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Torsional Buckling Optimization of Composite Drive Shafts

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Abstract: One interesting application of composite materials is the composite drive shafts as power transmission tubing which are used in many mechanical and structural systems; such as automobiles, marine and flight vehicles, gas and wind turbines…etc. In this paper, a composite drive shaft for an automotive application is optimized for maximizing the torsional buckling torque under mass constraint. Other constraints include bending natural frequency as well as interlaminar shear failure criterion. The selected design variables are the fiber volume fraction, fiber orientation angle and thickness of each composite layer. A case study for a simply supported drive shaft made of carbon/epoxy composite material is considered through the work of this paper. The attained optimum solutions are compared with a known baseline design having the same length, same cross section and same material properties. The optimization problem is built in a nondimensional form; and Global Optimization Toolbox in MATLAB program has been implemented for modeling the optimization problem. It was found that the cross-ply layup gives the best results for maximum buckling torque and bending natural frequency without mass penalty.

Key words: Drive shaft . optimization . composite materials . buckling torque

INTRODUCTION

Advanced composite materials such as graphite, carbon, Kevlar and glass with suitable resins are widely used because of their high specific strength and stiffness. Advanced composite materials seem ideally suited for long, power driver shafts applications. Their elastic properties can be tailored to increase the torque they can carry as well as the rotational speed at which they operate. Drive shafts can be used in several applications such as, automotive, aircraft, wind turbines and aerospace structures. The automotive industry is exploiting composite material technology for structural components construction in order to obtain the reduction of the weight without decrease in vehicle quality and reliability. Steel drive shafts are usually manufactured in two pieces to increase the fundamental bending natural frequency which is inversely proportional to the square of the beam length. The two piece steel drive shaft, shown in Fig. 1, consists of three universal joints, a center supporting bearing and a bracket, which increase the total weight of a vehicle. Power transmission can be improved through the reduction of inertial mass and increase of stiffness. Substituting composite structures for conventional metallic structures has many advantages because of higher specific stiffness and strength of composite material [1, 2]. Bijagare *et al*. [3] has applied genetic algorithm to minimize the weight of a composite shaft subjected to constraints imposed on torque transmission

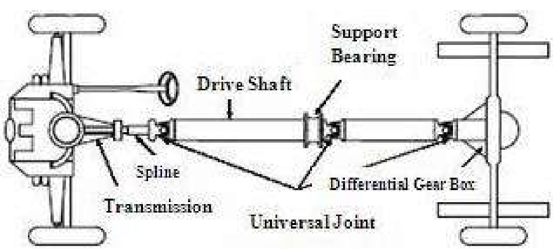

Fig. 1: Automotive metallic drive shaft

and fundamental natural frequency. Rangaswamy and Vijayarangan [4] showed that weight reduction of a drive shaft can have a certain role in the general weight reduction of the vehicle and is highly desirable goal, if it can be achieved without increase in cost and decrease in quality and reliability. Kim and Lee [5] investigated the optimal design of a press fit joint between the hybrid tube and aluminum yoke of a one-piece hybrid drive shaft, considering the number and shape of the steel teeth to obtain high torque capability. The calculated optimal solution was compared with known experimental results. The optimal design resulted in amass saving of about 50 % as compared with a two-piece shaft. Shokrieh *et al*. [6] studied the torsional stability of composite drive shaft using finite element analysis with ANSYS software and shell 99 elements. Results showed good agreement with known experimental results for carbon/epoxy composite shaft. Also, it was shown that in designing a composite shaft,

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the buckling torque must be properly higher than the static applied torque; the boundary conditions of the shaft do not have much effect on the buckling torque. In contrast, the fiber orientation angle and stacking sequence of the layers were found to strongly affect the buckling torque. Filament winding process is commonly used in the fabrication of composite drive shafts, in which fiber tows wetted with liquid resin are wound over a rotating male cylindrical mandrel. The angle, fiber tension and resin content can be varied under control software downloaded in the filament winding machine. Filament winding is relatively inexpensive, repetitive and accurate in fiber placement [7].

This paper aims to generate an optimization model for drive shaft manufactured from composite materials. The optimization model was constructed using MATLAB 2012b computer program and applying Optimization Toolbox with global search routines to obtain the required optimal solutions. Figure 2 shows a general flow chart describing the main stages of an optimization process.

