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Estimating of Undersaturated Oil Formation Volume Factor and Isothermal Compressibility Coefficient Using Artificial Neural Networks

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Abstract: The phase performance of hydrocarbons is a very complicated behavior that hydrocarbons show at the time of phase change or when they remain in a particular phase. Process design is almost impossible without a good understanding of this behavior. Artificial Neural Networks have been widely utilized for engineering applications during the last two decades. Therefore undersaturated oil formation volume factor and isothermal compressibility coefficient have been developed using the Artificial Neural Networks (ANNs) approach. Detailed comparison has also been made with various important correlations currently available in the literature. Sensitivity analysis of the developed models was also performed to determine the relative importance of various input parameters. It was found that the developed models outperformed most other existing correlations by giving significantly lower values of average absolute relative error for the parameters studied.

Key words: Neural networks · Formation volume factor · Isothermal compressibility coefficient · Undersaturated crude · Sensitivity analysis

INTRODUCTION

Calculations for material balance, estimation of oil reserves, inflow performance, well test analysis and numerical reservoir simulation constitute important problems in reservoir engineering. Such problems require knowledge of reservoir fluid properties, like formation volume factor, bubble/dew point pressure, viscosity and oil compressibility. The accuracy of the calculations depends on the exactness in predicting such properties. These properties vary with the geographical location of the oil/gas reserve and, ideally should be determined from laboratory analysis of the samples. But such sampling and consequent analysis involves considerable expenses and time, which is undesirable. So, theoretical computations, collectively known as oil-system correlations (or PVT correlations), based on easily measurable parameters, are used for the prediction of reservoir fluid properties. Such parameters include temperature, pressure, solubility and API and gas gravity.

Over the last six decades, various researchers have tried to develop appropriate correlations for predicting reservoir fluid properties with varying degrees of success. The first approach towards theoretical evaluation of PVT properties was made way back in 1942, when Katz [1] developed five methods for predicting crude oil shrinkage. Standing [2] presented graphical correlations for bubble point pressure, oil formation volume factor (OFVF) and total OFVF, based on 105 laboratory flash vaporization observations of California crude. He reported an average error of 4.8% for his correlations and later put them in the form of algebraic equations.

Vasquez *et al*. [3] presented correlations for solution GOR, saturated and undersaturated OFVF and undersaturated oil viscosity from a study of more than 6000 measurements from PVT analysis of 600 samples from all over the world using regression methods. Their study showed that gas gravity is a very important correlating parameter, but its measurement is seldom very accurate because it depends on conditions at which the

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gas/oil separation is made. In order to enhance accuracy of the predicted values, they recommended the use of gas gravity values at a separator pressure of 100 psig (790.83 kPa), as at this pressure oil shrinkage was minimum for the available separator tests. A formula was also provided to convert gas gravity value from any pressure to 100 psig (790.83 kPa). Two sets of equations were provided for each of the correlations, one for crudes above 30 API and another for those below, leading to better prediction. Linear regression was used to estimate isothermal compressibility and that was used to determine undersaturated OFVF. Average error reported was - 0.7% for bubble point pressure, - 0.4% for saturated OFVF and - 7.541% for undersaturated oil viscosity.

Glaso [4] extended Standing's correlation by taking into account crude oil paraffinicity and presence of non-hydrocarbons (CO_2, N_2 and H_2S) in reservoir surface gases. He used graphical and regression analysis for bubble point pressure, saturated OFVF, total OFVF and dead oil viscosity. He reported an average error of 1.28% for bubble point pressure, - 0.43% for saturated OFVF and - 4.56% for total OFVF.

