Free Convection Boundary Layer Flow on a Solid Sphere with Convective Boundary Conditions in a Micropolar Fluid

Hamzeh Taha Alkasasbeh, Mohd Zuki Salleh, Razman Mat Tahar, Roslinda Nazar and Ioan Pop

Abstract: In this paper, the problem of free convection boundary layer flow on a solid sphere in a micropolar fluid with convective boundary conditions, in which heat is supplied through a bounding surface of finite thickness and finite heat capacity is considered. The basic equations of boundary layer are transformed into a non-dimensional form and reduced to nonlinear systems of partial differential equations are solved numerically using an implicit finite difference scheme known as the Keller-box method. Numerical solutions are obtained for the wall temperature, the local heat transfer coefficient and the local skin friction coefficient, as well as the velocity, angular velocity and temperature profiles. The features of the flow and heat transfer characteristics for different values of the material or micropolar parameter, $K = 0$, (Newtonian fluid), $1$, $2$, $3$, (micropolar fluid), the Prandtl number, $Pr = 0.7$, $1$, $7$, the conjugate parameter, $\gamma = 0.05$, $0.1$, $0.2$ and the coordinate running along the surface of the sphere, $x$ between $0$ and $120^\circ$ are analyzed and discussed.

Key words: Boundary Layer • Convective Boundary Conditions • Free Convection • Micropolar Fluid • Solid Sphere

INTRODUCTION

The essence of the theory of micropolar fluid flow lies in the extension of the constitutive equation for Newtonian fluid, so that more complex fluids such as particle suspensions, liquid crystal, animal blood, lubrication and turbulent shear flows can be described by this theory. The theory of micropolar fluid was first proposed by Eringen [1]. Extensive review of the theory and applications can be found in the review article by Ariman et al. [2] and the Blasius boundary-layer flow of a micropolar fluid is considered by Rees and Bassom [3]. On the other hand, Chen and Mucoglu [4] studied the analysis of mixed, forced and free convection about a sphere in a viscous fluid. Further, Nazar et al. [5,6,7] considered the free and mixed convection boundary layer flows on a sphere in a viscous and micropolar fluid with constant wall temperature (CWT) and constant heat flux (CHF), respectively. The natural convection heat and mass transfer from a sphere in micropolar fluids with constant wall temperature and concentration were presented by Cheng [8]. On the other hand, the laminar mixed convection boundary layer flow about an isothermal solid sphere in a micropolar fluid was studied by Nazar et al. [9]. It should be pointed that all the papers above studied the boundary condition of two cases, i.e. CWT and CHF.

It is worth mentioning that the Newtonian heating conditions in which the heat transfer from the surface is proportional to the local surface temperature have been used by Merkin [10] and Lesnic et al. [11-13] where free convection boundary layer flow along vertical and
horizontal surfaces in porous medium were studied. Salleh et al. [14-17] studied the free and mixed convection boundary layer flows on a sphere with Newtonian heating in viscous and micropolar fluids. On the other hand, Aziz [18] used the convective boundary conditions recently and obtained the similarity solution for laminar thermal boundary layer over a flat plate by applying convective boundary conditions, in which heat is supplied through a bounding surface of finite thickness and finite heat capacity. Further, the similarity solutions for flow and heat transfer over a permeable surface and the radiation effects on thermal boundary layer flow over a moving plate with convective boundary conditions have been studied by Ishak [19] and Ishak et al. [20]. Merkin and Pop [21] studied the forced convection flow of a uniform stream over a flat surface with a convective surface boundary condition. Yao et al. [22] presented the heat transfer of a viscous fluid flow over a stretching/shrinking sheet with a convective boundary condition. Recently, the numerical solution for stagnation point flow over a stretching surface with convective boundary conditions using the shooting method, has been studied by Mohamed et al. [23].