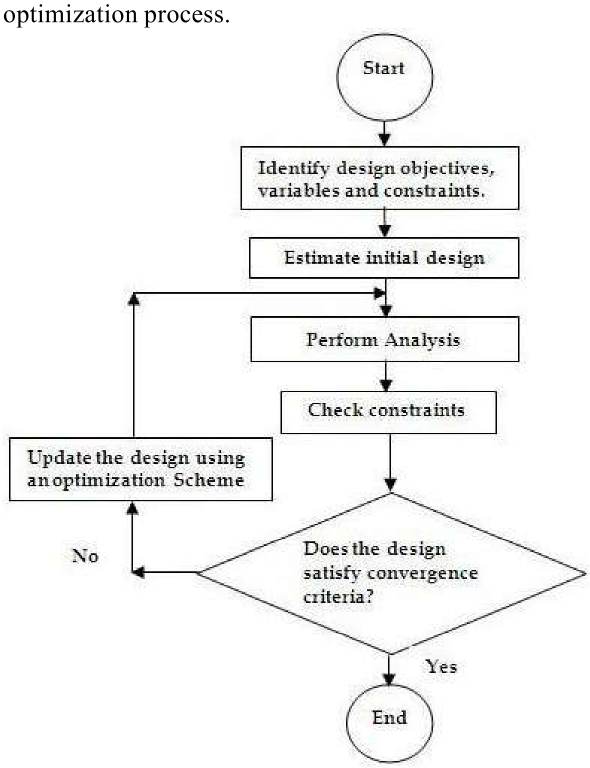


Fig. 2: Optimization modeling flow chart

DESIGN ASPECTS OF COMPOSITE DRIVE SHAFTS

Drive shafts should be designed to have enough torsional strength to carry the applied torque without failure. In addition, the possibility of torsional buckling must be considered for thin-walled shafts. The third major design requirement is that the drive shaft should have a sufficiently high bending natural frequency. An optimum design of the shaft is desirable, which is cheapest and lightest but meets all above requirements.

Table 1:Halpin-Tsai semi-empirical relations for calculating composite properties [8]

Elastic properties	Mathematical formula	
Longitudinal modulus E_{11}	$E_{1f}V_f + E_mV_m$	
Transverse modulus E_{22}	$E_m\dfrac{(1+\xi\eta V_f)}{(1-\eta V_f)}$	$\eta = \dfrac{(E_{2f}-E_m)}{(E_{2f}+\xi E_m)}$
Shear rmodulus G_{12}	$G_m\dfrac{(1+\xi\eta V_f)}{(1-\eta V_f)}$	$\eta = \dfrac{(G_{12f}-G_m)}{(G_{12f}+\xi G_m)}$
Poisson's ratio v_{12}	$\upsilon_{12f}V_f + \upsilon_mV_m$	
Mass density ρ	$\rho_fV_f + \rho_mV_m$	

Material properties: One of the most important factors in determining the properties of composites is the relative proportions of the matrix and reinforcing materials. The relative proportions can be given as mass fractions or volume fractions. The mass fractions are easier to obtain during fabrication or experimentally after fabrication. However, the volume fractions are exclusively used in the theoretical analysis of composite materials. For a two-phase composite (e.g. carbon/epoxy), the various material properties are defined in Table 1.

Assuming no voids is present, then $V_f + V_m = 1$, where V denotes volume fraction. Subscripts "m" and "f" refer to properties of matrix and fiber materials, respectively. ξ is curve fitting factor and is approximately 2 for E_{22} and 1 for G_{12} [8, 9].

Laminate Stiffness: The elements of the laminate stiffness matrices are defined in the following [8]:

Extensional stiffness:

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (h_k - h_{k-1})$$

Coupled extension-bending stiffness:

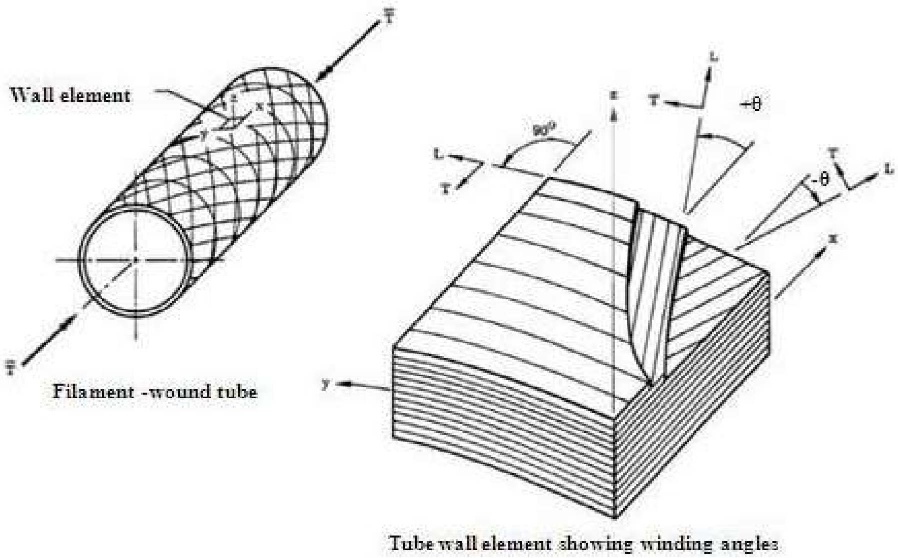
$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (h_k^2 - h_{k-1}^2) \tag{1}$$

Bending stiffness:

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (h_k^3 - h_{k-1}^3)$$

where h_k is the thickness of the k^{th} lamina and n is the total number of layers. Details of determining the elements of the k-th lamina stiffness matrix, $[\overline{Q}]$ are given in Appendix A.

Two types of lay-up are considered in the present study:



L: longitudinal fiber direction (direction 1), T: Transverse direction (direction 2)

Fig. 3: Notation and coordinate systems for a composite drive shaft

- Symmetric laminates: In which the plies of the laminate are a mirror image about the geometrical mid-plane. The B_{ij} matrix equal zero in this case.
- Symmetric & balanced laminates: where for each ply with +θ fiber orientation angle, there must be another layer with-θ angle with the same material properties and thickness. A_{16} and A_{26} equal zero in this case as shown in Fig. 3.

Structural analysis

Applied shear stress: The primary load carried by a drive shaft is torsion. The applied shear stress,τ, for shafts with thin-walled, circular cross section, can be determined from the relation [10]:

$$\tau = \frac{T}{2\pi R^2 H} \tag{2a}$$

where: T is the applied torque, R the mean radius and H the total wall thickness of the shaft given by:

$$R = (R_{ou} + R_{in})/2 \tag{2b}$$

$$H = \sum_{k=1}^{n} h_k \tag{2c}$$

R_{in} and R_{ou} are the inner and outer radii, respectively. The allowable shear stress, τ_{allow}, can be calculated according to the embedded material properties and volume fraction of the fiber from the following equation (11) (refer to appendix A):

$$\tau_{allow} = \frac{\max(\tau_{xy\_rupture})}{F.O.S} \tag{3a}$$

$$\tau_{xy\_rupture}(k) = 1 \Big/ \sqrt{4c^2 s^2 \left(\frac{2}{\sigma^2_{1t\_rup}} + \frac{1}{\sigma^2_{2t\_rup}}\right) + \frac{(c^2 - s^2)^2}{\tau^2_{12\_rup}}} \tag{3b}$$

where c=cos (θ_k), s=sin (θ_k) and F.O.S denotes the factor of safety taking according to design specifications.

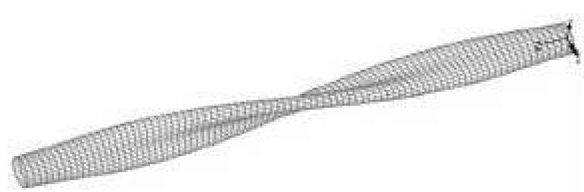


Fig. 4: Typical torsional buckling mode shape of a drive shaft

Torsional buckling: When a hollow shaft is subjected to torsion, at a certain amount of torsional load instability occurs (Fig. 4). This is called the torsional buckling load. Different expressions for calculating the buckling torque of thin-walled tubes can be found in the literatures. The most commonly used formula is [10]:

$$T_{cr} = (2\pi R^2 H)(0.272)(E_x)^{0.25}(E_y)^{0.75}(H/R)^{1.5} \tag{4}$$

In Eq. (4), it is postulated that buckling begins in the outer layers of the shaft first. E_x and E_y are the moduli of elasticity in the axial and transverse directions, respectively. They are given, for symmetric laminate by the expressions [12, 13]:

$$E_x = \frac{1}{Ha_{11}},\ E_y = \frac{1}{Ha_{22}} \tag{5a}$$

$$[a] = [A]^{-1} \tag{5b}$$

Bending natural frequency: Assuming that the drive shaft is simply supported at both ends, the fundamental bending natural frequency (f_n) can be determined from the simple formula [8, 13]:

$$f_n = \frac{\pi}{2}\sqrt{\frac{E_x I_y}{ML^3}} \tag{6a}$$

where: L is the total length of the shaft, M the total mass and I_y the second moment of area about y-axis. They can be determined using the expressions:

$$M = 2\pi RL \sum_{k=1}^{n} \rho_k h_k \tag{6b}$$

$$\rho_k = \rho_f V_{fk} + \rho_m (1 - V_{fk}) \tag{6c}$$

$$I_y = \pi H R^3 \tag{6d}$$

Tsai-Wu Failure Theory: This failure theory is based on the total strain energy failure theory of Beltrami. A lamina is considered to be failed if the following equation is violated [9].

$$FI = H_1\sigma_1 + H_2\sigma_2 + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \tag{7a}$$

where

$$H_1 = \frac{1}{\sigma_{1t\_rup}} - \frac{1}{\sigma_{1c\_rup}} \tag{7b}$$

$$H_{11} = \frac{1}{\sigma_{1t\_rup}\sigma_{1c\_rup}} \tag{7c}$$

$$H_2 = \frac{1}{\sigma_{2t\_rup}} - \frac{1}{\sigma_{2c\_rup}} \tag{7d}$$

$$H_{22} = \frac{1}{\sigma_{2t\_rup}\sigma_{2c\_rup}} \tag{7e}$$

$$H_{66} = \frac{1}{\tau_{12\_rup}^2} \tag{7f}$$

H_{12} is determined experimentally (according to Mises-Hencky theory):

$$H_{12} = -\frac{1}{2}\sqrt{H_{11}H_{22}} \tag{7g}$$

$$(FI)_{\max} = Max(FI)_{k=1,2,..n} \tag{7h}$$

OPTIMIZATION PROBLEM FORMULATION

As has been mentioned before, structural buckling failure due to torsion is a major consideration in designing composite drive shafts. Therefore, the present study seeks maximization of the buckling torque (T_{cr}) at which torsional instability might occur without mass penalty. Design variables include the fiber volume fraction (V_{fk}), fiber orientation angle (?$_k$) and thickness (h_k) of the individual k-th lamina. The total number of layers is assumed to be preassigned. In addition to the mass and frequency constraints, side constraints are always imposed on the design variables for geometrical, manufacturing or logical reasons to avoid having unrealistic odd shaped optimum designs.

Table 2: Definition of dimensionless quantities

Quantity	Notation	Dimensionless expression**
Mean radius	R	$\hat{R} = R/R_o$
Total wall thickness	H	$\hat{H} = H/H_o$
Thickness of kth lamina	h_k	$\hat{h}_k = h_k/H_o$
Density of kth lamina	ρ_k	$\hat{\rho}_k = \rho_k/\rho_o$
Total mass	M	$\hat{M} = M/M_o = \hat{R}\sum_{k=1}^{n}\hat{\rho}_k \hat{h}_k$
Modulus of elasticity:	E_x	$\hat{E}_x = E_x/E_{xo}$
	E_y	$\hat{E}_y = E_y/E_{yo}$
Buckling torque	T_{cr}	$\hat{T}_{cr} = T_{cr}/T_{cro}$
Fundamental frequency	f_n	$\hat{f}_n = 60 f_n/rpm$

**Baseline design parameters are denoted by subscript 'o': Density $\rho_o = 0.5(\rho_f + \rho_m)$, Mass: $M_o = 2\pi\rho_o R_o H_o L$. rpm is the maximum rotational speed of the drive shaft

Baseline design: It is convenient first to normalize all variables and parameters with respect to a known baseline design (Table 2), which has been selected to be made of cross-ply layup [90^o, 0^o]$_s$ with equal fiber and matrix volume fraction, i.e. V_{fo} = 50%. Optimized shaft designs, shall have the same transmitted power, length, boundary conditions and material properties of those known for the baseline design.