Al-Marhoun [5] developed correlations for Middle East crude, using 160 observations each for bubble point pressure and saturated OFVF and 1556 observations for total OFVF collected from 69 bottom hole samples from 69 Middle East oil reservoirs. Linear and non-linear multiple regression analysis were used to develop the correlations and nomographs were presented for each one of them. Labedi [6, 7] used 128 samples from Libya, Nigeria and Angola to predict OFVF, density and compressibility using multiple regression. He argued that all Standing based correlations used total GOR and total gas gravity, which cannot be obtained from production tests. He proposed using easily measurable field data such as first-stage separator pressure and temperature, producing gas/oil ratio, stock-tank oil gravity, reservoir pressure and temperature. Flash and differential vaporization data were combined and separator conditions were incorporated in the correlations to give an adequate description of the overall volume changes.Various other correlations were also proposed by different researchers from all over the world, with varying degrees of accuracy in terms of average error and based on crude samples from different oil fields. Most of these correlations were developed through linear or non-linear regression.

In the late 1990s, rapid growth of data mining techniques and computational capability led some researchers (Gharbi [8]; Elsharkawy [9]; Gharbi & Elsharkawy [10]; Gharbi, Elsharkawy, & Kartoub [11]) to use advanced soft computing tools for the development of PVT correlations, especially artificial neural networks (ANN). Gharbi [8] presented a model based on multilayer perceptron (MLP) trained by backpropagation with momentum algorithm for isothermal compressibility coefficient for undersaturated crude oil from the regions of Middle East. Elsharkawy [9] developed models based on radial basis function network for prediction of OFVF, GOR, oil viscosity, saturated oil density, undersaturated oil compressibility and evolved gas gravity. Input data used were reservoir pressure, temperature, stock tank oil gravity and separator gas gravity. Gharbi & Elsharkawy [10] presented models for prediction of bubble point pressure and OFVF as functions of solution gas–oil ratio, gas specific gravity, oil specific gravity and temperature for Middle Eastern crudes. Gharbi *et al*. [11] attempted universal MLP based models for bubble point pressure and OFVF. They used 5200 data points from all over the world and reported an average absolute relative error of 6.48% and 1.97% for bubble point pressure and OFVF respectively. Osmanand Al-Marhoun [12] have presented correlations for bubble point pressure and saturated OFVF using MLP trained by backpropagation with early stopping for Saudi Arabian crudes. They reported an average absolute relative error of 5.89% for bubble point pressure and 0.511% for saturated OFVF.

However, most of these correlations were found to be appropriate for the specific region where the parameters have been measured, but not for other regions. The unavailability of a single universal correlation suited for all kinds of crudes underlines the need for specific geographical area-based correlations, as discussed by Hanafy *et al*. [13] with reference to Egyptian crudes. The current study aims to present correlations for the undersaturated oil FVF and isothermal compressibility for Iranian crudes. We have presented two models for these parameters for Iranian crude based on MLP.

Data Acquisition and Analysis: Data used for this study was gathered from several Iranian crude oil samples. Each data set was checked for any missing data and if found, such points were rejected. After this, we were left with 575 data sets each for undersaturated oil OFVF and 203 data sets for undersaturatedisothermal compressibility coefficient of oil. A separate ANN model was formulated for each of these properties. The ranges of crude oil properties are shown in Table 1.

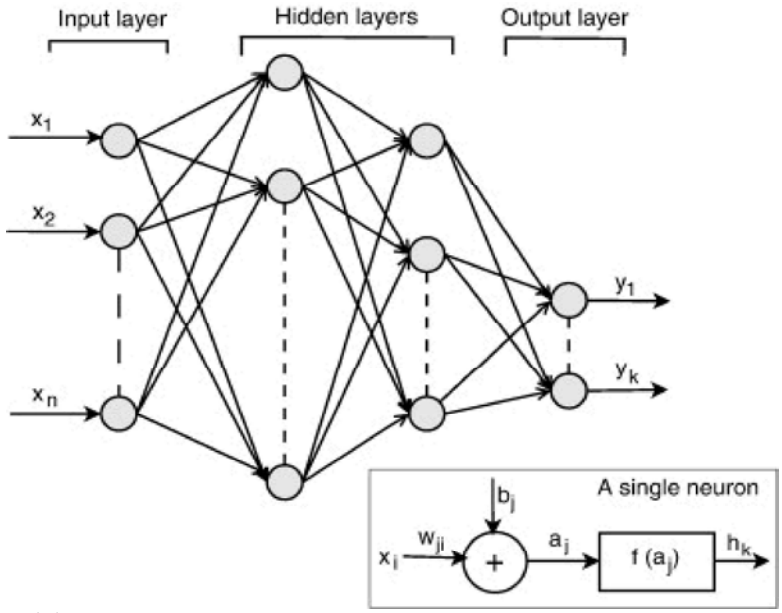

Fig. 1: A typical MLP artichetrue.