\[ \frac{\partial}{\partial \xi} (\varphi \eta) + \frac{\partial}{\partial \eta} (\varphi \eta) = 0, \tag{1} \]
\[ \rho \left( \eta \frac{\partial \varphi}{\partial \xi} + \varphi \frac{\partial \varphi}{\partial \eta} \right) = (\mu + \kappa) \frac{\partial^2 \varphi}{\partial \eta^2} + \rho \beta (\varphi - T_a) \sin \left( \frac{\pi}{a} \right) + \kappa \frac{\partial \theta}{\partial \eta}, \tag{2} \]
\[ \rho \left( \eta \frac{\partial \theta}{\partial \xi} + \varphi \frac{\partial \theta}{\partial \eta} \right) = -\kappa \left( 2 \theta + \frac{\partial \theta}{\partial \eta} \right) + \Phi \frac{\partial^2 \theta}{\partial \eta^2}, \tag{3} \]
\[ \frac{\partial \tau}{\partial \xi} + \frac{\partial \tau}{\partial \eta} = \nu \frac{\partial^2 \tau}{\partial \eta^2}, \tag{4} \]

subject to the boundary conditions of equations (1) to (4) (see Salleh et al. [16]; Aziz [18])

\[ \tau = 0, \quad -k \frac{\partial \theta}{\partial \eta} = h (\varphi - T) \eta = -n \frac{\partial \theta}{\partial \eta} \text{ at } \eta = 0 \]
\[ \tau \rightarrow 0, \quad \varphi \rightarrow T_a, \quad \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty. \tag{5} \]

Therefore, based on the above-mentioned studies, the aim of the present paper is to study the free convection boundary layer flow on a solid sphere in a micropolar fluid with convective boundary conditions. The governing boundary layer equations are first transformed into a system of non-dimensional equations via non-dimensional variables and then, into non-similar equations before they are solved numerically by the Keller box method as described in the book by Cebeci and Bradshaw [24]. To the best of our knowledge, this present problem (for the case of convective boundary condition) has not been presented before, so the reported results are new.

**Basic Equations:** A heated sphere of radius \( a \), which is immersed in a viscous and incompressible micropolar fluid of ambient temperature \( T_a \) which is subjected to convective boundary conditions (CBC) is considered as shown in Figure 1.

The gravity vector, \( g \) acts downward in the opposite direction, whereas the coordinates \( \varphi \) and \( \eta \) are chosen such that \( \varphi \) measures the distance along the surface of the sphere from the lower stagnation point and \( \eta \) measures the distance normal to the surface of the sphere.

We assume that the equations are subjected to convective boundary conditions (CBC) of the form proposed by Aziz [18]. Under the Boussinesq and boundary layer approximations, the basic dimensional equations of the flow are (see Eringen [1] and Salleh et al. [16]).
where \( \vec{u} \) and \( \vec{v} \) are the velocity components along the \( x \) and \( y \) directions, respectively, \( \vec{\Omega} \) is the angular velocity of micropolar fluid, \( \kappa \) is the vortex viscosity, \( T \) is the local temperature, \( T_h \) is the temperature of the hot fluid, \( g \) is the gravity acceleration, \( \beta \) is the thermal expansion coefficient, \( v = \mu \rho \) is the kinematic viscosity, \( \mu \) is the dynamic viscosity, \( \rho \) is the density, \( f \) is the microinertia density, \( \Pr \) is the Prandtl number and \( h \) is the heat transfer coefficient for the convective boundary conditions. It is worth mentioning that in boundary conditions (5), \( n \) is constant and \( 0 \leq n \leq 1 \). The value \( n = 0 \), which leads to \( \vec{\Omega} = 0 \) at the wall, represents concentrated particle flows in which the particle density is sufficiently great that microelements close to the wall are unable to rotate or is called “strong” concentration of microelements [25, 26]. The case corresponding to \( n = 1 \) results in the vanishing of antisymmetric part of the stress tensor and represents “weak” concentration of microelements [26]. In this case, the particle rotation is equal to fluid vorticity at the boundary for fine particle suspension. When \( n = 1 \), we have flows which are representative of turbulent boundary layer [27]. The case of \( n = 1/2 \), is considered in this paper.