Mathematical model: The optimization problem is constructed in nondimensional form, in which the equations of torsional buckling, mass and natural frequency are normalized with respect to the baseline design. The mean diameter is taken equal to that of the baseline design, i.e. $\hat{R} = 1$. The design variable vector, $\bar{X}$, which is subjected to change in the optimization process, is therefore, defined as:

$$\bar{X} = \left(V_f, \theta, \hat{h}\right)_{k=1,2,..n} \tag{8}$$

The final simplified torsional buckling optimization problem can be cast in the following:

$$\text{Minimize } F = -\hat{T}_{cr} \tag{9a}$$

Subject to Mass constraint: $\hat{M} \le 1$
Natural frequency:

$$\hat{f}_n \ge 1 \tag{9b}$$

Failure criterion constraint: $(FI)_{\max} < 1$
Side constraints: $\vec{X}_L \le \vec{X} \le \vec{X}_U$

$$\hat{H}_L \le \sum_{k=1}^{n} \hat{h}_k \le \hat{H}_U \tag{9c}$$

where $\vec{x}_L$ and $\vec{x}_U$ are the lower and upper bounds imposed on the design variable vector

$$\overline{X} = \left(V_f, \theta, \hat{h}\right)_{k=1,2,..n}$$

This optimization problem may be though as a search in an (3n) dimensional space for a point corresponding to the minimum value of the objective function and such that it lies within the region bounded by subspaces representing the constraint functions.

Optimization technique: Global search is a powerful technique used to find the needed optimum global design point [14]. It constructs a number of starting points and uses a local solver (e.g. fmincon) to find the local optimum in the basin of attraction of these starting points, then it give the global point from these local optimum points. The global search chooses the starting points randomly. Global search is distinguished with fast converging to local minimum even if it starts with a starting point far from the optimum. It eliminates non active starting points. The local solver (fmincon) uses the algorithm of sequential quadratic programming to find the local minimum. Sequential Quadratic Programming (SQP) is one of the most recently developed and perhaps one of the best methods of optimization. The method has a theoretical basis that is related to (1) the solution of a set of nonlinear equations using Newton's method and (2) the derivation of simultaneous nonlinear equations using Kuhn-Tucker conditions to the Lagrangian of the constrained optimization problem [15].

RESULTS AND DISCUSSION

The proposed optimization model has been implemented for obtaining the needed optimal designs of a composite drive shaft with a maximum torque capacity of 3000 Nm and maximum operating speed of 6500 rpm. For automotive applications (passenger cars and small trucks) with rear drive the horse power of the engine is approximately in the range of 100 to 400 Hp.

Table 3: Material properties of carbon-AS4/epoxy-3501-6 composite [9]

Property	Carbon fiber	Epoxy matrix
Mass density (g/cm^3)	ρ_f= 1.81	ρ_m=1.27
Young's moduli (Gpa)	E_{1f}= 235, E_{2f}= 15	E_m=4.3
Shear moduli (Gpa)	G_{12f} = 27, G_{23f} = 7	G_m=1.60
Poisson's ratio	ν_{12f}= 0.2	ν_m=0.35
Ultimate tensile strength (MPa)	3700	69
Ultimate compression Strength (MPa)	--	200
Ultimate shear strength (MPa)	--	100

The baseline design is made of cross-ply layup [90^o, 0^o]$_s$ with equal fiber and matrix volume fraction, i.e. V_{fo}=50%. The material of construction is selected to be carbon/epoxy with its properties given in Table 3. Other design data are given in the following:

Shaft length: L= 1.75 m
Mean diameter R_o= 54.6 mm
Wall thickness H_o=2.8 mm
Density ρ_o= 1.54 gm/cm^3
Mass M_o=2.588 kg
Modulus of elasticity: E_{xo}=E_{yo}=64.11 GPa
Buckling torque: T_{cro}= 10622 N.m
Bending frequency: f_{no} =127.8 Hz
Allowable Shear strength:τ_{allow}= 54.0 MPa (Safety factor =1.5)
The lower and upper bounds imposed on the design variables are selected to be:

$$\overline{X_L} = (0.25, 0.0, 0.01)_{k=1,2,\ldots n}$$

and

$$\overline{X_U} = (0.75, \pi/2, 0.1)_{k=1,2,\ldots n}$$

Table 4 presents the attained optimal solutions by implementing the developed program using the Mat Lab optimization tool box routines. A relationship between the number of layers and optimization gain ($OG = \frac{\hat{T}_{cr} - \hat{T}_{cro}}{\hat{T}_{cro}}$) to baseline is plotted in Fig. 5. It is observed from the previous results that the twelve

