research Methods and Procedure: The earlier work [20] was not a conventional research instrument in the sense that it set out to test a series of explicit hypotheses about the teaching of mathematics. Measures of teaching methodology within the survey questionnaires were superficial, so the survey itself was unable to address pedagogical and theoretical issues necessary for a formative development. Thus, to supplement the conclusions drawn from quantitative analysis of large scale data regarding teachers' influence on students achievement in math, it was decided to collect some qualitative information to explore the issue a little further.

Consequently, an opportunity sample of 32 Grade VIII mathematics teachers was sought from eight schools, both boys and girls within Lahore, a relatively more prosperous urban city of Pakistan. A larger, more nationally representative sample was not aimed for in this stage as the research was judged as exploratory. Should evidence emerge supporting the hypothesis of question 4, it was intended to mount an in-depth study before 'rolling out' a provincial development plan.

Teacher attitude data was collected by means of a self-administered questionnaire completed anonymously at the schools by the teachers. The questionnaire included items on Likert scales addressing mathematical content from the perspectives of the teacher understanding, student understanding and easiness to teach. A second major section required responses to statements of teaching methodology. A final open-ended section asked for views on 'The major problem in teaching mathematics at Grade VIII'. Questionnaire items are presented in the analysis tables in the results section.

RESULTS

Grade VIII Content: Responses to the first section of the teacher questionnaire were subjected to oblique rotation factor analysis to establish the three uni-dimensional scales of *Teacher Understanding of Grade VIII Content*, *Student Understanding of Grade VIII Content* and *Easiness of Teaching Grade VIII Content*. Oblique rotation is recommended for axis rotation in factor analysis investigations, because of the likely inter-correlation of attitudinal constructs [21]. After Alpha maximisation of scales constructed from extracted items [22], the scales were re-factorised to test for uni-dimensionality as shown by just a single factor solution [23, 24]. This final procedure adds statistical validity to reliability [25, 23].

Teacher Understanding of Grade VIII Content: The ranking list of the nine content topics of Grade VIII appearing in Table 1 is significant at $p<1\%$ and the effect size (calculated from the internal chi-square of the Friedman statistic) is large. *Algebra* is the most understood topic and *rational numbers,* although receiving a 'good' rating is much less so.

Factor analysis of the nine teacher ratings shows a single factor accounting for 66.9% of the total item variance. The computed scale of *Teacher understanding* has an Alpha reliability 0.93 (Table 1).

Student Understanding of Grade VIII Content: Table 2 shows the teachers' rating of their perceptions of students' understanding of the topics of the Grade VIII course on the ten-point rating scale. A rating of '5' indicates neither easy nor difficult. *Number systems of base 2 and 5* is rated the easiest of the topics and rational *numbers* the most difficult. The ranking list is significant at $p<5\%$ and the effect size (calculated from the internal chi-square of the Friedman statistic) is large.

A very strong factor accounting for 58.0% of the total item variance, but the topic of algebra is relatively weak and is the source of the accompanying minor factor. This suggests that algebra has specific characteristics in its assimilation not shared with the other topics. The computed scale of *Student understanding* (Table 2) of Alpha reliability 0.91 can include the algebra topic without loss in reliability but then loses some validity as it is no longer uni-dimensional.

Easiness of Teaching Grade VIII Content: Table 3 shows the teachers' rating of their ease in teaching the topics of the Grade VIII course on the ten-point rating scale. A rating of '5' indicates neither easy nor difficult to teach. *Algebra* is rated the easiest of the topics to teach and *rational numbers* the most difficult. However, it should be noted that the rating of *rational numbers* at 6.47 still places this topic in the 'easy' rather than 'difficult' category. The ranking list is significant at $p<1\%$ and the effect size (calculated from the internal chi-square of the Friedman statistic) is large. Despite *rational numbers* having the lowest mean rating in Table 3, the more appropriate non-parametric Friedman statistical test for testing ranks shows that *proportion* is the least easy topic to teach.

Table 1: Teacher understanding of mathematics content scale (N=32).