Let \( \tau(x) \) be the radial distance from the symmetrical axis to the surface of the sphere and \( \varphi \) is the spin gradient viscosity which are represented by

\[
\tau(x) = a \sin \left( \frac{x}{a} \right), \quad \varphi = (\mu + (\kappa/2)) \gamma
\]

We now introduce the following non-dimensional variables (Salleh et al. [16]; Aziz [18]):

\[
x = \frac{x}{a}, \quad y = Gr^{1/4} \left( \frac{y}{a} \right), \quad r = Gr^{1/4}
\]

\[
u = \left( \frac{a}{V} \right) Gr^{-1/2}, \quad \nu = \left( \frac{a}{V} \right) Gr^{-1/4}, \quad H = \left( \frac{a^2}{V} \right) Gr^{-3/4}, \quad \theta = T - T_\infty
\]

\[
\frac{T_f - T_\infty}{T_f} \]

where \( Gr = \beta \rho (T_f - T_\infty) \frac{a^3}{v^2} \) is the Grashof number for convective boundary conditions.

Substituting variables (7) into (1) to (4) leads to the following non-dimensional equations

\[
\frac{\partial}{\partial x} (ru) + \frac{\partial}{\partial y} (rv) = 0,
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = (1 + K) \frac{\partial^2 u}{\partial y^2} + \theta \sin x + K \frac{\partial H}{\partial y},
\]

\[
u \frac{\partial H}{\partial x} + \nu \frac{\partial H}{\partial y} = -K \left( 2H + \frac{\partial u}{\partial y} \right) + \left( 1 + \frac{K}{2} \right) \frac{\partial^2 H}{\partial y^2},
\]

\[
u \frac{\partial \theta}{\partial x} + \nu \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2},
\]

where \( K \) is the material or micropolar parameter defined as \( K = \frac{\kappa}{\mu} \). The boundary conditions (5) become

\[
u = v = 0, \quad \frac{\partial \theta}{\partial y} = -\gamma (1 - \Theta) \quad H = -\frac{1}{2} \frac{\partial u}{\partial y} \text{ at } y = 0
\]

\[
u \to 0, \quad \theta \to 0, \quad H \to 0 \text{ as } y \to \infty
\]
where and \( \gamma = ab f Gr^{-1/4} \) are the conjugate parameter for convective boundary condition. It is noticed that if we write the boundary condition \( \theta = 1 + \frac{\partial \theta_0}{\partial y} \) at \( y = 0 \) and when \( \gamma \to \infty \) we have \( \theta = 1 \), this mean the convective boundary conditions (CBC) becomes to constant wall temperature (CWT) at \( y \to \infty \) this case studied by Nazar et al [5].

To solve equations (8) to (11) subjected to the boundary conditions (12), we assume the following variables:

\[
\psi = x r f(x, y), \quad \theta = \Theta(x, y), \quad H = x h(x, y),
\]

where \( \psi \) is the stream function is defined as

\[
u = \frac{1}{r} \frac{\partial \psi}{\partial y}, \quad \frac{1}{r} \frac{\partial \psi}{\partial x}
\]

which satisfies the continuity equation (8). Substituting (14) into equations (9) to (11) and after some algebraic calculation, we get the following transformed equations

\[
(1 + K) \frac{\partial^2 f}{\partial y^2} + \left(1 + x \cot x\right) f \frac{\partial^2 f}{\partial y^2} \left(\frac{\partial f}{\partial y}\right)^2 + \frac{\sin x}{x} \frac{\partial f}{\partial y} \frac{\partial f}{\partial x} + \frac{x \frac{\partial f}{\partial y} \frac{\partial f}{\partial x} + \frac{\partial f}{\partial x} \frac{\partial f}{\partial y}}{x} \left(\frac{\partial f}{\partial y} \frac{\partial f}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial f}{\partial y}\right),
\]

\[
1 \frac{\partial^2 \theta}{\partial y^2} + (1 + x \cot x) f \frac{\partial^2 \theta}{\partial y^2} = x \left(\frac{\partial f}{\partial y} \frac{\partial f}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial f}{\partial y}\right) \left(\frac{\partial f}{\partial y} \frac{\partial f}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial f}{\partial y}\right),
\]

subject to the boundary conditions

\[
f = \frac{\partial f}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = -\gamma(1 - \theta), \quad h = \frac{1}{2} \frac{\partial^2 f}{\partial y^2} \text{ at } y = 0
\]

\[
\frac{\partial f}{\partial y} \to 0, \quad \theta \to 0, \quad h \to 0, \quad \text{as } y \to \infty
\]