Table 1: Properties of Iranian crudes.

Parameter	Minimum	Maximum	Mean	Standard Deviation
Res. pressure, psi	1313	9300	3924.2	1107.9
Bubble point pressure, psi	1071	4590	3546.9	1097.5
Res. temperature, F	116	274	220.8	43.12
Solution gas oil ratio, scf/stb	66	1995.9	1197.0	500.4
Oil API gravity	7.24	34.5	29.4827	5.57
Gas gravity (air=1)	0.774	1.610	0.997	0.151
Oil compressibility coefficient, 1/psi	6.2e-06	3.55e-05	1.71e-05	6.48e-06
Undersaturated OFVF, bbl/stb	1.04	2.41	1.59	0.373

Artificial Neural Networks: Artificial neural networks (ANNs) are massively parallel, distributed processors, constituting of numerous simple processing units, called neurons, developed by mimicking the behavior of the human brain. Like the human brain, ANNs gather information from the environment by a 'learning process' and store them in interneuron connection strengths, called synaptic weights. This learning process takes place through a learning algorithm, which modifies the synaptic weights to attain a desired objective. The detailed theory for solving these kinds of problems is discussed in detail by Haykin [14] andBishop [15] and only a basic overview of the topics pertaining to the problem of PVT properties will be provided below. PVT properties fall under the category of regression problem and it involves supervised learning paradigm.

There exist various such learning algorithms and one of the most popular is feed forward multilayer perceptron (MLP) with backpropagation. An example of MLP is shown in Figure 1. In this technique, network architecture is first chosen and several input and desired output (called target) are provided to the network. The network computes some values (called network output) depending on its processing units, which are compared with the targets.Figure 1 also shows a single neuron of the MLP. The response from each neuron is given by $y = f(w_{ji}x + b_j)$, where w_{ji} is the synaptic weight between jth neuron of one layer and ith neuron of the preceding layer, b_j is the bias of the jth neuron, x is the input and $f(.)$ denotes the activation function. An error function is chosen which computes the error between the network output and target and the synaptic weights are updated so as to minimize this error. The error function is usually the mean of the squared errors (MSE) over all the inputs and is denoted by E.

For a particular input, the error can be considered to be some function of the weights. To make it more amenable to mathematical analysis, a differentiable transfer function is chosen for a neuron, which ultimately renders the error function to be differentiable. Hence any

Table 2: Details of ANN architecture and input variables

Parameter	Input variables	Network architecture	# Data points	# Training points	# validate points	# Test points
Undersaturated OFVF	$T, P, \gamma_g, API,, R_s$	5-9-5-1	575	403	86	86
Oil compressibility coefficient	$T, P, \gamma_g, API,, R_s$	5-13-8-1	203	143	30	30

gradient based unconstrained optimization technique (e.g. steepest descent, conjugate gradient, BFGS, Marquardt–Levenberg) may be chosen to minimize the error. This process is carried on iteratively till the error cannot be minimized further without compromising the generalization capability of the network. After training is completed, the network is tested for inputs which have not been used during the learning process. If the errors are comparable for both training and testing, the net is finalized and is ready for generating outputs for inputs with unknown targets.