My understanding of this topic overall (10=Excellent, 1=very poor)	Corrected Item-Total Correlation	Item mean	Std. dev.
1. Algebra	0.71	8.84	1.27
2. Geometry	0.78	8.16	1.65
3. Information handling	0.76	7.88	2.00
4. Number systems of base 2 and 5	0.61	8.34	1.77
5. Percentages	0.86	8.22	1.79
6. Proportion	0.83	8.22	1.98
7. Rational numbers	0.70	7.34	2.35
8. Sets	0.74	8.22	1.93
9. Square roots	0.87	8.22	1.91

Table 2: Student understanding *of mathematics content* scale (N=32)

Students' ability to understand (10=very easy, 1=very difficult) N=32	Corrected Item-Total Correlation	Item mean	Std. dev.
1. Algebra	0.48	5.94	1.54
2. Geometry	0.66	5.47	1.98
3. Information handling	0.68	5.41	2.27
4. Number systems of base 2 and 5	0.70	6.25	1.83
5. Percentages	0.80	5.53	1.87
6. Proportion	0.81	5.50	1.72
7. Rational numbers	0.61	4.81	2.07
8. Sets	0.67	5.94	1.83
9. Square roots	0.76	5.88	2.32

Table 3: Easiness of teaching scale (N=32)

Teaching this topic (10=very easy, 1=very difficult)	Corrected Item-Total Correlation	Item mean	Std. dev.
1. Algebra	0.88	7.63	2.23
2. Geometry	0.93	7.06	2.71
3. Information handling	0.91	6.75	2.72
4. Number systems of base 2 and 5	0.88	7.06	2.29
5. Percentages	0.93	6.72	2.59
6. Proportion	0.94	6.56	2.68
7. Rational numbers	0.79	6.47	2.81
8. Sets	0.88	7.06	2.32
9. Square roots	0.90	6.84	2.77

Factor analysis of the nine teacher ratings shows a very strong, single factor accounting for 84.0% of the total item variance. The computed scale of *Easiness of teaching* (Table 3) has the high Alpha reliability 0.98.

Teacher Gender and Experience: Questionnaire data provided the teacher's gender and experience in years teaching Grade VIII mathematics. Experience ranged from one to 24 years with a median of three years. Those with less than three years were classified as *inexperienced*.

The first phase of this research had pointed out the significant effect of gender on teacher cognition [20]. Teacher experience might be expected to mediate the *second level* of Douady's concept of the *didactical contract* between the teacher and learner in mathematics [11]. Consequently, the three scale scores were investigated by two-way analyses of variance with respect to teacher gender and experience to show that;

- *Teacher Understanding* has a significant gender effect with female teachers scoring lower ($F=9.191$, $df=1$, $p<1\%$, large effect size);
- *Student Understanding* has a significant variation with teacher experience ($F=6.069$, $df=1$, $p<1\%$, large effect size), where the more experienced teachers rate student understanding higher and

- *Easiness of Teaching* has significant two-way effects for both experience (F=6.751, df=1, p<5%, large effect size) and gender (F=25.704, df=1, p<1%, large effect size) Experienced male teachers rate teaching easiest.

Teacher Content Mastery: The three Content scales are all highly significantly inter-correlated. Factor analysis shows a single factor accounting for 80.4% of variance of the three scale totals. Reliability analysis shows that *Student Understanding* adds nothing to the other two scales. The single factor comprising *Teacher Understanding* and *Easiness of Teaching* generates a scale of *Teacher Content Mastery* of Alpha reliability 0.86. A two-way analysis of variance of *Teacher Content Mastery* scores shows significant gender (F=20.286, df=1, p<1%, large effect size) and experience variation (F=5.874, df=1, p<5%, large effect size).

Teachers are ranked on their scores on this scale and those in the upper and lower quartiles identified. The upper quartile comprises the *High Content Masters* who are exclusively male. The quartile of lowest scorers are exclusively female (p<1%, chi square, large effect size). Highest scorers are most experienced in the ratio of 7:1. Lowest scorers are least experienced in the ratio of 6:2 (p<5%, chi square, large effect size).