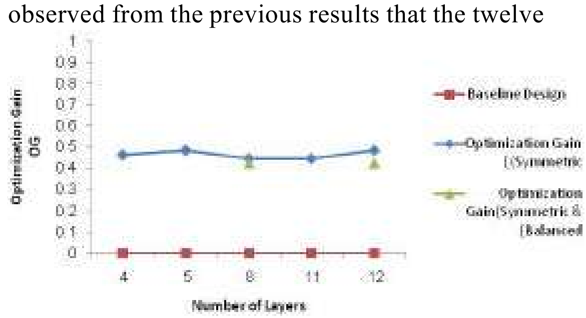


Fig. 5: Composite drive shaft buckling torque relative to baseline for different no. of layers

layers no. is the best number of layers for the buckling torque problem, which gives:

Table 4: Optimum solutions

No. of layers	Lay-up configuration	$x_{opt} = (V_f, \theta, h)_{k=1,2...n}$	$\hat{T}_{cr}$	$\hat{M}$
n=4	Symmetric	$(0.75, 90^o, 0.29)_1$	1.46	0.979
	(1=4), (2=3)	$(0.75, 0^o, 0.16)_2$		
n=5	Symmetric	$(0.75, 90^o, 0.25)_1$	1.482	1.0
	(1=5), (2=4)	$(0.75, 0^o, 0.17)_2$		
	--	$(0.75, 90^o, 0.08)_3$		
n=8	Symmetric	$(0.75, 90^o, 0.19)_1$	1.444	0.979
	(1=8), (2=7)	$(0.75, 0^o, 0.13)_2$		
	(3=6), (4=5)	$(0.75, 90^o, 0.09)_3$		
	--	$(0.75, 0^o, 0.04)_4$		
	Symmetric & Balanced	$(0.75, 90^o, 0.14)_1$	1.422	0.957
	(1=8), (2=7)	$(0.75, -90^o, 0.14)_2$		
	(3=6), (4=5)	$(0.75, 0^o, 0.08)_3$		
	$\theta_1 = -\theta_2$	$(0.75, 0^o, 0.08)_4$		
	$\theta_3 = -\theta_4$	--		
n=11	Symmetric	$(0.75, 90^o, 0.1)_1$	1.444	0.979
	(1=11), (2=10)	$(0.75, 0^o, 0.01)_2$		
	(3=9), (4=8)	$(0.75, 90^o, 0.09)_3$		
	(5=7)	$(0.75, 0^o, 0.1)_4$		
	--	$(0.75, 90^o, 0.09)_5$		
	--	$(0.75, 0^o, 0.06)_6$		
n=12	Symmetric	$(0.75, 90^o, 0.1)_1$	1.422	0.957
	(1=12), (2=11)	$(0.75, 0^o, 0.07)_2$		
	(3=10), (4=9)	$(0.75, 90^o, 0.09)_3$		
	(5=8), (6=7)	$(0.75, 0^o, 0.02)_4$		
	--	$(0.75, 90^o, 0.1)_5$		
	--	$(0.75, 0^o, 0.08)_6$		
	Symmetric & Balanced	$(0.75, 90^o, 0.1)_1$	1.482	1.0
	(1=12), (2=11)	$(0.75, -90^o, 0.1)_2$		
	(3=10), (4=9)	$(0.75, 0^o, 0.08)_3$		
	(5=8), (6=7)	$(0.75, 0^o, 0.08)_4$		
	$\theta_1 = -\theta_2$	$(0.75, 90^o, 0.04)_5$		
	$\theta_3 = -\theta_4$	$(0.75, -90^o, 0.04)_6$		
	$\theta_5 = -\theta_6$	--		

Mass (m) =1*2.588=2.588 kg,
Critical buckling torque (T_{cr}) =1.482*10622=15741 Nm
Natural frequency (f_n) =1.35*127.8=172.5 Hz.