We have studied two PVT properties as listed in Table 2. One backpropagation network was developed for each of these properties. Approximately 15% of the data was put in test set. For each property, the total number of data points, the number of points in the training and test set, network architecture and input variables are listed in Table 2. The transfer function in the hidden layer neurons is tansigmoidal and the output layer is linear. The tansigmoidal function is chosen because it is continuous, infinitely differentiable and bounded for any real number and hence can cause the error function to be differentiable everywhere. Also, *tansig*(x) ∈ [- 1, 1], $\forall x \in R$, where R is the set of real numbers, hence this can act as a squashing function. The number of neurons in each layer and the number of layers for each network were decided after extensive trials with varying numbers of neurons and layers. The network with the minimum error was chosen. The architecture of any network is expressed as, $I\text{-}H_1 \ldots H_i \ldots \text{-} H_n\text{-} O$, where I denotes the number of neurons in the input layer, i.e. the number of input variables, H_i the number of neurons in the i^{th} hidden layer and O the number of neurons in the output layer. For all the networks developed here O is equal to *1*, since there is only one output.

RESULTS AND DISCUSSION

The statistical analysis of outputs of the neural network models for oil FVF and isothermal compressibility coefficient are listed in Table 3 and Table 4 respectively. Results of the ANN model are compared with other PVT correlations in terms of statistical measures like average absolute relative error (E_{aar}), maximum error (E_{max}), minimum error (E_{min}), correlation coefficient and standard deviation of absolute error (σ_{are}). These measures are defined as follows

$$E_{aar} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{x_{exp} - x_{predicted}}{x_{exp}}\right| \times 100\% \quad (1)$$

$$E_{max} = \max\left(\left|\frac{x_{exp} - x_{predicted}}{x_{exp}}\right|\right) \times 100\% \quad (2)$$

$$E_{max} = \min\left(\left|\frac{x_{exp} - x_{predicted}}{x_{exp}}\right|\right) \times 100\% \quad (3)$$

The data presented in Table 3 and Table 4 is only for the test set, which were not used in model building. This allows us to check whether the model performs well for data not used in model building.

Oil Formation Volume Factor for Undersaturated Crude: For OFVF above bubble point pressure, the ANN model is compared with correlations of Ahmed [16], Standing [2] and Glaso [4]. The detailed results are shown in Table 3. Though the average absolute relative error for none of the correlations is high, the ANN model shows significantly better results than the rest. In terms of correlation

Table 3: Statistical analysis of undersaturated OFVF (test data set)

Model	E_{aar}, %	E_{max}, %	E_{min}, %	Correlation coefficient, %	$\sigma_{are}\times10^2$
Glaso	2.46	7.80	0.004	99.59	3.35
Standing	3.02	12.56	0.14	99.57	6.00
Ahmed	7.81	18.19	0.16	96.44	8.66
ANN	0.14	2.69	6.77e-05	99.98	0.59

Table 4: Statistical analysis of undersaturated oil isothermal compressibility coefficient (test data set)

Model	E_{aar}, %	E_{max}, %	E_{min}, %	Correlation coefficient, %	$\sigma_{are}\times10^6$
Petrosky-Farshad	12.1	30.7	0.26	96.96	1.46
Vasquez-Beggs	10.5	56.4	1.64	95.21	1.21
ANN	3.90	32.3	0.0052	99.18	0.55

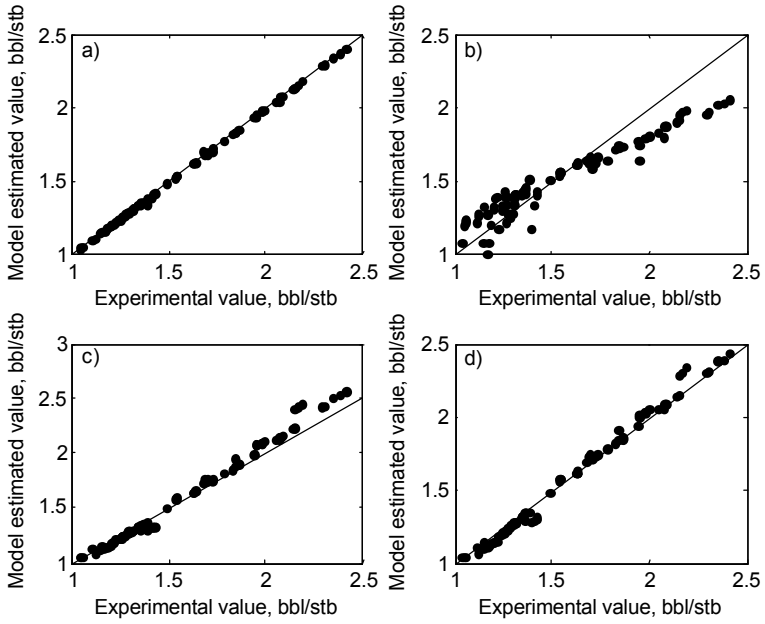

Fig. 2: Cross-plots for undersaturated OFVF correlations: a) ANN; b) Ahmed; c) Standing; d) Glaso.