Teaching Methodology: In the second section of the questionnaire, the 18 items of Table 4 describing aspects of mathematics teaching methodology were scored by the respondents on a scale of 1 *Not at all important* to 3 *Very important* prior to factor analysis. The large number of factors which emerged, eight in all accounting for 81.6% of the total item variance, suggests that the teaching methodology lacks a substantial theoretical base and consists of teachers meeting a range of largely psychologically disconnected, though traditional, procedures.

Mean scores show that asking questions (item 4), revising topics (item 8), using the chalkboard (item 11) and attending in-service courses (item 18) approach *Very important* ratings. Using non-textbook sources (item 17) is not at all popular. Although the value of using mathematical models/teaching aids (item 12), marking homework regularly (item 15) and applying learning in a new situation (item 3) are all recognised as being of some worth, these aspects are not rated very importantly.

Nevertheless, the strongest factor of *Homework-led teaching* (18.0% of the total item variance) does have some face as well as statistical validity with an acceptable reliability of 0.76 for a unidimensional, five-item scale [22]. The items of this scale appear in heavy type in Table 4. *Homework-led teaching* methodology is a pragmatic definition of mathematics teaching in terms of interesting lessons where use of the chalkboard leads to the regular setting of homework. Returned homework is marked, presumably to give feedback to both teachers and students and weekly testing consolidates the content imparted. Scores on *Homework-led teaching* show an association with a close study of the textbook (item 6, medium effect size).

Table 4: Mean score responses to 18 teaching methodology items

In the teaching of mathematics, how important is it that....	Mean (N=32)	Std. dev	Correlations with Homework-led teaching
1. The students understand what they are Learning	2.50	0.57	0.01
2. The students memorise what they are Learning	2.53	0.57	0.26
3. The students can apply their learning to a new situation	2.34	0.60	0.00
4. The students ask questions of the teacher when learning	2.81	0.47	- 0.05
5. The students discuss the topic with each other when learning	2.41	0.76	0.29
6. The textbook is followed closely.	2.38	0.75	0.39*
7. All the content in the textbook is Covered	2.56	0.67	- 0.16
8. The topics are often revised by the teacher to refresh students' learning	2.72	0.47	- 0.02
9. The students are tested on their learning every week	2.50	.0.57	0.42 *[a]
10. The students are tested on their learning at the end of each topic.	2.56	0.56	0.24
11. The chalkboard is used	2.72	0.46	0.54 **[a]
12. Mathematical models and other teaching aids are used	2.31	0.59	- 0.01
13. The lessons are made interesting	2.59	0.62	0.53 **[a]
14. Homework is set regularly	2.56	0.67	0.70 **[a]
15. Homework is marked regularly	2.34	0.65	0.48 **[a]
16. The teacher consults with other staff members about teaching mathematics	2.41	0.67	0.08
17. The teacher uses other non-text book material to help the students learn	1.66	0.65	0.32
18. The teacher attends in-service training Courses	2.69	0.47	0.07

Homework-led teaching items in heavy type

[a] correlation with scale total when this item score is removed

**p<1%, *p<5%

Table 5: Expansive mathematics as defined by *High Content Masters*

Item	Corrected Item-Total Correlation	Item mean	Std. dev.
4. The students ask questions of the teacher when learning.	0.97	2.88	0.35
5. The students discuss the topic with each other when learning.	0.95	2.75	0.71
7. All the content in the textbook is covered.	0.97	2.88	0.35
12. Mathematical models and other teaching aids are used.	0.80	2.50	0.76

High Content Masters are significantly stronger on *Homework-led teaching* than are the low scoring quartile (respectively, mean/item=2.85, s.d. =0.21, N=8 and mean/item=2.45, s.d.=0.32, N=8, p<1%, Mann-Whitney, large effect size).