At the lower stagnation point of the sphere, \( x = 0 \), equations (15) to (17) reduce to the following nonlinear ordinary differential equations:

\[
(1 + K) f'' + 2 f f'' - f'^2 + \Theta + Kh'' = 0
\]

\[
\left(1 + \frac{K}{2}\right) h'' + 2 f' h' - f'' h - K(2h + f'') = 0
\]

\[
\frac{1}{Pr} \theta'' + 2 f \theta' = 0
\]

and the boundary conditions (18) become

\[
f(0) = f'(0) = 0, \quad \theta'(0) = -\gamma(1 - \Theta(0)), \quad h(0) = -\frac{1}{2} f''(0),
\]

along with \( \theta'(0) = -\gamma(1 + \Theta(0)) \) (NH).

where \( \gamma \) is given by

\[
C_f = \left(1 + \frac{K}{2}\right) \frac{\partial^2 f}{\partial y^2} (x, 0), \quad Q_w(x) = \gamma(1 - \Theta(x, 0))
\]

where \( C_f = \tau_w/(\rho U_\infty^2) \) is the skin friction coefficient and \( \tau_w = [(\mu + \kappa)(\partial \sigma/\partial y) + \kappa H]_{y=0} \) is the wall shear stress. At the lower stagnation point of the sphere, \( x = 0 \), the skin friction coefficient and the heat transfer coefficient are measured by \( \frac{\partial^2 f}{\partial y^2} \) and \( \frac{\partial \theta}{\partial y} \), respectively.

RESULTS AND DISCUSSION

The nonlinear systems of partial differential equations (15) to (17) subject to the boundary conditions

1945
(18) are solved numerically using the Keller-box method for the case of convective boundary conditions (CBC) with four parameters considered, namely the material parameter $K$, the Prandtl number $Pr$, the conjugate parameter $\gamma$ and the coordinate running along the surface of the sphere, $x$.

The numerical solution starts at the lower stagnation point of the sphere, $x = 0$ and proceeds around the sphere up to the point of $x = 120^\circ$. Values of $K$ considered are $K = 0$ (Newtonian fluid), 1, 2, 3 (micropolar fluid) and values of $Pr$ considered are $Pr = 0$, 1, 2, 3 when $Pr = 0$ to the point of $x = 120^\circ$. It is worth mentioning that small values of $Pr(<1)$ physically correspond to liquid metals, which have high thermal conductivity but low viscosity, while large values of $Pr(>>1)$ correspond to high-viscosity oils. It is worth pointing out that specifically, Prandtl numbers $Pr = 0$, 1, 2, 3 correspond to air, electrolyte solution such as salt water and water, respectively.

The values of the wall temperature $\Theta(0,y)$ for the values of $K = 0$, 1, 2, 3 when $Pr = 0$, 1, 2, 3 and $\gamma = 1$ in the case of Newtonian heating (NH) are shown in Table 1. In order to verify the accuracy of the present method, the present results are compared with those reported by Salleh et al. [16]. It is found that the agreement between the previously published results with the present ones is very good.

Table 2 shows the values of the wall temperature, $\Theta(0,y)$ and the skin friction coefficient, $\frac{\partial^2 f}{\partial y^2}(0,y)$ for various values of $K$ when $Pr = 0.7$, $1$ and $\gamma = 1$ (NH) and the coordinate running along the surface of the sphere, $x$.

Table 3 to 5 show the values of the wall temperature $\Theta(0,y)$, the heat transfer coefficient $\frac{\partial \Theta}{\partial y}(0,y)$ and the skin friction coefficient $\frac{\partial^2 f}{\partial y^2}(0,y)$ for various values of $K$ when $Pr = 0.7$, 1, 2 and $\gamma = 1$. It is found that for fixed $Pr$, it is found that for fixed $\gamma$, as $K$ increases, the values of $\Theta(0,y)$ increase but the values of $\frac{\partial^2 f}{\partial y^2}(0,y)$ decrease. Also, it is found that for fixed $K$, as $\gamma$ increases, both $\Theta(0,y)$ and $\frac{\partial^2 f}{\partial y^2}(0,y)$ increase.