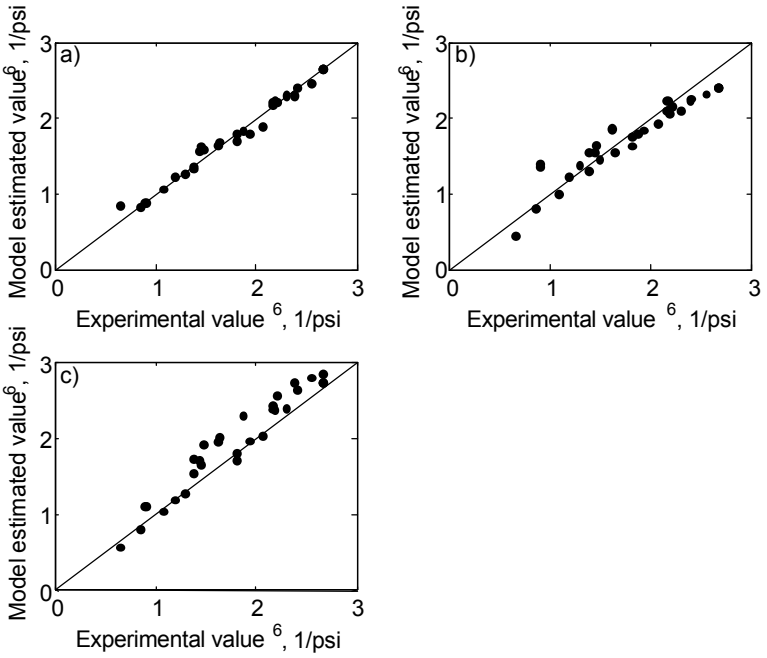

Fig. 3: Cross-plots for undersaturated oil isothermal compressibility coefficient: a) ANN; b) Vasquez-Beggs; c) Petrosky-Farshad.

coefficient, all the correlations compared have similar values, close to 100%. Cross-plots (Figure 2) for all the correlations show a near perfect fit and are almost indistinguishable. However, from Table 3 it can be observed that the accuracy of the models increases from Ahmed to ANN through Standing and Glasorespectively.

UndersaturatedOil Isothermal Compressibility Coefficient: For isothermal compressibility coefficient of oil, the ANN model is compared with existing correlations developed by Vasquez and Beggs [3] and Petroskyand Farshad [16]. The detailed results are shown in Table 4. As is evident from the cross-plots (Figure 3) and Table 4 correlations have almost similar performance. Correlation

coefficient is high for all the correlations. In terms of average absolute relative error, ANN model gives lowest value of 3.90%, whereas the empirical correlations show an error of around 11%.

Sensitivity Analysis: Artificial neural networks have traditionally been regarded as black-box models since the weights were not readily amenable to interpretation and the dependence of the various inputs on the outputs were not easily determinable. However, in the past few years, several techniques have been developed which allows determination of the contribution of a specific input on a specific output and also interpretation of the weights. A detailed review of various sensitivity analysis techniques has been provided by Gevrey *et al*. [17]. The authors recommend the partial derivative (PD) method as the most stable and this is the technique used for this study. The PD method computes the PD of a specific ANN output with respect to each input at each of the input vectors. For a network with n_i input neurons, one hidden layer with n_j neurons and tansigmoidal transfer function and linear output layer with n_k neurons, the k^{th} output may be expressed as

$$y_k = \sum_j {}^2w_{kj} \tan sig\left(\sum_i {}^1w_{ji}x_i + {}^1b_i \right) + {}^2b_k \quad (4)$$

where ${}^nw_{pq}$ denotes the connection weight from qth neuron in $(n-1)$th layer to pth neuron in nth layer and ${}^nb_m$ represents the bias value for the mth neuron in the nth layer (excluding the input layer). The equation 4 can be easily differentiated to give the PD of kth output with respect to ith input.