Discriminant function analysis of the teaching items from Table 4 extracts those which can distinguish most effectively between the *High Content Masters* and the others. This can be done with 87.5% efficiency, using just the scores of *Homework-led teaching*; item 3, *The students can apply their learning to a new situation* and item 17 *The teacher uses other non-text book material to help the students learn*. Items 3 and 17 ground the teaching at the higher cognitive levels [26].

There is a family of highly correlated teaching items exclusively for the *High Content Masters* (Table 5). The items form a unidimensional factor accounting for 92.8% of the total item variance (Alpha=0.92, N=8). This factor might be termed *expansive mathematics*. Scores are significantly higher for the *High Content Masters* on this factor when compared with all the other teachers (respective statistics are; mean/item=2.75, s.d. =0.52, N=8 and mean/item=2.45, s.d. =0.34, N=24, p<1%, Mann-Whitney, large effect size). Although in making this comparison, 'other' teachers do not meaningfully conceptualise an expansive view of mathematics.

Response to Open-Ended Items: 'The major problem in teaching mathematics at Grade VIII'.

Seventeen of the 30 respondents completed this section. Five of the eight *High Content Masters* did so. This proportion, although high, does not differ significantly from the lower proportion of 'other' teachers.

The remarks of the *High Content Masters* distinguish this group and, at the same time, validate the hypotheses emerging from the quantitative analysis. Four of the five respondents of this group refer to the inadequacy of training of Grade VIII mathematics teachers and their consequent lack of competence to handle the concepts and the teaching of the content. The remarks of *High Content Master* 'H' are typical:

'The main problem in teaching mathematics is that teacher should update their knowledge, they should use appropriate method for teaching and they should know the level of students.'

Two other *High Content Masters* referred to student problems in handling what is to them difficult content because of the lack of any command of the subject from earlier years and the burden of a lengthy textbook, which seems to go nowhere with its discontinuous content.

Only one of the twelve 'other' teachers made any reference at all to teacher competence, teacher preparation or training. Instead, this group referred to the student problems in handling content, which they did not understand. Nine of this group specifically mentioned the perceived difficulty of the subject and three spelt out that this was because students did not understand what they were doing. For example:

Teacher, 'N'

'At this grade, mathematics suddenly becomes difficult. Formulas are very difficult to retain, only few students are able to efficiently grasp them.'

Teacher, 'S'

'Mostly children are unable to understand the formulas and concepts but some student are not able to learn.'

Teacher, 'M'

'(The) major problem faced by teachers lies in the delivery of the lesson because many students face problems in understanding mathematics.'

Teachers in this group point out the de-motivation of the subject as it becomes more difficult for the students.

Teacher, 'Z'

'Most of the children feel bored because it is difficult.'

Teacher, 'F'

'Students are at lower level of learning and burden of studies called upon them as a result they do not give their 100% effort. Solution is that, burden of studies must not be so high that they start hating their studies.'

These responses of the *'Non-Masters'* show that they have a more restricted view of teaching subject content. To these teachers the problems lie with the students' responses to the information their teachers are attempting to convey. The teachers appear to have only a minimal awareness of the cognitive preparation needed by the students to assimilate the mathematical knowledge, concepts and rules. This pedagogical deficiency has been remedied to some extent by the *Masters*, who are appreciative of a more professional approach to the methodology of teaching content.

DISCUSSION

1(4): Do teacher attitudes support the hypothesis that Pakistani mathematics teaching is essentially a transmission of low-level knowledge?

The analyses of Tables 1, 2 and 3 provide the content context. Teachers' understanding of content is moderately good, overall, although *Information handling* and *rational numbers* are more troublesome. About one third of the sample give themselves an excellent rating on any one item and six teachers rate their understanding as excellent on all items. The situation is quite different with the *Student understanding* scale. The mean values tend to be close to the neutral response, the content being judged neither difficult nor easy for the students, but the range of values is large from 2.75 (difficult) to 8.25 (easy) according to the teacher. The strong correlation of *Easiness of Teaching* and *Teacher Understanding* means that the rankings of teachers on each scale are similar, but the difficulties that some teachers have in conveying mathematics to their students appears in the lower means of Table 3, which range widely from 10 to 2.89.