CONCLUSIONS AND FUTURE ASPECTS

In view of the practical use of advanced composite in several engineering applications, a model for optimizing stability performance of a composite drive shaft has been developed and applied to a thin-walled circular shaft made of carbon/epoxy laminates with simply supported boundary condition. The objective function is measured by maximization of the torsional moment at which buckling instability occurs. Design variables include the fiber volume fraction, fiber orientation angle and thickness of the individual layers composing the shaft cross section. Design constraints are imposed on the total structural mass, fundamental bending frequency and interlaminar shear strength limitation. Side constraints are also imposed on the values of the design variables in order to avoid having negative values or odd-shaped configurations in the resulting optimum solutions. The study assumes slender shaft configuration, which enables the use of simplified design formulas to calculate the buckling torque and natural bending frequency. Results have shown that the proposed optimization model succeeds in arriving at the optimum values of the selected design variables corresponding to the specific design case. It has been also demonstrated that the cross-ply construction is much better than the unidirectional one in achieving the highest possible buckling strength and frequency without the penalty of increasing structural mass. Conspicuous optimum trends have been obtained for good designs with different number of layers. There are still many factors and different approaches that can be considered in future optimization of automotive drive shafts. Research work can be extended to consider a more comprehensive optimization formulation involving many design parameters and applying multi-criteria optimization techniques in order to simultaneously minimize several design objectives such as, vibration, structural weight, buckling, fatigue and manufacturing cost.

Appendix A

The reduced form of Hooke's law for an orthotropic homogeneous lamina in a plane stress state may be written as [8]:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} \qquad \text{(A-1)}$$

where the elements of the matrix [Q] is defined in terms of material properties as follows:

$$\begin{aligned} Q_{11} &= \frac{E_{11}}{1-\nu_{12}^2 E_{22}/E_{11}} \\ Q_{22} &= \frac{E_{22}}{1-\nu_{12}^2 E_{22}/E_{11}} \\ Q_{12} &= \frac{\nu_{12} E_{22}}{1-\nu_{12}^2 E_{22}/E_{11}} \\ Q_{66} &= G_{12} \end{aligned} \qquad \text{(A-2)}$$

The elastic moduli E_{11}, E_{22}, G_{12} and υ_{12} are functions of the fiber volume fraction, as given in Table 1. For a generally orthotropic lamina, equation (A-1) should be transformed to reflect rotated fiber orientation angles and the relation between the membrane stresses and strains takes the matrix form:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \qquad \text{(A-3)}$$

The elements of the k-th lamina stiffness matrix, $[\bar{Q}]$, which is now referred to the reference axes of the shaft (x, y, z), are given by:

$$\begin{aligned} \bar{Q}_{11} &= U_1 + U_2\cos 2\theta + U_3\cos 4\theta \\ \bar{Q}_{22} &= U_1 - U_2\cos 2\theta + U_3\cos 4\theta \\ \bar{Q}_{12} &= U_4 - U_3\cos 4\theta \\ \bar{Q}_{16} &= 0.5U_2\sin 2\theta + U_3\sin 4\theta \\ \bar{Q}_{26} &= 0.5U_2\sin 2\theta - U_3\sin 4\theta \\ \bar{Q}_{66} &= 0.5(U_1 - U_4) - U_3\cos 4\theta \end{aligned} \qquad \text{(A-4)}$$

The angle θdenotes fiber orientation with x-axis. The terms U_i are solely function of the material properties and, hence the volume fractions. They are defined by the following expressions:

$$\begin{aligned} U_1 &= (3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})/8 \\ U_2 &= (Q_{11} - Q_{22})/2 \\ U_3 &= (Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})/8 \\ U_4 &= (Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})/8 \end{aligned} \qquad \text{(A-5)}$$

Ultimate Strengths of Unidirectional Lamina [9]

Longitudinal tensile strength:

$$\sigma_{1t\_rup(k)} = \sigma_{ft} V_f \qquad \text{(A-6)}$$

Longitudinal compressive strength:

$$\sigma_{1c\_rup(k)} = \frac{G_m}{1 - V_f\left(1 - \frac{G_m}{G_{12f}}\right)} \qquad \text{(A-7)}$$

Transverse tensile strength:

$$\sigma_{2t\_rup(k)} = \left[1 - \left(V_f^{0.5} - V_f\right)\left(1 - \frac{E_m}{E_{2f}}\right)\right]\sigma_{t\_m} \qquad \text{(A-8)}$$

Transverse compressive strength:

$$\sigma_{2c\_rup(k)} = \left[1 - \left(V_f^{0.5} - V_f\right)\left(1 - \frac{E_m}{E_{2f}}\right)\right]\sigma_{c\_m} \qquad \text{(A-9)}$$

In-plane shear strength:

$$\tau_{12\_rup(k)} = \left[1 - \left(V_f^{0.5} - V_f\right)\left(1 - \frac{E_m}{E_{2f}}\right)\right]\tau_m \qquad \text{(A-10)}$$

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