$$\frac{\partial y_k}{\partial x_i} = \sum_j {}^2w_{jk}\left(1 - h_j^2\right){}^1w_{ji} \quad (5)$$

where h_m is the response from mth neuron in the hidden layer. Similarly, for a network with two hidden layers, having j and m neurons in first and second hidden layer,

$$\frac{\partial y_k}{\partial x_i} = \sum_m \left[{}^3w_{km}\left(1 - \left({}^2h_m\right)^2\right) \sum_j \left\{ {}^2w_{mj}\left(1 - \left({}^1h_j\right)^2\right){}^1w_{jm} \right\} \right] \quad (6)$$

where ${}^nh_m$ is the response of the mth neuron of the nth hidden layer. If there are N observations, one will get N partial derivatives; which may be plotted versus each corresponding input variable and allow direct interpretation of the influence of the input variable on the output. If the PD is negative, it means the output will decrease with increase in the input variable and vice-versa. The relative contribution of each input variable on a specific output can be determined by computing the sum of the squares of the partial derivatives (SSD).

$$SSD_i = \sum_{j=1}^{N} \left(\frac{\partial y_k}{\partial x_i} \right)_j^2 \quad (7)$$

where $\frac{\partial y_k}{\partial x_i}$ is the PD for Jthobservation. The contribution of each input variable is given by

$$\text{Contribution} = \frac{SSD_i}{\sum_i SSD_i} \times 100\% \quad (8)$$

The variable having the highest SSD affects the output most. On this basis, the inputs may be ranked in order of their influence on the output. The Table 5 and Table 6 show sensitivity results for undersaturated oil FVF and isothermal compressibility coefficient.

Table 5: Sensitivity analysis for undersaturated oil FVF

Variables	SSD	Contribution, %	Rank
Pressure	11.5	0.39	5
Temperature	1514.3	50.86	1
Solution gas oil ratio	336.5	11.30	4
API gravity	375.8	12.62	3
Gas gravity	739.2	24.83	2

Table 6: Sensitivity analysis for undersaturated oil isothermal compressibility coefficient

Variables	SSD	Contribution, %	Rank
Pressure	27.90	24.51	3
Temperature	0.909	0.80	4
Solution gas oil ratio	50.445	44.30	1
API gravity	0.285	0.25	5
Gas gravity	34.311	30.14	2

CONCLUSION

In this study, two neural network models were used to determine the undersaturated oil formation volume factor and isothermal compressibility coefficient for Iranian hydrocarbon mixtures. The performance of the new model was compared to other existing correlations and was found to outperform them in terms of average absolute relative error, correlation coefficient, maximum and minimum absolute relative error and standard

deviation of absolute relative error. Mean absolute relative errors of 0.14 % for the volume factor and 3.9 % for the isothermal compressibility coefficient were obtained. Attempts were made to reach the best network architecture in terms of the number of neurons and network layers. It can be firmly stated that the Artificial Neural Network method gives better results than previously published methods when there is a sufficient amount of experimental data. Sensitivity analysis was also performed for each of the models developed and the input variables ranked on the basis of their contribution..