The gender effects in the results of the three content scales are consistent with the earlier research presented in the first phase of the current study [20]. Female teachers score significantly below males on Grade VIII cognitive testing, so what emerges from this latest analysis is that the former acknowledge their 'inadequacies' in the patriarchal society and rank themselves accordingly, perpetuating a trend identified in Pakistani primary schools [27]. The positive effect on attitudes of being a more experienced teacher is most likely in Pakistan a case of 'learning on the job'. Teacher cognition in Grade VIII mathematics is poor and in-service education, where it exists, is ineffective [20]. So as a teacher gains experience, familiarity with the text, difficulties encountered in past teaching and learning with earlier cohorts of students, this all builds up a pragmatic teaching approach which achieves a measure of recognisable success in the system. Hence the appearance of the *High Content Masters.* Despite the relatively poor cognitive achievements of female teachers, there is no fundamental reason why they should be absent entirely from the *Masters* classification and a larger sample of teachers almost certainly would have a modest female representation.

The almost ad-hoc teaching approach in the mathematics classroom is conveyed by the single coherent methodology factor of *Homework-led teaching* of Table 4. It is easily seen how mathematics learning follows a ritual of teaching content through the chalkboard; homework; feedback through checking (assessing) of homework and weekly testing. These tasks are made as interesting as possible, but the textbook is always near at hand. This learning cycle would have been refined over the years by the more experienced teachers and it is no surprise to see the superiority of the *High Content Masters* on this measure.

The existence of *Homework-led teaching* as the most integrated element of Pakistani mathematics is taken as strong support for the hypothesis that teaching is a transmission of low-level knowledge and that Douady's *second didactical contract* where, mathematics is not negotiated or explored but accepted as prescribed, is operating within the classrooms of this sample [11, p: 377]. It was pointed out in the Introduction to this paper that, internationally, a majority of students will be experiencing transmissive methods [28]. In the sociology of a predominantly Muslim society like Pakistan, such an approach is culturally acceptable and is characteristic of Pakistani teachers [22, 32]. That teachers are prepared to exist in such a comfort zone is not unusual, even in the international context, as it minimises the work that has to be done and eliminates the risk that goes with using initiative [31].

2(5): If the hypothesis of question 4 is supported, how can mathematics teaching and learning be moved to a higher level?

By analysing the responses of the *High Content Masters* as presented in Table 5, the associated discriminant function analysis and the open-ended comments, a logical pathway to a higher level of mathematics learning opens up. It is suggested that Pakistani teachers begin their careers as exponents of traditional transmissive mathematics, seeing failure as a student-centred inability to grasp the conceptual message. Although *Homework-led teaching* gives a holistic summary of the teachers approach to their task, they are not ignorant of the other teaching techniques of Table 4, rating them all helpful with the exception of item 17 'relying solely on the textbook'. Questioning, understanding, applying and discussing are not rejected, but are not assimilated into a substantial approach or theory of what they should be doing. Nevertheless, the concepts are there, which can form a foundation for education.

It appears that on-the-job experience can, for some teachers, stimulate the growth of a more expansive view of managing the students' learning. Building on a solid base of *Homework-led teaching,* the *High Content Masters* move teaching and learning to a higher cognitive level by fostering 'application' learning and a creative approach to the textbook. The *Masters* start to draw together the discrete elements of teaching/learning in the factor of *Expansive mathematics* (Table 5), but it should be noted that this progression still allows for a comprehensive study of the textbook material. As the *Masters* reflect upon their professional behaviour, they can see the errors perpetuated by inexperienced teachers and those happy to exist in the comfort zone of just having a job.

Not all experienced teachers progress to the *High Master* stage. Of 17 teachers defined as experienced in the sample, two remained at the Low stage and eight occupied the Intermediate stage. On the other hand, one inexperienced teacher was already at the *High Master* stage, which supports the idea that teachers progress along a *Mastery continuum* at a rate determined by their academic preparation, the quality of the pre-service education they receive as well as any in-service interventions.