Table 6 to 9 present the values of the local heat transfer coefficient $Q(x)$ and the local skin friction coefficient $C_f$ for various values of $x$ when $Pr = 0.7$, 1, 2, 3.
Table 3: Values of the wall temperature \( \theta(0, y) \) for various values of \( K \) when \( Pr = 0.7, 1, 7 \) and \( \gamma = 0.1 \)

<table>
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<td>Present</td>
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<tr>
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<tr>
<td>3</td>
<td>0.265189</td>
<td>0.246400</td>
<td>0.163335</td>
</tr>
</tbody>
</table>

Table 4: Values of the heat transfer coefficient \( \frac{\partial \theta}{\partial y}(0, y) \) for various values of \( K \) when \( Pr = 0.7, 1, 7 \) and \( \gamma = 0.1 \)

<table>
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</thead>
<tbody>
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Table 5: Values of the skin friction coefficient \( \frac{\partial f}{\partial y}(0, y) \) for various values of \( K \) when \( Pr = 0.7, 1, 7 \) and \( \gamma = 0.1 \)

<table>
<thead>
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<tbody>
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Table 6: Values of the local heat transfer coefficient \( Q_x(x) \) for various values of \( x \) when \( Pr = 0.7, 1, 7, K = 0 \) and \( \gamma = 0.5 \)

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Table 7: Values of the local skin friction coefficient, \( C \), for various values of \( x \) when \( Pr = 0.7, 1, 7, K = 0 \) and \( \gamma = 0.5 \)

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Table 8: Values of the local skin friction coefficient, \( C \), for various values of \( x \) when \( Pr = 0.7, 1, 7, K = 2 \) and \( \gamma = 0.5 \)

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<td>0.372073</td>
</tr>
<tr>
<td>90°</td>
<td>0.310761</td>
<td>0.313264</td>
<td>0.368721</td>
</tr>
<tr>
<td>100°</td>
<td>0.308944</td>
<td>0.310862</td>
<td>0.364647</td>
</tr>
<tr>
<td>110°</td>
<td>0.306907</td>
<td>0.308146</td>
<td>0.359724</td>
</tr>
<tr>
<td>120°</td>
<td>0.302652</td>
<td>0.304752</td>
<td>0.353513</td>
</tr>
</tbody>
</table>

Table 9: Values of the local skin friction coefficient, \( C \), for various values of \( x \) when \( Pr = 0.7, 1, 7, K = 2 \) and \( \gamma = 0.5 \)

<table>
<thead>
<tr>
<th>Pr</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
<td>Present</td>
<td>Present</td>
</tr>
<tr>
<td>0°</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>10°</td>
<td>0.063458</td>
<td>0.062299</td>
<td>0.047452</td>
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<tr>
<td>20°</td>
<td>0.126326</td>
<td>0.124072</td>
<td>0.094862</td>
</tr>
<tr>
<td>30°</td>
<td>0.188080</td>
<td>0.184866</td>
<td>0.142196</td>
</tr>
<tr>
<td>40°</td>
<td>0.248124</td>
<td>0.244147</td>
<td>0.189483</td>
</tr>
<tr>
<td>50°</td>
<td>0.305980</td>
<td>0.301498</td>
<td>0.236618</td>
</tr>
<tr>
<td>60°</td>
<td>0.359604</td>
<td>0.354899</td>
<td>0.282270</td>
</tr>
<tr>
<td>70°</td>
<td>0.411721</td>
<td>0.407061</td>
<td>0.329185</td>
</tr>
<tr>
<td>80°</td>
<td>0.460355</td>
<td>0.456068</td>
<td>0.375692</td>
</tr>
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<td>0.505152</td>
<td>0.501525</td>
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<tr>
<td>100°</td>
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<td>0.467708</td>
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<tr>
<td>110°</td>
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<td>0.579857</td>
<td>0.511386</td>
</tr>
<tr>
<td>120°</td>
<td>0.627322</td>
<td>0.614398</td>
<td>0.556060</td>
</tr>
</tbody>
</table>
Fig. 2: Variation of wall temperature, $\theta(x, 0)$ with Prandtl number Pr when $K = 2$ and $\gamma = 0.05, 0.1, 0.2$.