Nomenclature:

API API gravity (°API)
γ_g Gas specific gravity (air = 1)
σ_{are} Standard deviation of absolute error
b_j Bias value of the jth neuron of a layer
w_{ji} Synaptic weight to the jth neuron of a layer from the ith neuron of the preceding layer
B_o OFVF above bubble point pressure (rb/stb)
E Mean squared error
E_{aar} Average absolute relative error (%)
E_{max} Maximum average absolute error (%)
E_{min} Minimum average absolute error (%)
N Total number of weights in the network
P Reservoir pressure (psia)
R_s Solution GOR (scf/stb)
T Reservoir temperature (°F)

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1- Ahmed's Correlations:

$$B_o = B_{ob} \times EXP[D[EXP(aP) - EXP(aP_b)]] \quad \text{(A-1)}$$

$$D = [4.588893 + 0.0025999 \times R_s]^{-1} \quad \text{(A-2)}$$

$$a = -0.00018473 \quad \text{(A-3)}$$

$$B_{ob} = F + a_1 T + a_2 T^2 + a_3/T + a_4 P + a_5 P^2 + a_6/P + a_7 R_s + a_8 R_s^2 + a_9/R_s \quad \text{(A-4)}$$

$$F = a_{10} + (R_s^{a11} \times API^{a12} / \gamma_g^{a13}) \quad \text{(A-5)}$$

$$\begin{aligned} a_1 &= -4.5243973 \times 10^{-4} & a_2 &= 3.9063637 \times 10^{-6} \\ a_3 &= -5.5542509 & a_4 &= -5.7603220 \times 10^{-6} \\ a_5 &= -3.9528992 \times 10^{-9} & a_6 &= 16.289473 \\ a_7 &= 3.8718887 \times 10^{-4} & a_8 &= 7.0703685 \times 10^{-8} \\ a_9 &= -1.4358395 & a_{10} &= -0.12869353 \\ a_{11} &= 0.023484894 & a_{12} &= 0.015966573 \\ a_{13} &= 0.021946351 \end{aligned} \quad \text{(A-6)}$$

Bubble point pressure was calculated by Al-Marhoun (Al-Marhoun 1988) Correlation (where experimental P_b was not available).

$$P_b = a \times R_s^b \times \gamma_g^c \times \gamma_o^d \times T^e \quad \text{(A-7)}$$

$$\begin{aligned} a &= 5.38088 \times 10\text{-}3 & b &= 0.715082 \\ c &= -1.87784 & d &= 3.1437 \\ e &= 1.32657 \end{aligned} \quad \text{(A-8)}$$

2- Standing's Correlations(Standing 1947)*:*

$$B_o = B_{ob} \exp\left(C_o \left(P_b - P\right)\right) \quad \text{(A-9)}$$

$$C_o = \frac{-1433 + 5 \times R_s + 17.2 \times T - 1180 \times \gamma_g + 12.61 \times {}^\circ API}{10^5 \times P} \quad \text{(A-10)}$$

$$B_{ob} = 0.9759 + 0.000120\left[R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.5} + 1.25 \times T\right]^{1.2} \quad \text{(A-11)}$$

$$P_b = 18.2\left[\left(R_s/\gamma_g\right)^{0.83} 10^a - 1.4\right] \quad \text{(A-12)}$$

$$a = 0.00091 \times T - 0.0125 \times {}^\circ API \quad \text{(A-13)}$$

3- Glaso's Correlations(Glaso 1980)*:*

$$B_o = B_{ob}\exp\left(C_o\left(P_b - P\right)\right) \tag{A-14}$$

We calculated C_o from equation A-10.

$$B_{ob} = 1.0 + 10^{A} \tag{A-15}$$

$$A = -6.58511 + 2.91329\times \mathrm{Log}(B^*_{ob}) - 0.27683\left(\mathrm{Log}(B^*_{ob})\right)^2 \tag{A-16}$$

$$B^*_{ob} = R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.526} + 0.968\times T \tag{A-17}$$

Appendix B- Correlation for Undersaturated Oil Isothermal Compressibility Coefficient

1- The Vasquez-Beggs Correlation(Vasquez and Beggs 1980)*:*

$$C_o = \frac{-1433 + 5\times R_s + 17.2\times T - 1180\times\gamma_g + 12.61\times{}^\circ\mathrm{API}}{10^5\times P} \tag{B-1}$$

2- The Petrosky-Farshad Correlation(Tarek 2006)*:*

$$C_o = 1.705\times10^{-7} R_s^{0.69357}\gamma_g^{0.1885}\mathrm{API}^{0.3272}T^{0.6729}P^{-0.5906} \tag{B-2}$$