Although in this research, none of the *High Master*s are female, internationally, females are considered less likely than males to see teaching as the direct transmission of knowledge [32, p: 88]. From this, one would expect female mathematics teachers to progress more rapidly along the *Mastery continuum*. The reason that this has not happened in the present research is either because of the sample statistics or because in a male-dominated society the 'starting-point' is shifted [20].

To re-orient teacher education in mathematics away from direct transmission of content and an over-reliance on memory, the 2009 OECD [32] report suggests that an approach could be to use generic strategies of problem solving. This returns the discussion to the current state of research. Given that 'there is no such thing as a well-established, unifying theory...supported by a majority of researchers' [15, p:1308], it is recommended that teachers and learners are moved out from Level 2 of Douady's *didactic contract* on to Level 4, where knowledge is of central importance to both teacher and student to be negotiated and explored [11, p: 377]. This would be achieved at both the initial and in-service stages of teacher education by addressing the concept of *realistic mathematics education* [18], drawing upon Freudenthal's view of mathematics as the activity of solving problems and organising subject matter [19]. Working in Lahore, Pakistan, Pell, Iqbal and Sohail [33] have shown that guided-discovery science can be effectively deployed with rote-learners, so there is no cultural obstacle to introducing this mildly constructivist approach to mathematics teaching.

For curriculum reform to be effective, arguably there has to be input from the bottom of the system as well as the top. The quality of education at all levels in Pakistan is such that teachers or Government administrators are unlikely to be the agents of change required [34, 35, 29]. As Hallak [36] has pointed out, staged planned curricular reforms should be modest and consolidated before moving on. There is a history of too many ambitious projects funded by international donors proving unsuccessful [36, p: 256, 37]. It is suggested that a phased introduction of Teacher Professional Centres be set up, starting with the more urban areas of the relatively prosperous parts of the country. These Centres would be supervised by capable mathematics staff from the university Departments of Education or the Institutes of Education and Research, who by providing mathematics re-education off-campus would be present an in-service input to the *Mastery continuum.* The same staff would parallel their change agent roles by driving reform on-campus at the initial teacher education stage [38, 39].

CONCLUSIONS

This research has provided evidence that the sample of Pakistani mathematics teachers adopt an essentially, intellectually un-contested approach to mathematics where textbook exposition is reviewed by homework and weekly testing evaluation. This approach to mathematics teaching, where retention is through memorisation rather than understanding, is especially prevalent in Less Developed Countries. It is negatively associated with performance [28, 3] and is not helping Pakistan's scientific and technical wellbeing.

Small scale studies such as this provides data that deepens understanding of the teacher's role in conveying the essence of mathematics [13, p: 397]. The first phase of the research [20] identified the significance of teacher cognition in student performance. This second stage was, in its conception, judged as exploratory. It is considered essential that a large scale province-wide study should now be mounted to generalise and extend the findings of the present research. As it is, a surprisingly coherent picture has been revealed. Transmissive teaching of mathematics under the Douady *second didactical contract* is certainly present, but for some teachers a natural evolution towards a more expansive view of learning and teaching does take place along a *Mastery continuum* as the classroom years go by. However, without planned change agent intervention to provide the *High Content Masters* with the confidence to innovate and modify current practices, the expertise of this small group of teachers will remain locked within their classrooms and a rigid, monolithic system.

The key to transform mathematics teaching lies within the teacher education programme in Pakistan (as elsewhere), both pre-service and in-service. It is there that not only the course material for problem -solving *realistic mathematics* has to be developed, but more effective pedagogies of mathematics teaching as well. It is from there that the push toward the classrooms of the *fourth didactical contract* needs to come. Unfortunately, many of the universities reflect the schooling of the students they receive [35], so it will remain the task of those institutions with visionary leadership [40] and committed to the cause of education to support what in Pakistan would be an innovatory approach to policy input through the reform of mathematics education. In other words, both the curriculum and the mathematics teacher education programme needs a complete fresh outlook.

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