Fig. 3: Variation of the skin friction coefficient, $\frac{\partial^2 f}{\partial y^2}(x, 0)$ with Prandtl number Pr when $K = 2$ and $\gamma = 0.05, 0.1, 0.2$.

Fig. 4: Variation of wall temperature, $\theta(x, 0)$ with conjugate parameter $\gamma$ when Pr = 0.7, 1, 7 and $K = 2$.

Fig. 5: Temperature profiles $\theta(0, x)$ for some values of $\gamma = 0.05, 0.1, 0.2$ when Pr= 0.7 and $K = 2$.

Fig. 6: Velocity profiles $\frac{\partial f}{\partial y}(0, x)$ for some values of $\gamma = 0.05, 0.1, 0.2$ when Pr= 0.7 and $K = 2$.

$K = 0, 2$ and $\gamma = 0.5$, respectively. It is found that, for fixed $K$, as Pr increases, the $Q_\gamma(x)$ increase and $C_f$ decrease. From these tables, for a fixed Pr, as $x$ increases, i.e. from the lower stagnation point of the sphere, $x = 0$ and proceeds around the sphere up to the point $x = 120^\circ$, the values of $Q_\gamma(x)$ decrease and $C_f$ increase. On the other hand, the values of $C_f$ are higher for micropolar fluid ($K = 2$) than those for Newtonian fluid ($K = 0$).

The graphs of $\theta(x, 0)$ and $\frac{\partial^2 f}{\partial y^2}(x, 0)$ for some values of the Prandtl number Pr when $\gamma = 0.05, 0.1$ and 0.2 are plotted in Figures 2 and 3, respectively. It is found that, as Pr increases, both $\theta(x, 0)$ and $\frac{\partial^2 f}{\partial y^2}(x, 0)$ decrease. For small values of Pr($<<1$), the value of $\theta(x, 0)$ and $\frac{\partial^2 f}{\partial y^2}(x, 0)$ is higher than for large values of Pr($>>1$) and it is seen that the surface temperature is very sensitive to the Prandtl number variations.

Figure 4 illustrates the variation of the wall temperature $\theta(x, 0)$ with conjugate parameter $\gamma$ when Pr = 0.7, 1, 7 and $K = 2$. Furthermore, in order to get a physically acceptable solution, $\gamma$ must be less than or equal to some critical value, say $\gamma_c$, i.e. $\gamma < \gamma_c$, depending on Pr. It can be seen from this figure that $\theta(x, 0)$ becomes larger as $\gamma$ approaches the critical value of $\gamma_c = 0.5103$ when Pr = 0.7, $\gamma_c = 0.5592$ when Pr = 1 and $\gamma_c = 1.019$ when Pr = 7.

Figures 5 to 7 illustrate the temperature $\theta(x, 0)$, velocity $\frac{\partial f}{\partial y}(0, y)$ and angular velocity $h(0, y)$ profiles of the sphere for some values of $\gamma$, namely $\gamma = 0.05, 0.1, 0.2$ when Pr= 0.7 and $K = 2$, respectively. It is found that when $K$ is fixed, as $\gamma$ increases, the temperature, velocity and angular velocity profiles increase.
Fig. 7: Angular velocity profiles $h(0, y)$ for some values of $\gamma = 0.05, 0.1, 0.2$ when $Pr = 0.7$ and $K = 2$

Fig. 8: Temperature profiles, $\theta(0, x)$ when $K = 0, 1, 2, 3$, $Pr = 1$ and $\gamma = 0.1$

Fig. 9: Velocity profiles, $\frac{\partial f}{\partial y}(0, y)$ when $K = 0, 1, 2, 3$, $Pr = 1$ and $\gamma = 0.1$

Fig. 10: Angular velocity profiles, $h(0, y)$ when $K = 0, 1, 2, 3$, $Pr = 1$ and $\gamma = 0.1$

Fig. 11: Temperature profiles, $\theta(x, y)$ at $x = 0^\circ, 60^\circ, 90^\circ$ when $Pr = 0.7, 7, K = 2$ and $\gamma = 0.1$

Fig. 12: Velocity profiles, $\frac{\partial f}{\partial y}(x, y)$ at $x = 0^\circ, 60^\circ, 90^\circ$ when $Pr = 0.7, 7, K = 2$ and $\gamma = 0.1$

Fig. 13: Angular velocity profiles, $h(x, y)$ at $x = 0^\circ, 60^\circ, 90^\circ$ when $Pr = 0.7, 7, K = 2$ and $\gamma = 0.1$

Figures 8 and 9 display the temperature $\theta(0, y)$ and velocity $\frac{\partial f}{\partial y}(0, y)$ profiles for some values of $K$, namely $K = 0, 1, 2, 3$ when $Pr = 1$ and $\gamma = 0.1$, respectively. It is found that when $Pr$ is fixed, as $K$ increases, both the temperature and velocity profiles increase. Angular velocity $h(0, y)$ profiles, when $K = 0, 1, 2, 3$, $Pr = 1$ and $\gamma = 0.1$ are plotted in Figure 10. These figures show that the angular velocity is completely negative for $K = 0$, while it may be positive for $K > 0$.

Figures 11 to 13 display the temperature, velocity and angular velocity profiles at $x = 0^\circ, 60^\circ, 90^\circ$ when $Pr = 0.7, 7, K = 2$ and $\gamma = 0.1$. From Figure 11, it is found that as $Pr$ and $x$ increase, the temperature profiles decrease and the
thermal boundary layer thickness also decrease. This is because for small values of the Prandtl number \( \text{Pr} \ll 1 \), the fluid is highly conductive. Physically, if \( \text{Pr} \) increases, the thermal diffusivity decreases and this phenomenon leads to the decreasing manner of the energy transfer ability that reduces the thermal boundary layer. Furthermore, in these figures shown that for fixed \( K \), as \( \text{Pr} \) increases, the velocity profiles decrease and the angular velocity profiles decrease. In the same figures it has been found that when \( \text{Pr} \) is fixed and \( x \) increases, the temperature, velocity and angular velocity profiles increase.

**CONCLUSIONS**

In this paper, we have numerically studied the problem of free convection boundary layer flow on a solid sphere in a micropolar fluid with convective boundary conditions (CBC). We are interested to see how the material parameter \( K \), the Prandtl number \( \text{Pr} \) and the conjugate parameter \( \gamma \) affect the flow and heat transfer characteristics. We can conclude that (for the case of (CBC):

- When \( \text{Pr} \) and \( \gamma \) are fixed, as \( K \) increases, the value of the wall temperature \( \theta(0, y) \) increases but the skin friction coefficient, \( \frac{\partial^2 f}{\partial y^2}(0, y) \) decreases. On other hand, when \( K \) and \( \gamma \) are fixed, as \( \text{Pr} \) increases, the heat transfer coefficient, \( \frac{\partial \theta}{\partial y}(0, y) \), the skin friction coefficient, \( \frac{\partial^2 f}{\partial y^2}(0, y) \) and the angular velocity profiles, \( h(0, y) \) decrease but the heat transfer coefficient decreases
- When \( K \) is fixed, an increase in \( \gamma \) leads to an increase of the wall temperature \( \theta(0, y) \) skin friction coefficient \( \frac{\partial^2 f}{\partial y^2}(0, y) \), temperature profiles \( \theta(0, y) \) velocity profiles \( \frac{\partial f}{\partial y}(0, y) \) and angular velocity profiles \( h(0, y) \).
- When \( \text{Pr} \) and \( \gamma \) are fixed, the values of \( C_f \) are higher for micropolar fluids \( (K \neq 0) \) than those for a Newtonian fluid \( (K = 0) \);
- When \( \text{Pr} \) is fixed and \( x \) increases, the temperature profile, velocity profile and angular velocity profiles increase;
- When \( K \) and \( \gamma \) are fixed, as \( \text{Pr} \) increases, the values of the local heat transfer coefficient increase and the local skin friction coefficient decrease;
- To get a physically acceptable solution, \( \gamma \) must be less than \( \gamma \) depending on the \( \text{Pr} \).

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**REFERENCES**